



**Collusion or Sniping
in simultaneous ascending Auctions**

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A prisoner's dilemma

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Abstract:

In simultaneous ascending price auctions with heterogeneous goods Brusco and Lopomo (2002) derive collusive equilibria where bidders divide objects among themselves, while keeping the prices low. Considering a simultaneous ascending price auction with a fixed deadline, i.e. the Hard Close auction format, a prisoner's dilemma situation results and collusive equilibria do no longer exist, even for only two bidders. Hence, we introduce a further reason for sniping behavior in Hard Close auctions.

Keywords: collusion, sniping, multi unit auctions, prisoner's dilemma

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1 Introduction

Brusco and Lopomo (2002) (BL) consider collusive behavior in a "simultaneous ascending bid auction", i.e. a multi-object version of the English auction (see also Engelbrecht-Wiggans and Kahn 2005). They show that in a symmetric perfect Bayesian equilibrium the bidders split the markets through signaling their favorite market if private values are assumed, even if communication is not allowed¹. Therefore, collusion in simultaneous English auctions holds due to a punishment mechanism. Defection from an "collusive agreement" leads to a "bidding war", resulting in the standard equilibrium properties in separated English auctions. However, the low-price collusion equilibrium outcome is from the bidders' point of view superior to a high-price equilibrium with a "separated English auctions" strategy, i.e. bidding in all auctions up to the willingness to pay. Of course the seller favors the high-price equilibrium.

Kwasnica and Sherstyuk (2007) test this theoretic consideration experimentally and conclude that "collusion occurs in experimental auctions for multiple objects as long as the number of bidders is small" and that "outcomes of these auctions, when classified as collusive, often match the BL signaling model quite well".

This paper extends the literature by investigating whether collusion behavior holds when the simultaneous English auction is limited in time, i.e. when a "simultaneous Hard Close auction" is considered. In this auction format the punishment mechanism disappears and the fixed deadline turns the bidders' interaction into a prisoner's dilemma situation. Hence, collusion is dominated by sniping, i.e. submitting a bid in the last possible point of time. The Hard Close auction is one of the most frequently used auction formats in electronic markets (e.g. eBay) and has been studied both with single goods (see e.g. Roth and Ockenfels 2002) or multiple homogeneous goods (see e.g. Peter and Severinov 2006). Sniping

¹BL consider heterogeneous goods with and without complementarities. We just focus on the no complementarities cases.

is a widely accepted stylized fact² and many reasons for this behavior are found: sniping prevents incremental (Wintr 2004) and shill bidding (Engelberg and Williams 2005), it prevents information revelation in early stages (Rothkopf, Teisberg and Kahn 1990) or late bidding is due to uncertainty over one's own private valuation (Rasmusen 2003, Hossain 2006). We find a further possible explanation for sniping: late defection from seemingly collusive behavior, i.e. to appear to collude early in the auction and defect in the very last moment.

Section 1 introduces bidding behavior in the separated markets both in the Soft Close auction and the Hard Close auction. Section 2 sketches the the findings of BL and provide an analogue approach with hard close auctions. Afterwards we conclude.

2 Separated Soft and Hard Close Auctions

We consider bidding behavior in two different auction formats: the Soft Close auction, the "online" version of the English auction, and the Hard Close auction. According to the eBay pricing rule used in Ockenfels and Roth (2006) we model the auctions as a dynamic second-price format (Füllbrunn and Sadrieh 2006) in which at any time t the current price is equal to the second highest bid submitted in the previous stage³. The current holder(s) at time t is (are) the bidder(s) who has (have) submitted the highest bid. In each stage all bidders are informed on the current price and on their status as current holders. They are not informed on the bids of the other bidders.

When the auction ends, the current holder receives the item and pays the current price. Ties are broken by assigning the item with equal probabilities to one of the current holders.

²Evidence on sniping found Ockenfels and Roth (2006), Bajari and Hortaçsu (2003), Anwar et al. (2006) or Hayne et al. (2003).

³In contrast to their paper we do not use bidding increments. Therefore an equilibrium in weakly dominated strategies holds. Taking bid increments in consideration makes the theory unnecessarily complicated in our case.

The payoff of the buyer - the bidder who receives the item - depends on her valuation of the item. We consider a private value environment and, thus, the payoff of the buyer equals the difference between her induced private value and the price. All other bidders have a payoff of zero.

Since the auction is symmetric the distribution of the private values is identical for all. The private values are identically and independently drawn from a common uniform distribution that is common knowledge. The bidders face no liquidity constraints.

The Hard Close auction consists of T stages. In the last stage, bidders may submit a last bid and afterwards the auction ends⁴. The Soft Close auction consists of at least T stages. If no bid is submitted in T , the auction ends. As long as bids are submitted in stages $T, T + 1, \dots$ a further stage occurs. In this case the auction ends after a stage with no bids. The terminal stage equals a sealed bid second-price auction. Following the Vickrey argument (Vickrey 1961) the weakly dominant strategy is to submit a bid equal to the private value. Bids in previous stages are arbitrary as long as the value is not exceeded. Therefore, the bidder with the highest valuation receives the good and pays a price equal to the second highest valuation.

In the Hard Close auction *sniping*, i.e. submitting non informative low bids in previous stages and bid according to the weakly dominant strategy in the last stage, remains as a possible strategy and there is experimental evidence for this behavior (e.g. Füllbrunn and Sadrieh 2006). However, any other strategy yields the same outcome as long as all bidders place equilibrium bids at some stage and never bid higher⁵. In the Soft Close auction sniping is no longer a reasonable strategy due to the extension rule. As in the continuous English auction format the bidder may engage in bidding wars until their value is reached or they may bid the equilibrium bid in the first or in any stage. However, with private

⁴We do not consider bid transfer problems as in Ariely et al. (2005)).

⁵Actually, any other strategy yields the same outcome as long the second highest bid equals the second highest valuation and the bidder with the highest value receives the award.

values both auction formats lead to the same results and the revenue equivalence theorem holds.

3 Simultaneous Auctions

Two risk neutral bidders ($i = x, y$) compete for two heterogeneous goods ($j = 1, 2$). The bidders' valuation for each good, v_{ij} are identically and independently drawn from a uniform distribution with limits 0 and 1. Due to a continuous distribution we assume different valuations and thus, we do not pay attention to possible ties in valuations. Bidders cannot observe the other bidders' values. The payoff function for bidder x depending on final bids β_{x1}, β_{x2} and β_{y1}, β_{y2} is (respectively for bidder y)

$$\pi_x = \begin{cases} v_{x1} - p_1 + v_{x2} - p_2, & \text{if } \beta_{x1} > \beta_{y1} \text{ and } \beta_{x2} > \beta_{y2}; \\ v_{x1} - p_1, & \text{if } \beta_{x1} > \beta_{y1} \text{ and } \beta_{x2} < \beta_{y2}; \\ v_{x2} - p_2, & \text{if } \beta_{x1} < \beta_{y1} \text{ and } \beta_{x2} > \beta_{y2}; \\ 0, & \text{if } \beta_{x1} < \beta_{y1} \text{ and } \beta_{x2} < \beta_{y2}, \end{cases}$$

where p_j is the price in the respective market. All parameters aside from the bidders private values are common knowledge.

The bidders can submit bids in two simultaneous but separated dynamic second-price auctions. In the Hard Close auctions both markets simultaneously begin with stage 1 and end with stage T . In the Soft Close auctions both markets simultaneously begin with stage 1 and simultaneously end either after stage T , if no bid is submitted in T in either market, or in a stage $T + s$, if in any stage $T, T + 1, \dots, T + s - 1$ bidders submit bids and no bids in $T + s$ in either market.

We assume $T \geq 3$ in order to give coordination possibilities. The lowest bid is at least $b_0 = 0$ and the bidders have the possibility *not to bid*, i.e. bidding ω . A bid in stage t by

bidder i in market j is b_{ij}^t .

3.1 The simultaneous Soft Close Auction

According to BL's Proposition 0 (p. 8) the separated Soft Close auction (SSC) strategy, i.e. bidding in all auctions up to the valuation, forms a perfect Bayesian equilibrium. However, in the second-price environment this means submitting a bid equal to the private value in an arbitrary stage as long as the auction proceeds or overbid any price as long as positive payoffs are possible, i.e. the value exceeds the price⁶.

Proposition 1 (BL, p. 9) established the existence of a symmetric perfect Bayesian equilibrium which dominates the perfect Bayesian equilibrium in SSC strategies in terms of bidders' surplus. A simple collusion (CSC) strategy that forms this equilibrium⁷ is:

- if $v_{i1} > v_{i2}$ submit $b_{i1}^1 = b_0$ and $b_{i2}^1 = \omega$;
- if $v_{i1} < v_{i2}$ submit $b_{i1}^1 = \omega$ and $b_{i2}^1 = b_0$;
- if, in stage 2, $b_{xj}^1 \neq b_{yj}^1$ the bidders divide the markets among each other and take no further action in the following stages;
- if, in stage 2, $b_{xj}^1 = b_{yj}^1$ or if in a following stage someone defects from the bidding instructions given above, then all types revert to the SSC strategy.

Hence, the bidders only submit one lowest bid in the market with the higher valuation. If, in stage 2, there is no competitor in the market they take no further action in the following stages and the auction ends after stage T . Therefore the bidders profit equals their highest valuations. In all other cases the perfect Bayesian equilibrium in SSC strategies evolve.

⁶Assuming some time costs bidders finish the bidding process until stage T is reached.

⁷BL assume that $E[x] \geq \frac{1}{2}$ holds. This condition "can be interpreted as requiring that each bidder has to expect a sufficiently high degree of competition from her opponent, should the SEA [=SSC] strategies be triggered. Otherwise there is no point in colluding, since both objects can be obtained at a low expected price." A uniform distribution satisfies this condition.

Without loss of generality we assume $v_{x1} > v_{x2}$. Following the *borderline case* from Engelbrecht-Wiggans and Kahn (2005) the expected payoff for bidder x , if both bid according to the CSC strategy, is

$$E[\pi_x(CSC)] = \frac{v_{x1}}{2} + \frac{1}{2} \left[\int_0^{v_{x1}} (v_{x1} - v_{y1}) 2v_{y1} dv_{y1} + \int_0^{v_{x2}} (v_{x2} - v_{y2}) 2(1 - v_{y2}) dv_{y2} \right], \quad (1)$$

where the first summand equals the expected payoff if $v_{y1} < v_{y2}$ and the second summand equals the expected payoff if $v_{y1} > v_{y2}$. If both bid according to the SSC strategy the expected payoff for bidder x is

$$E[\pi_x(SSC)] = \int_0^{v_{x1}} (v_{x1} - v_{y1}) dv_{y1} + \int_0^{v_{x2}} (v_{x2} - v_{y2}) dv_{y2}. \quad (2)$$

Hence, the expected advantage from playing the CSC strategy for bidder x is

$$E[\pi_x(CSC)] - E[\pi_x(SSC)] = \frac{3v_{x1} + v_{x1}^3 - v_{x2}^3 - 3v_{x1}^2}{6}.$$

This advantage exceeds zero for $v_{x1} > v_{x2}$. Hence, the equilibrium in CSC strategies yields always a higher expected payoff than the equilibrium in SSC strategies.

Collusive behavior leaves lower revenues to the seller⁸. The expected revenue in the perfect Bayesian equilibrium in SSC strategies is

$$E[\Pi(SSC)] = 2 \int_0^1 2v(1 - v) dv = \frac{2}{3}.$$

The revenue in each market equals the lowest value. Therefore, the expected revenue is

⁸We assume a seller valuations of zero.

two times the last order statistic of two random draws within the interval.

The revenue in the symmetric perfect Bayesian equilibrium in CSC strategies is (i) zero if the highest valuations of the bidders are in separated markets, i.e. $v_{x1} > v_{x2}$ and $v_{y1} < v_{y2}$ or $v_{x1} < v_{x2}$ and $v_{y1} > v_{y2}$ and (ii) positive otherwise. The probability of case (i) is 50%. Hence, the expected revenue is 50%*(the expected price in the low-values-market + the expected price in the high-values-market). The expected price in the low-values-market, i.e. the object for which both bidders have the lower valuation, equals the last order statistic of 2 random draws. The price in the high-value-market, i.e. the object for which both bidders have the higher valuation, is either the second or the third highest order statistic of 4 random draws with equal probability. Hence, the expected revenue in the symmetric perfect Bayesian equilibrium in CSC strategies yields

$$\begin{aligned} E[\Pi(CSC)] &= \frac{1}{2} \left[\int_0^1 4v(1-v)^3 dv + \frac{1}{2} \left(\int_0^1 v(12(1-v)v^2 + 12(1-v^2)v) dv \right) \right] \\ &= \frac{7}{20}. \end{aligned}$$

Thus, the seller loses 47.5% if the bidder engage in collusive behavior.

Furthermore collusive behavior can decrease efficiency. Assume for example $v_{x1} > v_{x2}$, $v_{y1} < v_{y2}$, and $v_{x2} > v_{y2}$: CSC strategies leaves object 2 to bidder y although bidder x has a higher valuation for this object.

In the simultaneous Soft Close auction with two markets and two bidders collusive behavior is possible and forms a symmetric perfect Bayesian equilibrium in CSC strategies which dominates the perfect Bayesian equilibrium in SSC strategies in terms of bidders' surplus.

3.2 Simultaneous Hard Close Auction

In this part we show that collusion behavior, especially a kind of the CSC strategy, cannot be an equilibrium strategy⁹. Assume a good will "signaling" (CHC) strategy in the simultaneous Hard Close auction:

- if $v_{i1} > v_{i2}$ submit $b_{i1}^1 = b_0$ and $b_{i2}^1 = \omega$;
- if $v_{i1} < v_{i2}$ submit $b_{i1}^1 = \omega$ and $b_{i2}^1 = b_0$;
- if, in stage 2, $b_{xj}^1 \neq b_{yj}^1$ the bidders divide the markets among each other and take no further action in the following stages;
- if, in stage 2, $b_{xj}^1 = b_{yj}^1$ or if in a following stage someone defects from the bidding instructions given above, bid $b_{i1} = v_{i1}$ and $b_{i2} = v_{i2}$ at the latest in stage T .

Contrary to the CHC strategy assume a sniping (SHC) strategy in the simultaneous Hard Close auction:

- if $v_{i1} > v_{i2}$ submit $b_{i1}^1 = b_0$ and $b_{i2}^1 = \omega$;
- if $v_{i1} < v_{i2}$ submit $b_{i1}^1 = \omega$ and $b_{i2}^1 = b_0$;
- if, in stage 2, $b_{xj}^1 \neq b_{yj}^1$ take no further action in the following stages until stage T and bid $b_{i1} = v_{i1}$ and $b_{i2} = v_{i2}$ in stage T ;
- otherwise bid $b_{i1} = v_{i1}$ and $b_{i2} = v_{i2}$ at the latest in stage T .

In the following we assume $v_{x1} > v_{x2}$ and $v_{y1} < v_{y2}$ because the other cases lead to corresponding results of the strategies, CHC and SHC.

Therefore, this auction can be considered as a prisoner's dilemma with strategies $C =$ Cooperation, i.e. submit a bid in the market with the highest value in the first stage and

⁹We do not consider side payments.

take no action further in the following stages if the competitor signals cooperation as well (CHC strategy), and $D =$ Defection, i.e. submit a bid in the market with the highest value and take no action further in the following stages if the competitor signals cooperation as well, but submit bids that equal the valuations in both markets in the last stage (SHC strategy).

Due to symmetry we consider bidder x . The (expected) payoffs for bidder x depending on strategy combinations $(S_x; S_y)$ where $S_i = (C, D)$ are

$$E[\pi_x(\text{CC})] = v_{x1},$$

$$E[\pi_x(\text{CD})] = 0,$$

$$E[\pi_x(\text{DC})] = v_{x1} + v_{x2},$$

$$E[\pi_x(\text{DD})] = \int_0^{v_{x1}} (v_{x1} - v_{y1})2(1 - v_{y1})dv_{y1} + \int_0^{v_{x2}} (v_{x2} - v_{y2})2v_{y2}dv_{y2}.$$

If both bidder cooperate the bidders receive a payoff that equals their highest valuation. If only one bidder defects he receives a payoff that equals the sum of her valuations and the other receives nothing. If both bidders defect, the payoff depends on the valuation constellation in the separated markets. In market 1 the expected payoff for bidder x equals the expected positive difference between the valuations in that market conditional v_{x1} is highest and v_{y1} is the lowest statistic order of the valuations of bidder y . In market 2 the expected payoff for bidder x equals the expected positive difference between the valuations in that market conditional v_{x2} is highest and v_{y2} is the highest statistic order of the valuations of bidder y . The normal form of the simultaneous Hard Close auction with collusive behavior is to be found in table 1.

For $v_{x1} > v_{x2}$ the expected payoff with mutual defection exceeds the payoff with mutual cooperation. Thus, the dominant strategy is to defect due to the fact that if bidder y

		Bidder y			
		C		D	
Bidder x	C	v_{x1}	v_{y2}	0	$v_{y1} + v_{y2}$
	D	$v_{x1} + v_{x2}$	0	$\frac{1}{3}(3v_{x1}^2 - v_{x1}^3 + v_{x2}^3)$	$\frac{1}{3}(v_{y1}^3 + 3v_{y2}^2 - v_{y2}^3)$

Table 1: Prisoner’s Dilemma payoff matrix

cooperates, x is better off to defect because she gains her whole possible surplus in both auctions, and if bidder y defects, x is better off to defect as well because otherwise she gains zero payoff. Hence, in equilibrium the expected payoff in the simultaneous Hard Close auction equals the expected payoff in the perfect Bayesian equilibrium in SSC strategies in the simultaneous Soft Close auction¹⁰.

Hence, due to the lack of a punishment facility in a simultaneous Hard Close auction collusive strategies cannot sustain. At the latest in T , the bidders are better off to bid their valuations in both markets, i.e. to snipe. In this setting the seller can increase her expected revenue by over 90% by choosing the Hard Close auction in contrast to the Soft Close Auction¹¹.

4 Conclusion

We show that, unlike in a simultaneous Soft Close auction, in simultaneous Hard Close auctions collusive behavior has no chance to sustain even with only two bidders. Just before the deadline arrives the bidders can increase their payoff by means of sniping. Even with communication¹² the last stage in the Hard Close auction provides incentives to snipe.

¹⁰In Füllbrunn and Neugebauer (2006) we experimentally test whether in three bidders-three markets simultaneous Hard Close auctions the bidders engage in collusive agreements. Generally, no collusion has been found.

¹¹The expected revenue in the Soft Close auction is $7/20$ and in the Hard Close auction $2/3$. The relative increase in expected revenue is $2/3 * 20/7 - 1 = 90.48$.

¹²For experimental evidence in multiple object auctions with communication see Sherstyuk (2002) or Kwasnica (2000).

In infinitely repeated simultaneous auctions, collusion facilities may sustain (see Kwasnica and Sherstyuk 2007), but in finitely repeated auctions the backward induction argument holds and with the first auction the bidders defect. Hence, if the seller has the possibility to choose the auction format she is better off to choose the Hard Close auction.

However, with more bidders and/or more markets the possibility of collusion decreases in simultaneous Soft Close auctions due to less theoretical collusion possibilities and due to coordination failure¹³. Hence, from the sellers point of view the advantage of the Hard Close auction format decreases, especially when the Soft Close auction format results in higher revenues¹⁴

In this paper we introduce a new reason for sniping behavior in Hard Close auctions. Bidders signal to cooperate in early stages but defect in the last stage to grab the entire surplus, i.e. they snipe.

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¹³For a discussion see Kwasnica and Sherstyuk (2007).

¹⁴Some support for higher revenues in Soft Close auctions is found in Ariely et al. (2005).

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