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Abstract:

We consider a moral hazard type agency problem. Two tasks need to be performed within the agency. The principal can either delegate both tasks to the agent or perform one of the tasks himself. In the latter case the principal can choose which task to delegate but doing both personally is not feasible.

As firm value is not contractible by assumption the incentive contract offered to the agent needs to be based on a possibly non-congruent performance measure. Allowing for both of the players to be risk averse, agency costs can arise from a trade-off in allocating incentives and risk as well as from a congruity problem.

While full delegation results in a standard two task agency problem, partial delegation creates a double moral hazard problem as neither the principal can observe the agent's effort nor vice versa. We find that full delegation is more favorable the more risk is optimally allocated to the agent. Accordingly partial delegation is beneficial if the principal has a relatively low degree of risk aversion. Moreover, full delegation allows the principal to scale incentives provided to the agent but not to fine tune the intensity of incentives for each effort separately. With partial delegation fine tuning is possible but increasing incentives for one effort implies reducing them for the other. If scaling is more effective in minimizing agency costs than fine tuning incentives, the principal tends to prefer full delegation to partial delegation and vice versa.

Keywords: Delegation, agency problem, congruity, risk sharing

I. Introduction

Most of the literature on principal-agent relationships assumes somewhat exogenously that the principal delegates one or several tasks to an agent as he is unable to perform any task himself. A compelling example to justify this assumption is a corporation with vastly diversified ownership. Its shareholders are typically unable to manage the firm's daily business themselves and need to delegate these tasks to one or several professional manager(s).

The necessity for complete delegation, however, is likely to be absent in other business environments. E.g. an owner-manager of a firm may be forced due to limited time to delegate tasks or decision rights to subordinates. However, the *degree* of delegation as well as the specific tasks to be delegated are subject to some discretion. The same reasoning is likely to apply to principal-agent relationships on lower hierarchical levels. Even in a corporation of the type described in the first paragraph, e.g. a division manager exhibits discretion when deciding whether or not to delegate a task, and which particular tasks to delegate, to subordinated managers or employees.

In this paper we focus on the latter type of settings. Delegation is inevitable but its degree and specifics are subject to managerial choice. In particular we consider a linear moral hazard type agency model. Two tasks need to be performed. A manager (principal) is unable to perform both tasks personally. He employs an agent and decides whether to delegate both tasks to the agent or only one. We allow for both, the principal and the agent, to be risk averse in our analysis. The principal maximizes firm value, but firm value is non-contractible in our setting. Rather, a performance measure that is positively correlated with firm value is available for contracting in order to incentivize the agent. While firm value and the performance measure are assumed to be increasing in effort, performance sensitivities of both measures may differ for each task. Accordingly, the owner's objectives are potentially captured only imperfectly by the performance signal. This creates a setting in which a risk and incentive trade-off and a congruity problem are present simultaneously. We study the effect of both types of agency problems on the delegation choice.

The optimal compensation contract prescribes the delegation mode along with the payment parameters. Technically, we proceed in two steps. We determine the optimal contracting parameters for each delegation mode. Comparing the principal's surplus in each mode determines the optimal delegation scenario. We distinguish three delegation modes. Under full delegation both tasks are delegated to the agent. Under partial delegation either task 1 or task 2 is delegated to the agent and the remaining action will be performed by the manager himself. Note that with partial delegation we face a double moral hazard problem. Due to unobservability of effort, both, the agent and the principal, are unable to commit to a specific effort ex ante. An incentive compatibility constraint needs to be specified not only for the agent but for the principal. Importantly, the principal chooses effort trading off an increase in firm value and the disutility from effort as well as compensation costs. The latter results from the fact, that additional effort not only increases expected firm value, but also expected value of the performance measure determining the payment to the agent. Of course the increase in expected compensation is stronger, the higher the incentive rate agreed upon in the contract. A higher incentive rate thus decreases equilibrium effort of the principal.

Having determined the optimal contract parameters and payoffs in each delegation mode we continue investigating the risk-incentive problem separately from the congruity problem in a ceteris paribus analysis. To do so we assume a fully congruent performance measure in the first step. Given the risk incentive trade-off is the only rub, we find that full delegation is preferred if the agent's optimal exposure to risk is high while partial delegation is optimal if this exposure is low. Intuitively, with full delegation the principal motivates both efforts to be performed by the agent using a single incentive rate. With partial delegation he motivates only one effort and, moreover, limits the equilibrium effort for his own task by determining the incentive rate. As a result the optimal incentive rate with full delegation is always higher than with partial delegation. From a risk sharing perspective, however, a high incentive rate is beneficial only if most of the risk should be allocated to the agent. Otherwise a low rate would be preferred. As a result full delegation becomes more favorable, the lower the agent's risk aversion relative to the principal's

In the second step we focus purely on the congruity issue assuming two risk neutral players. If the signal is not fully congruent, partial delegation may become optimal. The optimal choice of the delegation mode follows a trade-off between scaling and alignment of incentives. With full delegation the principal can increase or decrease incentives by adapting the incentive rate but cannot affect the relative size of both efforts. In contrast, under partial delegation additional effort with respect to one task can be motivated at the expense of the other task. Precisely, increasing the incentive coefficient for the agent increases the effort in the delegated task but at the same time decreases the equilibrium effort in the task performed by the principal as described above. Whether full or partial delegation is optimal depends on whether scaling or alignment of incentives is more efficient in approaching first best. Intuitively, the ability to scale efforts is helpful if the sensitivity of the performance measure with regard to both tasks is either too high or too low. In contrast the ability to align incentives is helpful if the sensitivity of the performance measure is too high for one task and too low for the other as compared to the effect the tasks have on firm value.

Our paper is closely related to Itoh (1994) and may be considered an extension of his work. Similar to our scenario, Itoh considers a setting where either two tasks or one task can be delegated to an agent. However, he employs a risk neutral principal and a risk averse agent. The principal in his model maximizes firm value, which at the same time is used as the performance measure in the agent's incentive contract. Further, in Itoh (1994) both tasks are equally productive such that the principal is indifferent which one to delegate. As a consequence the risk and incentive trade-off in Itoh (1994) is different from ours. More importantly, a congruity problem in his paper does not arise nor does the question of which task to delegate most effectively.

A comparison of the impact of full versus partial delegation is also carried out in De Paola/Scoppa (2009). Using a two-task agency with risk neutral principal and agent under limited liability, they allow for the principal and the agent to have different abilities and assume that transmission costs exist. In contrast to our paper, the principal always performs the first task in their model and transmits its outcome to the agent to perform the second task.

Finally, Gürtler (2008) analyzes partial delegation in a somewhat related paper. He considers, however, relational contracts rather than explicit (formal) incentive contracts used in this paper.

Moreover, our paper ties in with at least three more streams in the literature. First, it is related to the literature investigating the interplay between performance measure congruity and the risk and incentive trade-off. Second, it is related to papers analyzing the value of delegation as such. Third, our paper connects with the literature on optimal task assignment in agency conflicts.

Seminal papers belonging to the first stream of literature mentioned above are Holmström/Milgrom (1991) and Feltham/Xie (1994). Both analyze frictions that occur in multi-task agency, if the agent's contribution to firm value is not contractible. In this context the papers by Datar et al. (2001) and Baker (2002) specifically analyze how performance measure risk and (lack off) congruity are traded-off in optimal incentive contracts. Similar to these papers, we assume that firm value is not contractible and a risk averse agent has to be motivated by a (potentially non congruent) performance signal. While in those papers optimal incentive weights are determined by the performance measure risk and its congruity, firm value risk is of additional relevance in our paper. This results from allowing for the principal to be risk averse. The main difference, however, between the afore mentioned papers and our approach is, that they exogenously assume full delegation of a given number of tasks while in our model the principal may perform one task himself and the number of tasks delegated is endogenous.

Referring to the second stream of literature, different explanations for a positive value of delegation, or a benefit from decentralization, have been offered. E.g. in Melumad/Reichelstein (1987) costly communication can render delegation beneficial. Melumad/Mookherjee (1989) and Schöndube-Pirchegger/Schöndube (2012) show that the delegation of decision rights may act as a commitment device. In this paper neither a communication nor a commitment problem are present. Rather, the optimal delegation mode trades-off risk sharing and the congruity of the performance measure with firm value.

In Bushman et al. (2000) delegation may be optimal as the agent receives pre-decision information in a decentralized system. They show that the value of delegation is positive if the congruence of the performance measure exceeds a threshold that depends on the agent's risk aversion. Important differences from our paper are threefold: First, we do not consider pre-decision information. Second, in Bushman et al. delegation results in non-observability of the agent's effort while in our approach effort is always unobservable but the principal can perform one task himself. Third, in Bushman et al. the principal has to motivate the effort in one task across different states of the world while we consider a setting where the principal has to control the effort to be devoted to two different tasks.

Finally, with respect to the task assignment literature, our paper is related to some extent to Zhang (2003) and Hughes et al. (2005). In a model in which a principal can assign tasks to two identical agents, Zhang compares specialized and broad task assignment when tasks are complementary and Hughes et al. investigate optimal task assignment when the payoff from production is not contractible such that the congruity of performance signals matters. Reichmann/Rohlfing-Bastian (2014) consider a setting where the principal delegates the task assignment decision to one of two agents. Schöttner (2008) analyzes the task assignment problem under relational contracts. Again, all these papers differ from our setting as they assume that all tasks have to be delegated to agents while in our model partial delegation is feasible.

II. Model description

We consider a single period LEN-type principal-agent model. Two tasks need to be performed.
The manager/principal can delegate both tasks to the agent. Alternatively he can perform the first or the second task himself. For exogenous reasons, e.g. time constraints, it is impossible for the principal to perform both tasks on his own. In other words some delegation is inevitable in our setting while full delegation is possible. All effort is privately observed by the performing party.
The effort in both tasks $e_i,\ i = 1,2,$ affects long run firm value x as follows

$$x = b_1 e_1 + b_2 e_2 + \varepsilon_x.$$

$b_i > 0$ is the contribution per unit of effort in task i to the firm value, we call it task i's productivity. We assume that x is non verifiable and therefore not available for incentive contracting. Rather, principal and agent can contract on a performance measure y that is affected by both tasks

$$y = m_1 e_1 + m_2 e_2 + \varepsilon_y.$$

$m_i > 0$ is the contribution per unit of effort in task i to the performance measure, we call it task i's sensitivity. ε_x and ε_y are (jointly) normally distributed$\varepsilon_j \sim N(0, \sigma_j^2)$, with $j = x, y$. We assume that firm value and performance measure are positively correlated with $Cov(\varepsilon_x, \varepsilon_y) = \rho\sigma_x\sigma_y$ and correlation coefficient between x and y $\rho \in (0,1]$.
Performing a task creates identical disutilities no matter whether the principal or the agent provides the effort:

$$C(e_i) = \frac{e_i^2}{2} \text{ for } i = 1,2.$$

We restrict ourselves to linear incentive contracts of the form

$$s(y) = f + vy$$

where f determines a fixed compensation and v an incentive rate that relates the agent's compensation to performance measure y.
Both, the agent and the principal might be risk averse. Risk aversion parameters are denoted r^A and r^P for agent and principal, respectively. Given our LEN-setup the certainty equivalents of both players are given by

$$CEA = E\big(s(y)\big) - I_1C(e_1) - I_2C(e_2) - \frac{r^A}{2}Var(s(y))$$

and

$$CEP = E(x) - E\big(s(y)\big) - (1 - I_1)C(e_1) - (1 - I_2)C(e_2) - \frac{r^P}{2}Var(x - s(y)).$$

Here CEA denotes the agent's and CEP the principal's certainty equivalent. $I_i \in \{0,1\}$ with $i = 1,2$ is equal to one if the task is delegated to the agent and zero if the principal acts himself. The agent's certainty equivalent of reservation utility is normalized to zero without loss of generality.
Note that the principal offers a contract to the agent that specifies which task(s) to be performed by the agent along with the agent's compensation. Thus delegation and control choices are made simultaneously at the beginning of the game. The agent either accepts the contract or refuses to do so and the game ends. If the agent accepts, he performs all the tasks delegated to him according to the contract. Once y is realized the agent is paid.
In our multi task setting with risk averse players agency costs occur due to two types of overlapping effects. For once agency costs arise due to possibly suboptimal risk sharing between principal and agent when the principal needs to trade off risk and incentives. The other type of agency costs results from a lack of congruity of the performance measure available. Costs here are related to suboptimal allocation of effort to both tasks. Both combined effects are relevant with respect to the optimal degree of delegation.
In the next section we determine the first best (symmetric information) solution to our problem to provide a benchmark. Section IV contains second best solutions for each delegation mode. The results are analyzed in detail in sections V and VI.

III. First best solution

If asymmetric information with regard to effort is absent it does not matter how many tasks are delegated given that principal and agent perform both tasks at identical costs. All that is necessary to achieve first best is a forcing contract specifying the effort level for each task and the player to perform the effort. We randomly assume below that both tasks are delegated to the agent resulting in the following optimization problem.

$$\max_{e_1,e_2,v,f} CEP = E(x) - E\big(s(y)\big) - \frac{r^P}{2} Var\big(x - s(y)\big) \quad (1)$$

s.t.

$$E\big(s(y)\big) - C(e_1) - C(e_2) - \frac{r^A}{2} Var\big(s(y)\big) \geq 0. \quad (2)$$

(2) is the participation constraint for the agent. By inserting the binding participation constraint, (1) reduces to

$$\max_{e_1,e_2,v} Z = E(x) - C(e_1) - C(e_2) - \frac{r^A}{2} Var\big(s(y)\big) - \frac{r^P}{2} Var(x - s(y)) \,. \quad (3)$$

Z is the net surplus of the agency. It comprises the benefits $E(x)$ less the cost to generate the benefits.

Solutions to the above problem are presented in lemma 1.

Lemma 1: The optimal effort levels in the first-best solution are $e_1^{FB} = b_1$ and $e_2^{FB} = b_2$. The optimal incentive rate equals $v^{FB} = \frac{r^P \rho \sigma_x \sigma_y}{\sigma_y^2 (r^A + r^P)}$ and the principal's surplus is $Z^{FB} = \frac{(r^P + r^A)(b_1^2 + b_2^2) - r^P \sigma_x^2 (1 - \rho^2) - r^P r^A \sigma_x^2}{2(r^P + r^A)}$.

Proof. See the Appendix.

As argued above, delegation of only one task would result in identical incentive rate and payoff to the principal. Note that in the first best setting the incentive rate is determined in order to allocate risk between principal and agent optimally. In other words it minimizes the risk premium the principal has to bear in equilibrium, which is

$$RP = \frac{r^A}{2} Var\big(s(y)\big) + \frac{r^P}{2} Var\big(x - s(y)\big) = \frac{r^A}{2} v^2 \sigma_y^2 + \frac{r^P}{2} (\sigma_x^2 + v^2 \sigma_y^2 - 2\rho v \sigma_x \sigma_y).$$

As x and y are positively correlated the incentive rate is always positive. It can be either below or above one. It is increasing in the principal's risk aversion r^P and becomes zero if the principal is risk neutral. With the first best solution in place we continue analyzing second best.

IV. Second-best solutions

Full delegation

If the principal decides to delegate both tasks to the agent a standard multi-task agency problem with linear contracts is present, see, e.g., Feltham/Xie (1994). For a given incentive coefficient v the agent selects his effort as

$$e_1 = vm_1$$
$$e_2 = vm_2. \qquad (4)$$

To determine the optimal contract the principal maximizes (3) subject to the incentive compatibility constraints in (4).

Lemma 2: The optimal bonus coefficient under full delegation is

$$v^D = \frac{m_1 b_1 + m_2 b_2 + r^P \rho \sigma_x \sigma_y}{m_1^2 + m_2^2 + \sigma_y^2 (r^P + r^A)}.$$

The principal's corresponding surplus is

$$Z^D = \frac{1}{2} \frac{2b_2 m_2 (b_1 m_1 + r^P \rho \sigma_x \sigma_y) + b_1^2 m_1^2 + b_2^2 m_2^2 - (m_1^2 + m_2^2) r^P \sigma_x^2 + 2 m_1 b_1 r^P \rho \sigma_x \sigma_y - \sigma_x^2 \sigma_y^2 [(r^P)^2 (1 - \rho^2) + r^A r^P]}{m_1^2 + m_2^2 + \sigma_y^2 (r^P + r^A)}.$$

Proof: See the Appendix.

Partial delegation

If the principal delegates only one task to the agent and performs the other one himself, incentives change. As we assume that the principal cannot commit to a specific effort ex ante, we face a double moral hazard problem with partial delegation. Thus, we have to consider an incentive compatibility constraint for the principal, too. If the principal decides to delegate task i to the agent and performs task $-i$ himself, $i \in \{1,2\}, i \neq -i$, the incentive compatibility constraint for the agent applies as before

$$e_i = vm_i. \qquad (5)$$

The incentive compatibility constraint for the principal equals

$$e_{-i} = b_{-i} - vm_{-i}. \qquad (6)$$

The principal's incentive compatibility constraint is derived from maximizing his certainty equivalent CEP as defined in section II with respect to e_{-i}. Note that the principal chooses effort trading off the marginal increase in firm value, b_{-i}, and marginal cost in terms of marginal expected compensation to the agent, vm_{-i}. Thus whenever the principal performs some effort personally, his effort not only increases expected firm value, but also expected compensation paid to the agent. The latter constitutes a cost related to effort in addition to disutility $C(e_{-i})$.

As the agent's participation constraint is binding at the optimum, independently of the delegation mode the principal's objective function to determine incentive contracts is always given by (3).

Thus, under partial delegation the principal maximizes (3) subject to the incentive constraints (5) and (6). The solution to the problem is stated in Lemma 3.

Lemma 3: Under partial delegation, if task i is delegated to the agent, the optimal incentive coefficient is

$$v_i^* = \frac{m_i b_i + \rho r^P \sigma_x \sigma_y}{m_1^2 + m_2^2 + \sigma_y^2 (r^P + r^A)}.$$

The principal's corresponding surplus is given by

$$Z_i^* = \frac{1}{2} \frac{b_2^2 \left[m_i^2 + \sigma_y^2 (r^A + r^P)\right] + b_1^2 m_1^2 + b_2^2 m_2^2 - (m_1^2 + m_2^2) r^P \sigma_x^2 + 2 m_i b_i r^P \rho \sigma_x \sigma_y - \sigma_x^2 \sigma_y^2 [(r^P)^2 (1 - \rho^2) + r^A r^P]}{m_1^2 + m_2^2 + \sigma_y^2 (r^P + r^A)}.$$

Proof: See the Appendix.

Comparison of the results

Opposing the results from above we find that full delegation induces higher incentive rates as is stated below.

Lemma 4: The optimal incentive coefficient with full delegation is always strictly higher than with partial delegation.

Proof. From Lemma 2 and Lemma 3 we know that

$v_i^* = \frac{m_i b_i + \rho r^P \sigma_x \sigma_y}{m_1^2 + m_2^2 + \sigma_y^2 (r^P + r^A)}$ and $v^D = \frac{m_1 b_1 + m_2 b_2 + \rho r^P \sigma_x \sigma_y}{m_1^2 + m_2^2 + \sigma_y^2 (r^P + r^A)}$ such that

$v^D - v_i^* = \frac{m_{-i} b_{-i}}{m_1^2 + m_2^2 + \sigma_y^2 (r^P + r^A)} > 0.$

The intuition for this result is straightforward. With full delegation an increase in v increases the agent's incentives to perform both efforts. In contrast with partial delegation higher v increases the agent's incentive to perform the delegated task but at the same time reduces the equilibrium effort of the principal as is observed from (6). As a consequence increasing v is more effective in terms of effort motivation with full delegation than with partial delegation and the optimal incentive rate is always higher with full delegation.

Calculating the difference in objective function values our results are less clear cut.

$$Z^D - Z_i^* = \frac{1}{2}\frac{2b_{-i}m_{-i}\left(b_i m_i + r^P\rho\sigma_x\sigma_y\right) - b_{-i}^2\left(m_i^2 + \sigma_y^2(r^A + r^P)\right)}{m_1^2 + m_2^2 + \sigma_y^2(r^A + r^P)}$$

can be either positive or negative depending on parameter values. Full delegation is optimal in some settings and partial delegation in others. Optimality is affected simultaneously by the risk-incentive trade-off and the congruity problem present. In what follows we disentangle both types of agency problems and analyze them separately. We start in section V focusing on the risk and incentive trade-off. In section VI we investigate the effects of the congruity problem.

V. Delegation and contracting choices with risk averse players

In order to concentrate on the risk and incentive trade-off we assume that there is no congruity problem present in this section. To achieve this we set $m_1 = b_1$ and $m_2 = b_2$.[1]

Referring to Itoh (1994) once again, note that perfect congruity is present in his model as well but he assumes $b_1 = b_2 = 1$ in addition. Doing so implies that partial delegation of task 1 and task 2 yields the same equilibrium outcome which is not the case in our analysis. Moreover, Itoh assumed contractibility of firm value and an absence of an additional performance measure. Accordingly in his setting firm value x is the only variable exposing the players to risk while in our paper the risk jointly induced by x and y turns out to be important.

Applying $m_1 = b_1$ and $m_2 = b_2$ to the optimal incentive rates for full and partial delegation (Lemma 2 and 3) we obtain

$$v^D = \frac{b_1^2 + b_2^2 + r^P\rho\sigma_x\sigma_y}{b_1^2 + b_2^2 + \sigma_y^2(r^P + r^A)} \qquad (7)$$

and

[1] The performance measure is congruent only if $\frac{m_1}{m_2} = \frac{b_1}{b_2}$, see Feltham/Xie (1994).

$$v_i^* = \frac{b_i^2 + \rho r^P \sigma_x \sigma_y}{b_1^2 + b_2^2 + \sigma_y^2 (r^P + r^A)}. \qquad (8)$$

Also, recall from Lemma 1 that the first best incentive rate equals

$$v^{FB} = \frac{r^P \rho \sigma_x \sigma_y}{\sigma_y^2 (r^A + r^P)}\,. \qquad (9)$$

v^{FB} is unaffected by the now assumed absence of a congruity problem and, as stated above, minimizes the overall risk premium to be borne by the principal. Low (high) v^{FB} is tantamount to optimally allocating most of the risk to the principal (agent).

While v^{FB} is either below or above one, it follows from (7), (9), and Lemma 4 that whenever $v^{FB} > 1$, it holds that $v_i^* < v^D < v^{FB}$. Thus, if v^{FB} is sufficiently high, the optimal incentive rate is closer to the optimal risk allocation rate with full delegation than with partial delegation. Distortion from the optimal risk allocation therefore is lower with full delegation.

On the other hand $v^{FB} < v_i^* < v^D$ from (9), (8), and again Lemma 4 requires $v^{FB} < \frac{1}{2}$.[2]

Thus, for the distortion from first best risk allocation to be lower with partial delegation than with full delegation v^{FB} has to be sufficiently low. With these observations in place we continue contrasting full and partial delegation.

Calculating the payoff differences for the modes of delegation we obtain

$$Z^D - Z_i^* = \frac{1}{2} \frac{b_{-i}^2 (b_i^2 + h)}{b_1^2 + b_2^2 + (r^A + r^P)\sigma_y^2}, \qquad (10)$$

and

$$Z_i^* - Z_{-i}^* = \frac{1}{2} \frac{h (b_i^2 - b_{-i}^2)}{b_1^2 + b_2^2 + (r^A + r^P)\sigma_y^2}, \quad i = 1,2 \qquad (11)$$

with $h = 2 r^P \rho \sigma_x \sigma_y - (r^A + r^P)\sigma_y^2$.

This leads us directly to Proposition 1.

Proposition 1: If $h \geq 0$ full delegation is optimal. If partial delegation is optimal, the less productive task is delegated.

[2] Note that $\frac{b_i^2}{b_1^2 + b_2^2} \leq \frac{1}{2}$ needs to hold for task i to be optimally delegated as will be shown in Proposition 1.

Whenever h as defined above is positive, (10) is positive and full delegation is preferred to partial delegation. While $h \geq 0$ is sufficient for full delegation to be preferred, it is not a necessary condition. Rather, with $h < 0$ all we require is b_i^2 to be sufficiently high for (10) to be positive as well. Moreover, Proposition 1 implies that h needs to be negative for partial delegation to be optimal. Thus, if partial delegation is optimal at all, according to (11) the less productive task is optimally delegated to the agent.

To build some intuition for the above results it helps to relate h to the previously discussed optimal risk allocation rate v^{FB}. Note that $h \geq 0$ is equivalent to $v^{FB} \geq \frac{1}{2}$. Thus whenever the optimal risk allocation rate is above a threshold of $\frac{1}{2}$, full delegation is preferred. This finding results from two aspects developed earlier in this paper. First, with full delegation, effort motivation via v is more effective than with partial delegation and thus the optimal incentive rate is always higher with full delegation. Second, if the optimal risk allocation rate v^{FB} is high, full delegation leads to less distortion from the optimum as opposed to partial delegation. As a result full delegation is favorable for v^{FB} (or h) sufficiently high. Likewise, with partial delegation the optimal incentive rate is lower than with full delegation which is in line with the principal's risk sharing objectives if he wants to shield the agent from too much risk. Accordingly, partial delegation is preferred if v^{FB} (or h) is sufficiently low.

If partial delegation is preferred at all, the principal always performs the more productive task himself. To see this, recall that the agent's effort equals $e_i = v_i^* b_i$ at the optimum and the principal's equilibrium effort is characterized by $e_{-i} = (1 - v_i^*) b_{-i}$. For partial delegation $v^{FB} < \frac{1}{2}$ needs to hold which implies $v_i^* < \frac{1}{2}$ from (8) and (11). With $v_i^* < \frac{1}{2}$ it follows that the principal's effort incentives are higher than the agent's whenever partial delegation is used. As this is the case, it is optimal for the principal to delegate the less productive task and to perform the more productive one personally.

Finally, glance once more at (10). While the productivity of the delegated task, b_i, is relevant for full or partial delegation to be preferred, the productivity of the non-delegated task, b_{-i}, affects the absolute size of the difference. No matter whether full or partial delegation is preferable, the difference is increasing in b_{-i}. In fact the higher b_{-i} the lower the optimal incentive rate v_i^* and the higher the incentives the principal provides himself with. This increases the profitability of partial delegation if v^{FB} is sufficiently low but decreases it if v^{FB} is high and full delegation is optimal.

VI. Delegation and the congruity of performance measures

In this section we focus on the value of delegation when risk sharing between the contracting parties is irrelevant and agency costs arise only from limited congruity of the performance measure. To do so we assume that both contracting parties are risk-neutral. Inserting $r^P = r^A = 0$ into the solutions from section 4 we obtain the following results.

$$v^D = \frac{m_1 b_1 + m_2 b_2}{m_1^2 + m_2^2}$$

$$\text{and} \quad v_i^* = \frac{m_i b_i}{m_1^2 + m_2^2}.$$

Both incentive rates are positive and can be below or above one. Of course $v^D - v_i^* > 0$ still holds.

With respect to the difference in objective function values we obtain

$$Z^D - Z_i^* = \frac{1}{2} \frac{2b_{-i} m_{-i} - b_{-i}^2 m_i^2}{m_1^2 + m_2^2} \tag{12}$$

$$\text{and} \quad Z_i^* - Z_{-i}^* = \frac{1}{2} \frac{b_{-i}^2 m_i^2 - b_i^2 m_{-i}^2}{m_1^2 + m_2^2}. \tag{13}$$

From (13) we derive the necessary and sufficient condition for task i to be optimally delegated in the case of partial delegation as stated in Proposition 2.

Proposition 2: For $Z_i^* - Z_{-i}^* > 0$, that is task i rather than $-i$ is optimally delegated, $\frac{m_i}{m_{-i}} > \frac{b_i}{b_{-i}}$ holds.

Proof: See the Appendix.

The relation provided in Proposition 2 holds for one of the two tasks only. If $\frac{m_i}{m_{-i}} > \frac{b_i}{b_{-i}}$ holds true the principal prefers to delegate task i whereas the delegated task can be the more or the less sensitive task. If, e.g. $1 > \frac{m_i}{m_{-i}} > \frac{b_i}{b_{-i}}$ holds, the less productive task is delegated as $\frac{b_i}{b_{-i}} < 1$ holds. With $\frac{m_i}{m_{-i}} > \frac{b_i}{b_{-i}} > 1$ the more productive task is delegated.

Assuming that task i is the one to be delegated, the principal uses (12) in order to decide whether full delegation or partial delegation is optimal. A necessary and sufficient condition is presented in Proposition 3.

Proposition 3: The principal prefers partial delegation over full delegation only if $\frac{m_i}{m_{-i}} > 2\frac{b_i}{b_{-i}}$ holds.

Proof: The proof follows directly from (12).

Corollary 1 follows immediately from Proposition 3.

Corollary 1: If the performance measure is perfectly congruent the principal prefers full delegation to partial delegation.
For a performance measure to be perfectly congruent $\frac{m_i}{m_{-i}} = \frac{b_i}{b_{-i}}$ needs to hold such that the condition $\frac{m_i}{m_{-i}} > 2\frac{b_i}{b_{-i}}$ will be violated.

With a perfectly congruent performance measure in place, first best is achieved with full delegation. In fact the principal can choose v such that $e_1 = vm_1 = b_1 = e_1^{FB}$ and $e_2 = vm_2 = b_2 = e_2^{FB}$ by using $v = \frac{b_1}{m_1} = \frac{b_2}{m_2}$.

In contrast first best can never be achieved with partial delegation. Rather, an incentive rate $v = \frac{b_i}{m_i}$ is required to provide the agent with first best effort incentives such that $e_i = vm_i = b_i$. To provide a similar equilibrium effort for the principal, however, $e_{-i} = b_{-i} - vm_{-i} = b_{-i}$ requires $v = 0$. Accordingly, with partial delegation first best cannot be achieved even if a perfectly congruent performance measure is present and thus agency costs occur.

This observation provides the starting point for a broader interpretation of our results.

With full delegation the principal controls both efforts in identical fashion when determining the incentive rate v. An increase in the rate increases both efforts in equilibrium while a decrease reduces them at a similar rate. He is able to affect the overall amount of effort provided but the relation of effort, $\frac{e_1}{e_2}$, remains constant. In contrast with partial delegation the principal cannot motivate additional effort in one task without decreasing incentives in the other one. Thus the principal can control the relation of efforts but cannot increase or decrease both of them.

Whether full or partial delegation is preferred in a particular setting depends on what is more effective in reducing agency costs: A delegation mode that motivates both efforts simultaneously in the sense of scaling or one that is able to affect relative incentives. Intuitively, scaling helps to move effort towards first best, if the sensitivity of both tasks is either too low or too high as opposed to their productivities. In contrast if the sensitivity of one task is too low while the one of the other task is too high being able to align opposing incentives appears to be useful.

Formally the optimal second best effort is chosen to minimize the gap to first best effort in each delegation mode. To see this note that according to Pythagoras' Theorem the distance between first and second best effort equals

$$\Delta = \sqrt{(e_1^{FB} - e_1^{SB})^2 + (e_2^{FB} - e_2^{SB})^2}\ .$$

While $e_i^{FB} = b_i$ from Lemma 1, e_i^{SB} is determined according to (4), (5), and (6) for the different modes of delegation.

Minimizing Δ we obtain the results presented in Lemma 5.

Lemma 5: The minimum distance from first best equals

$$\Delta^D = \sqrt{\frac{(b_2 m_1 - b_1 m_2)^2}{m_1^2 + m_2^2}} \text{ with full delegation and}$$

$$\Delta_i^* = \frac{b_i m_{-i}}{\sqrt{m_1^2 + m_2^2}} \text{ with partial delegation of task } i.$$

Proof: See the Appendix.

Now comparing the minimum distances for the different modes of delegation reestablishes the conditions identified in Proposition 2 and 3. In fact

$\Delta_i^* > \Delta_{-i}^*$ holds if and only if $\frac{m_i}{m_{-i}} > \frac{b_i}{b_{-i}}$ and

$\Delta_i^* > \Delta^D$ holds if $\frac{m_i}{m_{-i}} > 2\frac{b_i}{b_{-i}}$ applies.[3]

To provide further intuition for the line of reasoning above we use the following example:

$$b_1 = 3, b_2 = 2,\ m_1 = 0.5, m_2 = 2.$$

It follows that $v^D = 1.29$ $\quad e_1^D = 1.29,\ e_2^D = 2.58$

$v_1^* = 0.35$ $\quad e_1^* = 0.18, e_2^* = 1.40$ with delegation of task 1

and $v_2^* = 0.94$ $\quad e_1^* = 2.53,\ e_2^* = 1.88$ with delegation of task 2.

Note that inserting into the conditions identified in Proposition 2 and 3 we obtain

[3] In fact Δ^D is equal to $|\Delta_1^* - \Delta_2^*|$. Thus, full delegation is optimal whenever $|\Delta_1^* - \Delta_2^*| < min\{\Delta_1^*, \Delta_2^*\}$. If $\Delta_1^* > \Delta_2^*$ the condition becomes $\Delta_1^* - \Delta_2^* < \Delta_2^*$ or $\Delta_1^* < 2\Delta_2^*$. For $\Delta_2^* > \Delta_1^*$ the condition becomes $\Delta_2^* - \Delta_1^* < \Delta_1^*$ or $\Delta_2^* < 2\Delta_1^*$. This is why the factor "2" in the optimality condition from Proposition 3 shows up.

$\frac{m_2}{m_1} = 4 \;\; and \;\; \frac{b_2}{b_1} = \frac{2}{3}.$

Thus $\frac{m_2}{m_1} > \frac{b_2}{b_1}$ as well as $\frac{m_2}{m_1} > 2\frac{b_2}{b_1}$ hold and the principal optimally delegates task 2 only.

Depicting the optimal pairs of effort derived above in the e_1/e_2 plane allows to compare the three modes with regard to Δ.

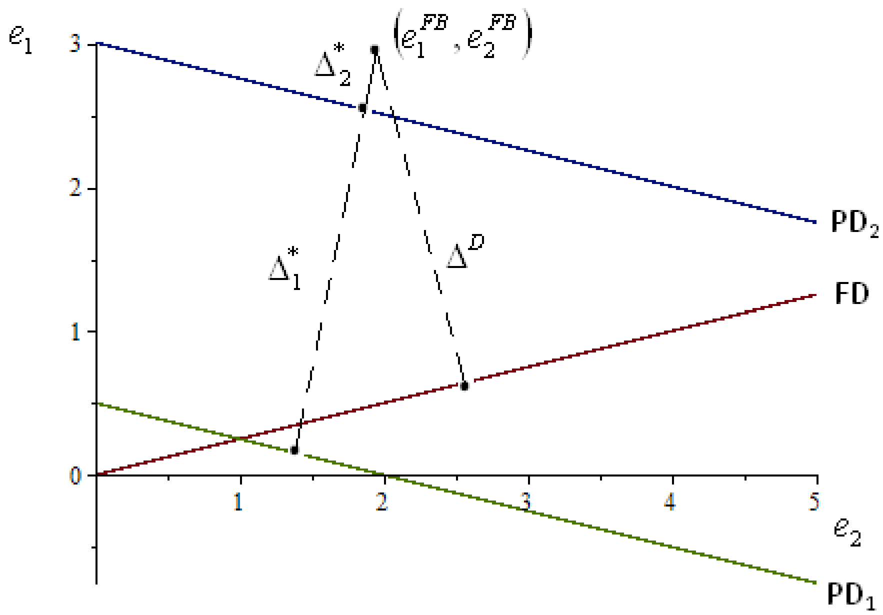

Figure 1: Scaling versus alignment

The graphs indicated as FD, PD1, and PD2 denote the combinations of effort that can be induced by the principal varying the incentive rate v, for full delegation, delegation of task 1, and delegation of task 2, respectively. (e_1, e_2) denote the optimal combinations with superscript D indicating a delegated effort.

Calculating Δs in our example results in $\Delta_1^* = 2.91, \Delta_2^* = 0.49, \Delta^D = 2.34$.

Note that under partial delegation of task 2 the principal minimizes $\Delta_2 = \sqrt{(b_1 - (b_1 - vm_1))^2 + (b_2 - vm_2)^2} = \sqrt{(vm_1)^2 + (b_2 - vm_2)^2}$. The second term in brackets is minimized for $v = 1$ as $b_2 = m_2 = 2$. The first term is minimized for $v = 0$. First best obviously cannot be achieved but using $v_2^* = 0.94$ we get fairly close.

In contrast, if the first task is delegated the principal minimizes $\Delta_1 = \sqrt{(vm_2)^2 + (b_1 - vm_1)^2}$. As b_1 is large in our example compared to m_1 a high incentive rate of $v = 6$ would be required to motivate first best effort in task 1. Again $v = 0$

would achieve first best effort with regard to the non-delegated task. The optimal incentive rate of $v_1^* = 0.35$ inevitable leads to a massive downward distortion of effort with respect to the delegated task which produces agency costs. As a result the principal prefers to delegate task 2 rather than task one.

Comparing delegation of task 2 to full delegation, note that in the latter case the principal minimizes $\Delta = \sqrt{(b_1 - vm_1)^2 + (b_2 - vm_2)^2}$. With regard to the first term in brackets the argument from above applies. Due to high b_1 and low m_1 a high incentive rate would be required to motivate first best in task 1. At the same time a more moderate incentive of $v = 1$ would implement first best with regard to task 2. Even though the difference in individually optimal v's is lower than with delegation of task 1 it is far higher than with delegation of task 2 and partial delegation is preferred to full delegation.

VII. Concluding remarks

In this paper we consider a principal who needs to delegate at least one of the two tasks to be performed in a firm to an agent. While full delegation is possible, no delegation is not. The principal needs to specify contractually which task(s) to delegate and offers an incentive contract to the agent. Due to non-contractibility of firm value the contract needs to be based on a performance measure that is positively correlated to firm value. Allowing for risk averse players in this setting two types of agency problems arise in our setting. A risk-incentive problem results from possible suboptimal allocation of risk between principal and agent along with distorted incentives. A congruity problem occurs due to the use of a not fully congruent performance measure. Both types of problems affect the optimal mode of delegation. In order to disentangle overlapping effects from both problems we analyze them separately in this paper.

Focusing on the risk and incentive trade-off, and assuming a fully congruent performance measure to do so, we find that partial delegation can only be favorable if the optimal exposure to risk of the agent is sufficiently low. If the principal prefers partial delegation he always delegates the less productive task. This is straightforward as low risk exposure and low incentives are imposed on the agent by a low incentive rate. In contrast, a low rate motivates high effort of the principal. Thus, if from a risk sharing perspective a low rate is desirable, it makes sense to delegate the task that is less productive.

To analyze the congruity problem we refrain from any risk aversion. Doing so, we find that partial delegation allows the principal to align effort incentives in the sense that he can motivate the optimal relation of efforts but not the optimal level. As opposed to that with full delegation the principal controls the level but not the relation of efforts. It hinges on the relation of effort sensitivities to effort productivities whether scaling or alignment allows to minimize agency costs. The higher the sensitivity of one task relative to the other the more favorable it is for the principal to delegate this task only.

Trying to look at combined effects we find that high risk aversion of the principal and a performance measure that is characterized by both tasks either more or less sensitive to the performance measure as compared to firm value work in favor of full delegation.

High risk aversion of the agent relative to the principal and a performance measure that is highly sensitive to one task and remotely sensitive to the other as compared to productivity render partial delegation more attractive.

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Appendix

Proof of Lemma 1:

The principal's problem in the first-best solution is

$\max_{e_1,e_2,v} Z = E(x) - C(e_1) - C(e_2) - \frac{r^A}{2}Var\big(s(y)\big) - \frac{r^P}{2}Var\big(x - s(y)\big).$ (A1)

With $\frac{r^A}{2}Var\big(s(y)\big) + \frac{r^P}{2}Var\big(x - s(y)\big) = \frac{r^A}{2}v^2\sigma_y^2 + \frac{r^P}{2}\big(\sigma_x^2 + v^2\sigma_y^2 - 2\rho v\sigma_x\sigma_y\big)$, (A1) can be written as

$$\max_{e_1,e_2,v} Z = b_1e_1 + b_2e_2 - \frac{e_1^2}{2} - \frac{e_2^2}{2} - \frac{r^A}{2}v^2\sigma_y^2 - \frac{r^P}{2}\big(\sigma_x^2 + v^2\sigma_y^2 - 2\rho v\sigma_x\sigma_y\big).$$

From the first-order conditions $\left\{\frac{\partial Z}{\partial e_1} = 0, \frac{\partial Z}{\partial e_2} = 0, \frac{\partial Z}{\partial v} = 0\right\}$ we obtain $e_1^{FB} = b_1,\ e_2^{FB} = b_2, v^{FB} = \frac{r^P Cov(x,y)}{\sigma_y^2(r^A+r^P)} = \frac{r^P\sigma_x\rho}{\sigma_y(r^A+r^P)}$, and the principal's first-best surplus is $Z^{FB} = \frac{(r^P+r^A)(b_1^2+b_2^2) - r^P\sigma_x^2(1-\rho^2) - r^Pr^A\sigma_x^2}{2(r^P+r^A)}$.

Proof of Lemma 2:

In the second-best solution under full delegation the principal maximizes (A1) subject to $e_1 = vm_1, e_2 = vm_2$. By inserting the incentive constraints into the objective function we obtain the following problem

$\max_v Z = v(b_1m_1 + b_2m_2) - \frac{1}{2}v^2(m_1^2 + m_2^2) - \frac{r^A}{2}v^2\sigma_y^2 - \frac{r^P}{2}(\sigma_x^2 + v^2\sigma_y^2 - 2\rho v\sigma_x\sigma_y).$

From the first-order condition $\frac{\partial Z}{\partial v} = b_1m_1 + b_2m_2 - v(m_1^2 + m_2^2) - r^Av\sigma_y^2 - r^P\big(v\sigma_y^2 - \rho\sigma_x\sigma_y\big) = 0$ we obtain $v^D = \frac{m_1b_1 + m_2b_2 + \rho r^P\sigma_x\sigma_y}{m_1^2 + m_2^2 + \sigma_y^2(r^P+r^A)}$ with a corresponding principal surplus of

$Z^D = \frac{1}{2}\frac{2b_2m_2(b_1m_1 + r^P\rho\sigma_x\sigma_y) + b_1^2m_1^2 + b_2^2m_2^2 - (m_1^2+m_2^2)r^P\sigma_x^2 + 2m_1b_1r^P\rho\sigma_x\sigma_y - \sigma_x^2\sigma_y^2[(r^P)^2(1-\rho^2) + r^Ar^P]}{m_1^2 + m_2^2 + \sigma_y^2(r^P+r^A)}.$

Proof of Lemma 3:

In the second-best solution under partial delegation of task i the principal maximizes (A1) subject to $e_i = vm_1, e_{-i} = b_{-i} - vm_{-i}$. By inserting the incentive constraints, the principal's objective function obtains as

$\max_v Z = v(b_im_i - b_{-i}m_{-i}) + b_{-i}^2 - \frac{1}{2}v^2m_i^2 - \frac{1}{2}(b_{-i} - vm_{-i})^2 - \frac{r^A}{2}v^2\sigma_y^2 - \frac{r^P}{2}(\sigma_x^2 + v^2\sigma_y^2 - 2\rho v\sigma_x\sigma_y)$.

From the first-order condition $\frac{\partial Z}{\partial v} = b_im_i - b_{-i}m_{-i} - vm_i^2 + m_{-i}(b_{-i} - vm_{-i}) - r^Av\sigma_y^2 - r^P\big(v\sigma_y^2 - \rho\sigma_x\sigma_y\big) = 0$ it follows

$$v_i^* = \frac{m_i b_i + \rho r^P \sigma_x \sigma_y}{m_1^2 + m_2^2 + \sigma_y^2(r^P + r^A)}$$

and $Z_i^* = \frac{1}{2}\frac{b_2^2[m_i^2+\sigma_y^2(r^A+r^P)]+b_i^2 m_i^2+b_{-i}^2 m_{-i}^2-(m_1^2+m_2^2)r^P\sigma_x^2+2m_i b_i r^P \rho\sigma_x\sigma_y-\sigma_x^2\sigma_y^2[(r^P)^2(1-\rho^2)+r^A r^P]}{m_1^2+m_2^2+\sigma_y^2(r^P+r^A)}$.

Proof of Proposition 2:

For $Z_i^* - Z_{-i}^* > 0$ to hold

$$\frac{m_i}{m_{-i}} > \frac{b_i}{b_{-i}} \quad (1A)$$

needs to hold. If the latter holds,

$$\frac{m-_i}{m_i} > \frac{b-_i}{b_i} \quad (2A)$$

will not hold simultaneously.

To see that, note that $\frac{m_i}{m_{-i}} > \frac{b_i}{b_{-i}} \quad \leftrightarrow \quad \frac{b_{-i}}{b_i} > \frac{m_{-i}}{m_i}$ which contradicts (2A).

Proof of Lemma 5:

Note that using Pythagoras' Theorem we can define the distance between first best and second best efforts as follows:

$\Delta = \sqrt{(e_1^{FB} - e_1^{SB})^2 + (e_2^{FB} - e_2^{SB})^2}$.

With partial delegation we obtain

$$\Delta_i = \sqrt{\left(b_i - e_i^{SB}\right)^2 + (b_{-i} - e_{-i}^{SB})^2} \quad (A3)$$

and $e_i^{SB} = vm_i$ and $e_{-i}^{SB} = b_{-i} - vm_{-i}$.

Solving the latter equation for v and inserting into the former we get

$$e_i^{SB}\left(e_{-i}^{SB}\right) = \frac{m_i}{m_{-i}}\left(b_{-i} - e_{-i}^{SB}\right). \quad (A4)$$

Inserting into (A3) results in

$$\Delta_i = \sqrt{\left(b_i - \frac{m_i}{m_{-i}}\left(b_{-i} - e_{-i}^{SB}\right)\right)^2 + (b_{-i} - e_{-i}^{SB})^2}. \quad \text{(A5)}$$

Deriving the first order condition for a minimum and solving for e_{-i}^{SB} we obtain:

$e_{-i}^{SB} = b_{-i} - b_i \frac{m_1 m_2}{m_1^2 + m_2^2}$.

And from (A4)

$$e_i^{SB} = \frac{b_i m_i^2}{m_1^2 + m_2^2}.$$

Inserting this result into (A5) we finally obtain $\Delta_i^* = \frac{b_i m_{-i}}{\sqrt{m_1^2 + m_2^2}}$.

With full delegation we need to minimize

$$\Delta = \sqrt{(b_1 - e_1^{SB})^2 + (b_2 - e_2^{SB})^2} \quad \text{(A6)}$$

with $e_i^{SB} = v m_i$ for $i = 1,2$.

Solving $e_2^{SB} = v m_2$ for v and inserting into $e_1^{SB} = v m_1$ we get

$$e_1^{SB}(e_2^{SB}) = \frac{m_1}{m_2} e_2^{SB}. \quad \text{(A7)}$$

Inserting into (A6) we get

$$\Delta = \sqrt{\left(b_1 - \frac{m_1}{m_{-2}} e_2^{SB}\right)^2 + (b_2 - e_2^{SB})^2}. \quad \text{(A8)}$$

Deriving the first order condition for a minimum and solving for e_2^{SB} we obtain:

$$e_2^{SB} = \frac{b_1 m_1 m_2 + b_2 m_2^2}{m_1^2 + m_2^2}.$$

And from (A7)

$$e_1^{SB} = \frac{b_2 m_1 m_2 + b_1 m_1^2}{m_1^2 + m_2^2}.$$

Inserting this result into (A6) we obtain $\Delta^D = \frac{\sqrt{(b_2 m_1 - b_1 m_2)^2}}{\sqrt{m_1^2 + m_2^2}}$.

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