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On the alignment of lot sizing decisions in a remanufacturing system in the presence of random yield

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Abstract

In the area of reverse logistics, remanufacturing has been proven to be a valuable option for product recovery. In many industries, each step of the products' recovery is carried out in lot sizes which leads to the assumption that for each of the different recovery steps some kind of fixed costs prevail. Furthermore, holding costs can be observed for all recovery states of the returned product. Although several authors study how the different lot sizes in a remanufacturing system shall be determined, they do not consider the specificity of the remanufacturing process itself. Thus, the disassembly operations which are always neglected in former analyses are included in this contribution as a specific recovery step. In addition, the assumption of deterministic yields (number of reworkable components obtained by disassembly) is extended in this work to study the system behavior in a stochastic environment. Three different heuristic approaches are presented for this environment that differ in their degree of sophistication. The least sophisticated method ignores yield randomness and uses the expected yield fraction as certainty equivalent. As a numerical experiment shows, this method already yields fairly good results in most of the investigated problem instances in comparison to the other heuristics which incorporate yield uncertainties. However, there exist instances for which the performance loss between the least and the most sophisticated heuristic amounts to more than 6%.

keywords: reverse logistics, remanufacturing, lot sizing, disassembly, random yield

1 Introduction

The reuse field has grown significantly in the past decades due to its economical benefits and the environmental requirements. Remanufacturing which represents a sophisticated form of reuse focusses on value-added recovery and has been introduced in many different industry sectors such as automotive, telecommunication, electrical equipment, machinery, etc. Within the process of remanufacturing, products that are returned by the customers to the producer are disassembled to obtain functional components. The obtained components are afterwards cleaned and reworked until a "good-as-new" quality is assured. Having met the required quality standards, these components can be used for the assembly of a remanufactured product that is delivered to the customers with the same warranty as a newly produced one. In addition to the economic profitability, as a part of the embedded economic value can be saved by remanufacturing, there is an increasingly legislative restriction that assigns the producers the responsibility for their used products, for instance the Directive 2002/96/EC related to Waste Electrical and Electronic Equipment and the Directive 2002/525/EC related to End of Life Vehicles. Because of that, remanufacturing has become an important industry sector to achieve the goal of sustainable development. Therefore, the management and control of inventory systems that incorporate joint manufacturing and remanufacturing options has received considerable attention in recent literature contributions.

One of the main topics in these contributions is the assessment of joint lot sizing decisions for remanufacturing and manufacturing which has been thoroughly investigated in recent years. One of the first authors who established a basic modelling approach was Schrady [8] who developed a simple heuristic procedure for determining the lot sizes of repair and manufacturing lots. He assumes in his work that a constant and continuous demand for a single product has to be satisfied over an infinite planning horizon. Furthermore, a constant return fraction is established that describes the percentage of used products that return to the producer. By using that assumption a constant and continuous return rate is ensured. Presuming fixed costs for remanufacturing and manufacturing as well as different holding costs for repairable and newly manufactured products, a simple EOQ-type formula is proposed that minimizes the sum of fixed and holding costs per time unit. As a result, an efficient cyclic pattern is established which is characterized by the fact that within each repair cycle a number of repair lots of equal size succeed exactly one manufacturing lot. By solving the proposed EOQ-formula which can be applied because an infinite production and repair rate is presumed as well, the number of repair lots and the length of a repair cycle can be determined. Teunter [10] generalized the results of Schrady in a way that he examined different structures of a repair cycle. His analysis concludes that it is not efficient if more than one repair lot and more than one manufacturing lot are established in the same repair cycle. This result extends the efficient cycle patterns by a cycle in which several manufacturing lots of equal size are followed by exactly one repair lot. The assumption of equal lot sizes is among other aspects critically studied in the contribution of Minner and Lindner [4]. They show that a policy with non-identical lot sizes can outperform a policy with identical lot sizes. However, the structure of an efficient repair cycle prevails also when the assumption of equal lot sizes is lifted.

Next to the analysis of the basic model context several extensions have been proposed that relax some of the assumptions made so far. Teunter [11], for instance, relaxes the assumption of an instantaneous manufacturing and repair process in order to derive more general expressions for the number of manufacturing and repair lots and their corresponding lot sizes. Since only a heuristic procedure was introduced on how to determine these values, Konstantaras and Papachristos [3] extended Teunter's work by developing an algorithm that leads to the optimal policy for certain parameter classes. By incorporating stochastic leadtimes and thereby including the possiblity of backorders, Tang and Grubbstrom [9] extend the basic model. Two general options are recommended on how such a system can be dealt with, a cycle ordering model and a dual sourcing ordering policy. Both approaches are compared in a numerical study that indicates certain parameter specifications under which conditions one approach outperforms the other. Furthermore, several papers have been published by Richter and Dobos (e.g. [5] and [6]) that relax the assumption of a constant rate of return. In their work they derived for several situations that a so called pure strategy is always optimal. In this context, a pure strategy means that either every returned product is repaired or everything is disposed of immediately. Therefore, a mixed strategy in which a part of the returned products is repaired and the rest is disposed of is always dominated by one of the pure strategies. Finally, the assumption of continuous demand and return rates has been relaxed by several authors. Consequently, the formerly EOQ-type model becomes a dynamic lot sizing problem. The contribution of Teunter et al. [12] shall be named representatively for this research area in which they extended well-known dynamic lot sizing heuristics such as the Silver Meal or the Part Period algorithm in order to test their performance in a remanufacturing environment.

Common to all contributions is that they do not consider the remanufacturing process explicitly. Although some authors speak of remanufacturing, they analyze a remanufacturing system in the same way as a repair system. This may lead to wrong conclusions as it is not regarded that the remanufacturing process itself consists of two different subprocesses, a disassembly process in which the returned products are disassembled and a rework process in which the obtained components are brought to an as-good-as-new quality (for a definition see Thierry et al. [14]). By explicitly incorporating both subprocesses in this contribution, the decisions that need to be made regarding disassembly and rework are decoupled which generalizes the basic models used so far.

Next to this generalization, this contribution will relax furthermore the assumption of a deterministic yield. This means that the number of components obtained by disassembly is not known with certainty beforehand. Considering stochastic yields has attained significant interest in the scientific literature as the basic work of Yano and Lee [16] as well as the overview of Grosfeld-Nir and Gerchak [1] present. However, most of the contributions presented in [1] describe purely manufacturing environments which can not be entirely translated to a remanufacturing system as this inherits greater risks to be dealt with [15]. Nevertheless, stochastic yields have also been studied in a remanufacturing environment. Inderfurth and Langella, for instance, have concentrated their analysis specifically on the yield risk within the disassembly process [2]. Yet, they focussed on a multi-product multi-component problem setting in which a given discrete demand for components needs to be satisfied by either disassembling used products or manufacturing new components. The authors develop in their contribution heuristic methods on how to deal with such a problem in which they neglected the presence of fixed costs for the disassembly and the remanufacturing process.

After this short introduction the problem assumptions and the nomenclature used in the remainder of the paper are illustrated in section 2. Section 3 presents thereafter two solution procedures to find the optimal solution in the deterministic yield scenario before section 4 widens the scope to a stochastic yield problem and presents three heuristic approaches that facilitate the decision making process in such an environment. The fifth section conducts a numerical experiment in order to test the heuristics' performance. Finally, a conclusion and an outlook are given in the last section.

2 Problem setting and model formulation

A company engaged in the area of remanufacturing that remanufactures several used products (e.g. engines) coming back from their customers shall be the background for the problem setting. To keep the analysis simple, the focus shall be restricted to only one specific remanufactured product named A. Figure 1 presents the general structure of this simplified system which is modeled as a multi-level inventory system containing three stages. Further simplifications are made regarding the fact that there are neither lead nor processing times. Furthermore, no disposal option is included in the problem setting.

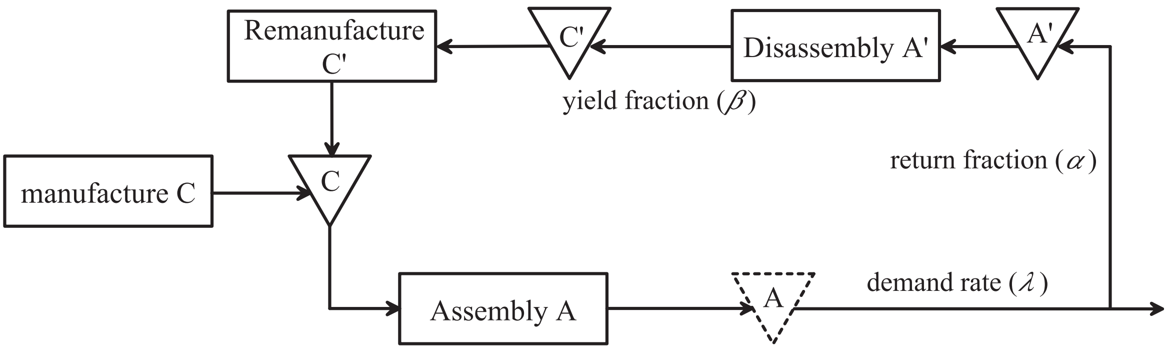

Figure 1: Inventory system in a remanufacturing environment

The customers' demand for the final product A is assumed to be constant and depletes the finished goods inventory continuously by a constant rate of λ units per time unit. In order to satisfy that demand, the company manufactures the final product by using component C which represents the most important component of the product. For the sake of simplicity only the most important component C is included in the analysis. However, the proposed model could be easily extended to a multi-component setting. The assembly process is supposed to be a flow line process at which the final product is assembled continuously and immediately delivered to the customers. When the customers have no further use for their product A (e.g. it is broken or its leasing contract ends) they have the opportunity to return the product to the company. However, only a fraction (named α) of those products in the market returns to the producer. For the subsequent analysis, the return flow of used products (which are denoted A') fills the used product inventory by the constant and continuous rate of $\lambda\alpha$. By disassembling A' the worn component C' is obtained. Although the process of disassembly typically consists of manual work, fixed costs prevail for setting up required disassembly tools and/or measuring devices that allow an improved assessment of the reusability of components before disassembly. Within this model K_d represents the fixed costs for a disassembly batch while h_d is the holding cost incurred for storing one unit of A' for one time unit. Due to different stages of wear, not all returned products contain a reworkable component C'. The ratio of the number of reworkable items obtained from the disassembly of A' to the rate of product returns $\lambda\alpha$ is denoted by β.

Assuming that at most one reworkable component C' can be obtained by disassembling one unit of A' the ratio β must not exceed one while being non-negative. As the released components C' can not be used directly for the assembly of the final product A since they usually do not meet the designated quality standards, these components have to be remanufactured. Since the remanufacturing process incurs fixed costs of K_r for setting up the cleaning and mechanical rework tools, a batching of reworkable components takes place as well. Hence, some reworkable components need to be stored before the next remanufacturing batch is started resulting in costs of h_r per unit and time unit. It is furthermore assumed that every component that is remanufactured is brought to an as-good-as-new condition. All successfully reworked components are held in a serviceables inventory at a cost of h_s per unit and time unit. In order to secure the final product assembly of A, some components of C have to be manufactured in addition (as α and β are usually smaller than one). The decision relevant fixed costs are denoted by K_m representing the cost for setting up a manufacturing lot for component C. Newly manufactured components are held in the same serviceables inventory as remanufactured ones and it is supposed that the holding costs do not differ between both sourcing options. A detailed discussion on the topic on how to set the holding cost parameters can be found in [13]. In general, the holding costs (when interpreted as costs for capital lockup) of all levels are connected by the following inequality since more value is added to the component on each level, i.e. $h_d < h_r < h_s$.

Balancing fixed and holding costs shall be achieved by applying an average cost approach to this model. This is commonly done for one-level inventory systems as for the well known EOQ-model formulation but can be easily extended to a multi-level environment by respecting the stipulated assumptions of the EOQ-model (e.g. infinite planning horizon with constant costs over time). As a result an optimal cyclic pattern is obtained by minimizing the average cost per time unit. In order to control the entire system, three decision variables are required. Firstly, the length of the disassembly cycle T determines the lot size of each disassembly batch ($\lambda\alpha T$) under the assumption that there is only one disassembly lot per cycle. This assumption is made for the sake of simplicity as an additional decision variable (number of disassembly lots per cycle) would complicate the analysis significantly. However, if we consider high fixed costs of disassembly, we conjecture that this assumption of one disassembly lot per cycle assures the optimality of the introduced deterministic policy. Furthermore, by fixing the number of remanufacturing lots R per disassembly cycle, their equal lot size can be computed by $\lambda\alpha\beta T/R$. Finally, the number of manufacturing lots M per disassembly cycle determines the lot sizes of the manufacturing lots to be $\lambda(1-\alpha\beta)T/M$. The subsequent chapter presents the optimal solution of a completely deterministic setting in which all parameters are known with certainty.

3 Deterministic yields

In this section a model is introduced that permits the evaluation of the optimal number of manufacturing and remanufacturing lots in a disassembly cycle. Before expanding the scope to stochastic yields from disassembly which represents the core issue of this contribution, the deterministic setting is studied in order to gain insight into the interrelations of the whole system. Figure 2 illustrates the behavior of the relevant inventory levels for three consecutive disassembly cycles. As a matter of fact, the optimal decision variables (T, R, and M) remain constant over time in a deterministic environment. As shown in the figure below the manufacturing lots are positioned always after the remanufacturing lots in the serviceables inventory. This is obvious as this strategy strictly dominates the strategy of starting a cycle on the serviceables level with a manufacturing lot due to the increased holding costs on the remanufacturables level.

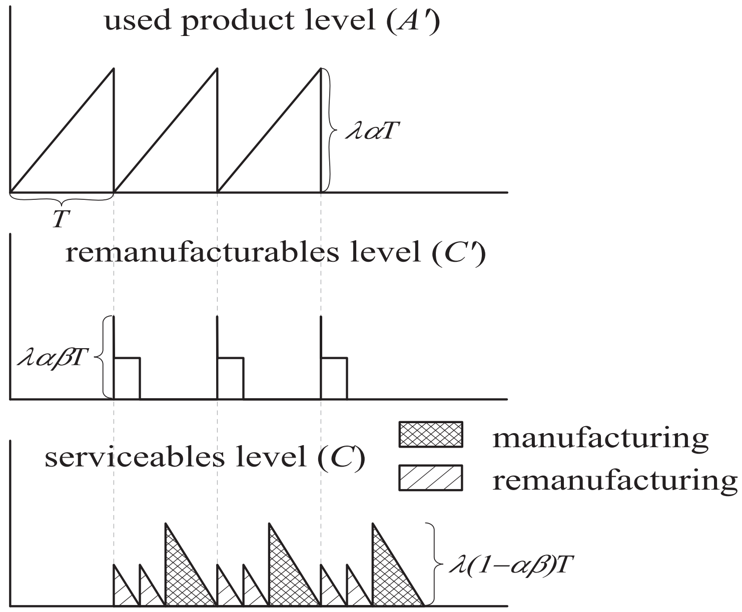

Figure 2: Used product, remanufacturables, and serviceables inventory in a deterministic yield environment (with R=2 and M=1)

By minimizing the total average cost per time unit, this specific example shows for the optimal cycle length T two remanufacturing lots ($R = 2$) which split the remanufacturables inventory inflow equally and one manufacturing lot ($M = 1$) which satisfies the remaining demand of the assembly process for component C. To analyze the total cost function (TC^D) only two main types of costs have to be considered, the fixed costs F^D and the holding cost H^D in which the index D indicates the deterministic setting. A detailed discussion on how this formula can be obtained is presented in appendix A while appendix B discusses the optimality of equal lot sizes in this setting. The total cost function in the deterministic setting can be formulated as follows:

$$TC^D = \frac{F^D}{T} + \frac{\lambda T H^D}{2} \tag{1}$$

with $F^D = K_d + RK_r + MK_m$ and
$H^D = \alpha h_d + \frac{R-1}{R}\alpha^2\beta^2 h_r + (\frac{\alpha^2\beta^2}{R} + \frac{(1-\alpha\beta)^2}{M})h_s$.

In order to minimize the total cost function which is a mixed-integer non-linear optimization problem two procedures can be applied. The first procedure that can be used is a simple enumerative procedure. Since R and M need to be integer valued only a finite number of calculations (in which are R and M are set to an integer value) has to be compared if R and M are restricted to certain intervals. The original objective function simplifies for given values of R and M to a non-linear convex function that only depends on T. Such a problem can be solved easily by using the subsequent equations:

$$T^{D^*} = \sqrt{\frac{2F^D}{\lambda H^D}} \tag{2}$$

$$TC^{D^*} = \sqrt{2\lambda F^D H^D}. \tag{3}$$

The formulas presented above are comparable to the determination of the economic order interval. However, the optimality of this solution approach can only be guaranteed if the optimal total cost TC^{D^*} is determined for every combination of realization of R and M which leads to a quite large number of calculations. Nevertheless, a good solution can be obtained in a fast manner by restricting the number of possible realizations.

After introducing an enumerative procedure another promising approach will be presented next. By relaxating the original objective function (1) such that R and M need not to take on integer values, one can prove that the total cost function has only a single local minimum in the relevant area (for T, R, $M > 0$). Appendix C focusses on this specific aspect. Yet, by evaluating the Hessian matrix in this area, it can be shown that the total cost function is not entirely convex in all variables which is presented in appendix D. This leads to the significant problem that a simple rounding procedure can not be used to obtain the optimal solution for the integer valued number of remanufacturing and manufacturing lots. Therefore, a solution algorithm could be implemented that can globally determine the minimum cost of this mixed-integer non-linear optimization problem. The BARON algorithm, as implemented in the GAMS software package, proved to be a valuable tool for this problem setting. In general, BARON implements deterministic global optimization algorithms of the branch-and-reduce type in order to determine the optimal solution for a mixed-integer non-linear optimization problem. For a detailed description of the algorithm see [7].

The subsequent chapter extends the deterministic model of this section to incorporate stochastic yields.

4 Stochastic yields

One of the main problems for many practical applications in the area of remanufacturing is that they have to deal with stochastic yields which means that the amount of remanufacturable components obtained from disassembling used returned products is not known with certainty (see also [2]). Due to the significance of that problem in a remanufacturing planning environment, we will now put forth the extension of the deterministic model that was introduced in the last section to incorporate stochastic yield fractions resulting from the disassembly process. Although being uncertain, it can be assured that the lowest possible yield fraction β_l can not be smaller than zero as negative yields would not be reasonable. The largest possible yield fraction β_u, however, can not exceed the value of one since this describes the situation that from every disassembled used product more than one remanufacturable component is obtained which is ruled out by the assumptions made. Within the range from β_l to β_u a specific distribution function can be defined which will be denoted in the following analysis by $\varphi(\beta)$. As the number of returned products disassembled per cycle corresponds to $\lambda\alpha T$, the independence of $\varphi(\beta)$ with respect to T reflects the fact that the subsequent analysis assumes stochastic proportional yields (for a definition see [16]). Therefore, the formerly used total cost function for a deterministic yield scenario (formula (1)) has to be extended in order to incorporate any possible yield outcome. Hence, the total cost of a given stochastic yield scenario can only be formulated as an expected total cost (which will be further denoted as TC^S) that is presented in the following equation:

$$TC^S = \frac{F^S}{T} + \frac{\lambda T H^S}{2} \tag{4}$$

with $F^S = K_d + \int_{\beta_l}^{\beta_u} (R(\beta)K_r + M(\beta)K_m) \cdot \varphi(\beta)d\beta$ and

$$H^S = \alpha h_d + h_s \int_{\beta_l}^{\beta_u} \frac{1}{M(\beta)} \cdot \varphi(\beta)d\beta - 2\alpha h_s \int_{\beta_l}^{\beta_u} \frac{1}{M(\beta)} \cdot \beta \cdot \varphi(\beta)d\beta + \alpha^2 \int_{\beta_l}^{\beta_u} (h_r - \frac{h_r}{R(\beta)} + \frac{h_s}{R(\beta)} + \frac{h_s}{M(\beta)}) \cdot \beta^2 \cdot \varphi(\beta)d\beta.$$

The fact that for any possible yield realization β an integer number of R and M has to be defined complicates the analysis of the total cost function TC^S significantly. In this setting, $R(\beta)$ describes the optimal number of remanufacturing lots for a given yield fraction β. Likewise, $M(\beta)$ represents the optimal number of manufacturing lots if the yield fraction β is fixed. Due to the fact that β is not known with certainty the total cost per time unit can only be formulated as an expectation over all different yield realizations. In contrast to the total cost function of the deterministic case (1), F^S and H^S can be regarded as an expectation of their corresponding deterministic equivalents F^D and H^D. As finding the optimal solution for any problem setting can not be guaranteed, which will be shown later in this chapter, three different heuristic policies will be presented in the succeeding paragraphs that differ in their degree of sophistication. The first and least complex policy is introduced in the following:

Policy I

The easiest option on how to handle a stochastic problem is to neglect the underlying stochastics in order to derive a deterministic equivalent of the stochastic problem. The firstly introduced policy proceeds exactly in this manner as it neglects the fact that R and M depend on the yield realization β. Thus, only one value for R and M needs to be derived that is valid for every yield realization between β_l and β_u. To obtain these values, one can insert a specific yield fraction into the deterministic total cost function of the last chapter (1) and apply the recommended solution procedures to obtain R and M. As any yield fraction can be inserted that lies in the range of possible yield realizations and therefore many different combinations of R and M may prevail, we limit the focus of policy I on inserting only the mean yield fraction into the deterministic model since the mean yield is one of the most important characteristics of the underlying yield distribution. As a result we obtain the values of R^D and M^D that replace $R(\beta)$ and $M(\beta)$ for every possible yield realization β in formula (4). The expected total cost of the first policy (TC_I) can therefore be easily calculated by the subsequent equation:

$$TC_I = TC^S(T, R^D, M^D). \tag{5}$$

Since policy I is a very simple approach the decision maker can improve the expected total cost by incorporating the underlying stochastics in the decision making process which is introduced in policy II.

Policy II

Contrary to the first policy, the second policy does not neglect the dependence of R and M on the realization of the random yield fraction β any more. Nevertheless, in order to keep this policy simple, the disassembly cycle length T is kept constant which reduces the policies' complexity significantly. For the sake of simplicity the length of the disassembly cycle T will be set to the optimal deterministic cycle length T^{D^*} obtained by formula (2) assuming that the mean yield fraction has been inserted as the deterministic equivalent for the underlying yield distribution. The assumption of fixing the cycle length to a specific value can be further used to draw some basic conclusions that can only be drawn for a given cycle length. The stochastic yield realization β determines for every disassembly cycle the number of remanufacturable items. As the number of remanufacturable and manufactured components per cycle always adds up to the value of λT, the number of manufactured items depends as well on the yield realization. However, for both options of demand fulfillment it can be observed that if more items are processed (either by manufacturing or remanufacturing) the number of respective lots in a cycle does not decrease. Therefore, when comparing two different yield realizations with all other parameters set equally it can be said: For the larger yield realization the number of remanufacturable components increases which means that the number of remanufacturing lots per cycle does not decrease. On the other hand, the number of newly manufactured components decreases with larger yield realizations which means that the number of manufacturing lots per cycle does not increase. The figures 3 and 4 compare both heuristic policies introduced so far for three consecutive disassembly cycles. On the left hand side (figure 3), it can be observed for policy I that regardless of the realized yield fraction the same number of R and M is applied in every cycle (R=2 and M=1). Figure 4 on the right hand side shows policy II that reacts for the same cycle length T on the different realizations of β which is supported by the fact that for a small yield realization the number of remanufacturing lots is smaller than for a large yield realization (R=1 in the first cycle compared to R=3 in the third cycle). An opposing behavior can be observed for the number of manufacturing lots per cycle that does not increase as smaller the yield realization is.

These general conclusions can not only be formulated verbally but also in a mathematical form by introducing so-called transition yield fractions which have the property that either the number of remanufacturing lots or the number of manufacturing lots changes when optimizing the deterministic equivalent problem. For the calculation of the specific yield fraction that is characterized by a switch of the optimal policy from R to R+1 remanufacturing lots, one needs to equate the deterministic total cost functions for R and R+1 as presented in the following equation:

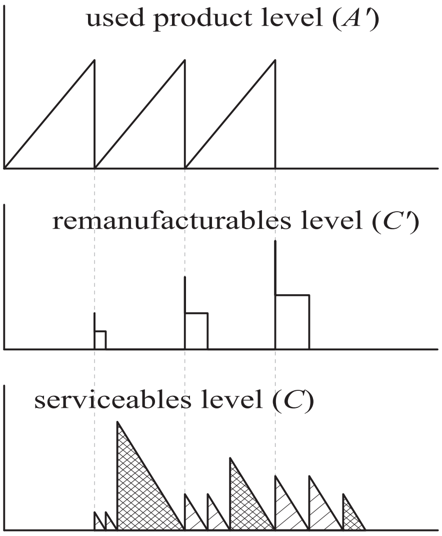

Figure 3: Inventory system in a stochastic yield environment applying policy I (R=2 and M=1)

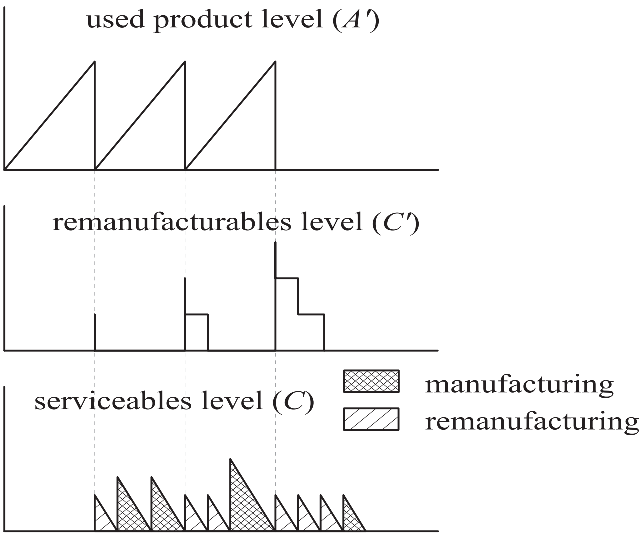

Figure 4: Inventory system in a stochastic yield environment applying policy II

$$\frac{F^D(R)}{T} + \frac{\lambda T H^D(R)}{2} = \frac{F^D(R+1)}{T} + \frac{\lambda T H^D(R+1)}{2}$$

with $F^D(R) = K_d + RK_r + MK_m$ and
$H^D(R) = \alpha h_d + \frac{R-1}{R}\alpha^2\beta^2 h_r + (\frac{\alpha^2\beta^2}{R} + \frac{(1-\alpha\beta)^2}{M})h_s$.

This equation can be solved with respect to β in order to obtain the transition yield fraction $\beta(R)$ at which the optimal decision in the deterministic case switches from R to R+1 for a given cycle length T:

$$\beta(R) = \frac{1}{\alpha T} \cdot \sqrt{\frac{2K_r R(R+1)}{\lambda(h_s - h_r)}}. \quad (6)$$

Not only is this function monotonously increasing in R which corresponds to the findings above that the number of remanufacturing lots does not decrease for larger values of β but it also does not depend on the number of manufacturing lots per cycle M. Thus, the same analysis can be carried out independently for the transition from M to M-1 manufacturing lots per cycle by equating both total cost functions in order to obtain the transition yield fraction $\beta(M)$:

$$\beta(M) = \frac{1}{\alpha} - \frac{1}{\alpha T} \cdot \sqrt{\frac{2K_m M(M-1)}{\lambda h_s}}. \quad (7)$$

Because this function is monotonously decreasing in M, the insight that a larger yield fraction does not lead to less manufacturing lots in a cycle is approved. Consequently, the lowest and highest values for R and M can be determined by exploiting the two formulas given above. Thus, for the lowest possible yield fraction β_l R_{min} and

M_{max} can be computed by the following procedure (analogously R_{max} and M_{min} can be computed for the highest possible yield fraction β_u):

$$R_{min} = \min_{R}\{\beta(R) \geq \beta_l\} \qquad\qquad M_{max} = \max_{M}\{\beta(M) \geq \beta_l\} \tag{8}$$

As the disassembly cycle length is fixed to a given value, the distribution function of the stochastic yield fraction can be subdivided into several intervals j being defined in the interval $l_j \leq \beta \leq u_j$. The main characteristic of such an interval is the fact that within this interval only one number of remanufacturing and manufacturing lots induces the optimal solution for any possible yield fraction within this interval. The optimal number of remanufacturing and manufacturing lots in a certain interval j are furthermore denoted by R_j and M_j, respectively. For the identification of the respective interval bounds the following pseudocode can be used:

start $j = 1$, $l_j = \beta_l$, $R_j = R_{min}$, $M_j = M_{max}$, $\beta(0) = \infty$

while $min\{\beta(R_j + 1), \beta(M_j - 1)\} < \beta_u$ **do**

  if $\beta(R_j + 1) < \beta(M_j - 1)$ **then**

    $u_j = \beta(R_j + 1)$

    $j = j + 1$, $l_j = u_{j-1}$, $R_j = R_{j-1} + 1$, $M_j = M_{j-1}$

  else

    $u_j = \beta(M_j - 1)$

    $j = j + 1$, $l_j = u_{j-1}$, $R_j = R_{j-1}$, $M_j = M_{j-1} - 1$

  end if

end do

$u_j = \beta_u$, J = j

end

After the initialization in which the first interval j=1 is opened (l_1=β_l) and given the values R_{min} and M_{max} the procedure evaluates if the transition to R_{min}+1 or M_{max}-1 is closer to β_l. For the lower one of these two values, the upper bound of the first interval u_1 is fixed to the transition rate and the next interval is opened ($l_2 = u_1$). This procedure stops when both next transitions to R+1 and M-1 are larger than the highest possible yield fraction β_u. At this point the total number of intervals into which the yield distribution can be separated is determined by the index j which is set to the number of intervals J. As a result the total yield distribution is separated into several intervals which is depicted for an example in figure 5. In this example (with $\beta_l = 0$ and $\beta_u = 1$) it can be observed that the solution of policy I would have been R=3 and M=4 as this would solve the deterministic equivalent to optimality for β=0.5 .

As the interval bounds vary with a changing disassembly cycle length T, the expected total cost function for policy II can be formulated as follows using the optimal

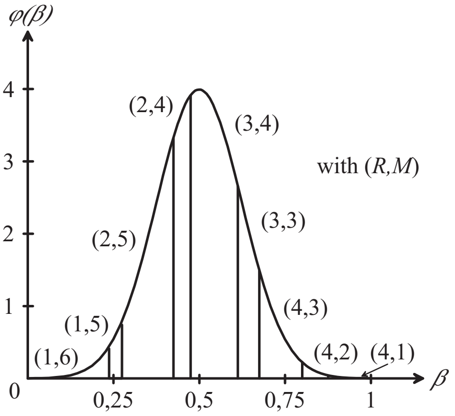

Figure 5: Exemplary separation of a yield distribution according to policy II

disassembly cycle length T^{D*} obtained by inserting the mean yield fraction into equation (3):

$$TC_{II} = \frac{F^S}{T^{D^*}} + \frac{\lambda T^{D^*} H^S}{2} \tag{9}$$

with $F^S = K_d + \sum_{j\in J}(R_jK_r + M_jK_m) \cdot \int_{l_j}^{u_j} \varphi(\beta)d\beta$ and

$$H^S = \alpha h_d + h_s \sum_{j\in J} \tfrac{1}{M_j} \cdot \int_{l_j}^{u_j} \varphi(\beta)d\beta - 2\alpha h_s \sum_{j\in J} \tfrac{1}{M_j} \cdot \int_{l_j}^{u_j} \beta \cdot \varphi(\beta)d\beta + \alpha^2 \sum_{j\in J}(h_r - \tfrac{h_r}{R_j} + \tfrac{h_s}{R_j} + \tfrac{h_s}{M_j}) \cdot \int_{l_j}^{u_j} \beta^2 \cdot \varphi(\beta)d\beta.$$

In comparison to formula (4) only a finite number of R and M has to be considered in order to determine the solution using policy II. The formerly required $R(\beta)$ which represents the optimal number of remanufacturing lots for any given yield fraction β can be replaced with R_j after separating the yield distribution into intervals in which only one R is optimal for each yield realization. Consequently, the same simplification holds for the number of manufacturing lots M as well. However, this solution can be further improved by varying the disassembly cycle length T which shall be done in the most sophisticated heuristic approach of this contribution.

Policy III

As the convexity of the expected total cost function of policy II (9) regarding the only remaining variable T can not be proven for any possible yield distribution we face the fact that obtaining the optimal solution for this system can not be guaranteed. Nevertheless, a simple local search heuristic can be implemented that alters the disassembly cycle length T from its initial value of policy II in order to check whether the expected total cost increases or decreases. The expected total cost function is evaluated by applying the procedure of policy II for any chosen parameter T. The local search procedure stops when both an increase or a decrease of T results in an increasing expected total cost meaning that at least a local minimum has been found that improves the solution of policy II at the expense of an increased complexity. Two heuristic approaches on how a lower and an upper bound for T can be obtained are presented in appendix E.

The following chapter elaborates a numerical experiment in which all three introduced heuristic approaches are tested in order to evaluate their performance in a stochastic yield environment.

5 Numerical experiment

The main objective of the numerical experiment conducted in this section is to evaluate the error that can be made when the simplest approach (policy I) is used compared to the more complex ones (policies II and III). In order to estimate the error several numerical tests have been conducted using randomly generated instances. All parameters were drawn from a discrete uniform distribution $DU(a, b)$ with a as the lower bound and b representing the upper bound of the distribution. Some parameters were multiplied after the random draw with a constant term in order to obtain reasonable values. Table 1 lists all parameters that were randomly drawn in this experiment:

Table 1: Parameters generated randomly in numerical experiment

Parameter	Generation method
Demand rate	$\lambda \sim DU(1, 10) \cdot 100$
Return fraction	$\alpha \sim DU(6, 18) \cdot 0.05$
Fixed cost for disassembly	$K_d \sim DU(0, 50)$
Fixed cost for remanufacturing	$K_r \sim DU(1, 100)$
Fixed cost for manufacturing	$K_m \sim DU(1, 100)$
Holding cost for used product	$h_d \sim DU(1, 10) \cdot 0.01$
Holding cost for remanufacturable component	$h_r \sim DU(5, 15) \cdot 0.01$
Holding cost for serviceable component	$h_s \sim DU(10, 20) \cdot 0.01$

The return fraction α, for instance, can take on values between 30 % and 90 %, only limited by the fact that the percentage must be an integer multiple of 5 %. Regarding the fixed costs, we restricted the possible region on integer values between 0 and 50 for the disassembly process and 1 to 100 for setting up a remanufacturing or a manufacturing lot. For the disassembly lot we established smaller values as these processes are done manually in some industrial applications and do not necessarily require a specific setup time. With respect to the holding costs we implicitly assumed that the holding cost increase from level to level as more effort has been put into the components. This means that every randomly generated instance has to fulfill the presumed inequality $h_d < h_r < h_s$. From these probability distributions 1,000 instances were drawn and tested for different yield distributions. Generally, the yield distribution followed a symmetric beta-distribution within the limits β_l=0 and β_u=1. The parameter that altered the yield distribution was the coefficient of variation ρ that was changed in the limits between 0.05 and 0.55 which is motivated by our experience with an automotive remanufacturer regarding its yield fractions. While a coefficient of variation of 0.05 indicates that almost the entire probability mass is centered around the distribution's mean, a coefficient of variation of 0.55 indicates for a beta-distribution within the interval 0 to 1 an approximately uniform yield distribution.

The three introduced heuristic approaches were tested for all instances. Figure 6 illustrates, for instance, the percentage deviation of the total costs of policies I and II. $\Delta_{I->II}$ denotes this percentage deviation and is calculated by $\Delta_{I->II}=TC_I/TC_{II}$-1. In detail, this deviation shows the percentage loss in performance if policy I (at which only the mean yield fraction is considered) is applied instead of policy II. The deviation with respect to the coefficient of variation of the underlying yield distribution is presented with the aid of box plots that do not only show the maximum and minimum deviation but also where half of the deviations are located inside the shaded area around the median.

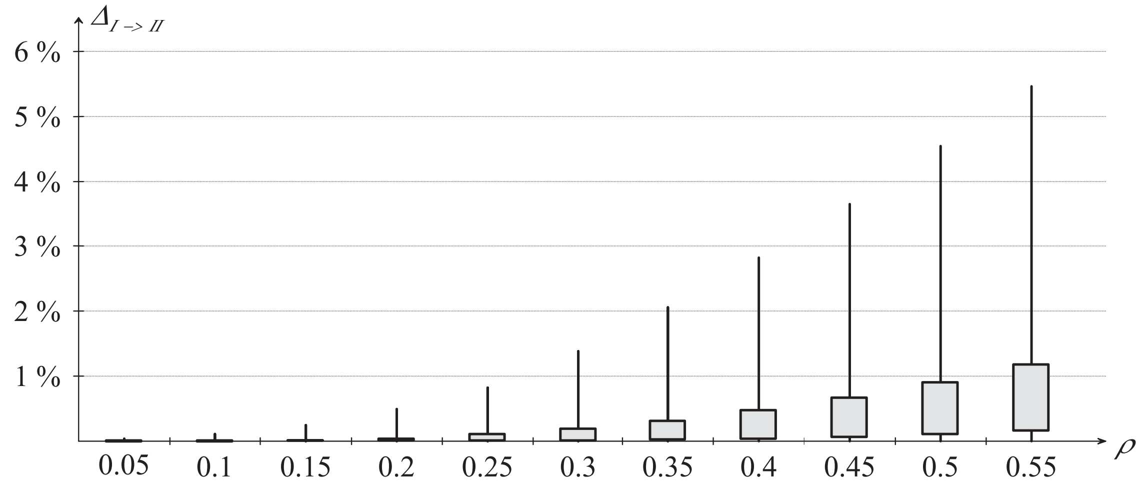

Figure 6: Percentage deviation of policy I compared to policy II

For very small coefficients of variation that are characterized by the fact that almost the entire probability mass is centered around the mean, the deviation between policy I and policy II is almost negligible. The reason for that is easy to be found. Although the yield distribution is defined in the interval between 0 and 1, the range of realizations that have a significant probability is very small. If the optimal number of remanufacturing and manufacturing lots per cycle that is determined by policy I is also optimal for a wide range of yield fractions around the distribution's mean both policies arrive at the same result. However, if the coefficient of variation grows larger the deviations increase as well. For an approximately uniformly distributed yield, for instance, the maximum deviation between policy I and II is around 5.4 %. On the other hand, the minimum deviation is 0 % which means that the optimal cycle pattern of policy I is still optimal for every yield realization between 0 and 1 even for such a widespread distribution. Although many instances have been tested, the effect of every parameter on the deviation can not be observed without doubt. Yet, some general trends can be derived from the experiments. For instance, it seems to be the case that the percentage gap in the total cost between policy I and II increases in most scenarios for instances with an increasing return rate α. Additionally, the different fixed costs seem to influence this gap as well. For high fixed costs for disassembly and remanufacturing (K_d and K_r) as well as for small fixed costs for manufacturing (K_m) the observed percentage gap increases for a large coefficient of variation of the yield distribution. The same analysis can be conducted for the different holding cost parameters, too. The percentage gap between policy I and II increases if the holding costs h_d, h_r, and h_s deviate significantly. Furthermore, it can be said as larger the difference between R_{min} and R_{max} as well as the difference between M_{min} and M_{max} is as larger is the percentage gap. Finally, no considerable influence on the percentage gap can be observed for the demand per time unit λ.

Figure 7 presents the deviation of policy II from policy III which means that the cycle length T is variied in order to decrease the total cost function. By $\Delta_{II->III}$ this deviation is represented. Regarding the coefficients of variation the same can be

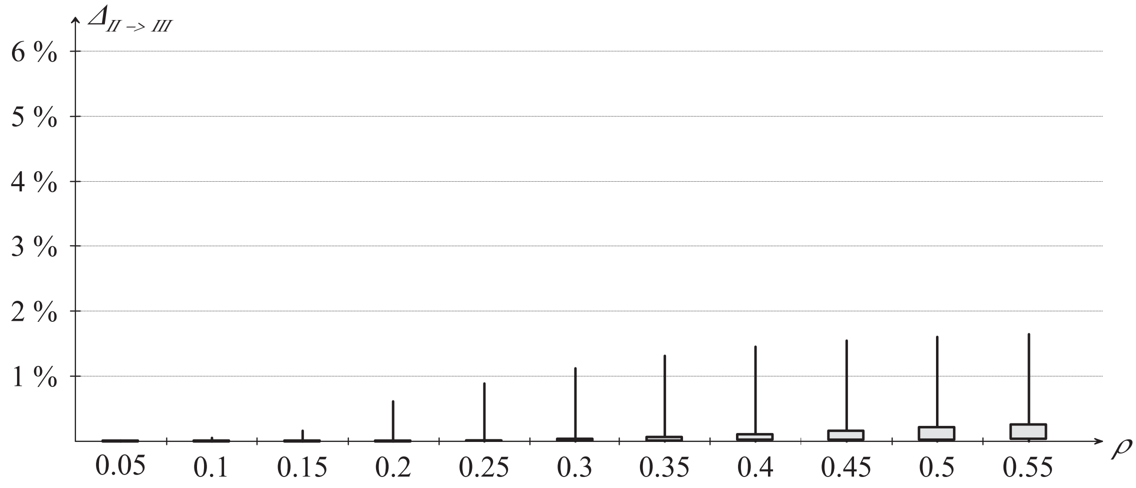

Figure 7: Percentage deviation of policy II compared to policy III

observed as for the first examined deviation. For small coefficients of variation there is almost no improvement possible by changing the cycle length. On the other hand, for larger coefficients the percentage gap grows larger which means that an adaption of T can improve the total cost function. However, these improvements are relatively small (in 97.4 % of all cases smaller than 1 % for ρ=0.55). Regarding the cost deviation between policy II and III, it is even more difficult (in comparison to the deviation between policy I and II)to define parameter areas at which the deviation is typically high or low. Yet, two general trends can be noticed. The largest deviations can be observed for instances with a large α and a wide spread of the holding cost levels. However, this observation can not be generalized for all instances with this parameter constellation.

Another interesting question that can be analyzed with this numerical experiment is whether the optimal cycle length increases or decreases in comparison to the cycle length of policy I and II that remains constant for all coefficients of variation. In 69.1% of all instances the cycle length decreased while it increased in the remaining 31.9%. Therefore, no general conclusion can be drawn regarding this aspect as no specific parameter constellation can be identified that increases or decreases the cycle length in general.

6 Conclusion and outlook

This contribution outlined an approach on how to handle deterministic and stochastic yield fractions within a multi-level remanufacturing system that considers the disassembly process explicitly. While being restricted to a single disassembly lot per cycle, simple derivations are made with respect to the three necessary parameters, the optimal disassembly cycle length as well as the optimal number of remanufacturing and manufacturing lots per disassembly cycle. By examining both the stochastic and the deterministic case, the error that can be made by neglecting the underlying stochastics is evaluated. The numerical experiment in section 5 has confirmed a quite straightforward assumption. The less variability of the random yield fraction is faced, the smaller is the error that is made by using the mean yield policy I instead of the more sophisticated ones. However, there exist situations in which using the simple policy I results in performance losses of more than 5 %. Nevertheless, in most cases the decision maker will obtain fairly good results if he neglects the underlying yield distribution and follows the deterministic mean yield fraction approach of policy I.

Finally, an outlook regarding future research efforts shall be given. The proposed model can be extended in several ways. For both the stochastic and the deterministic one, the option of allowing more than one disassembly lot per disassembly cycle is a promising extension of the model presented in this contribution. Especially for instances showing a small fixed cost of disassembly this might provide a valuable option to decrease the average costs per time unit. Furthermore, it can be studied how a multi-product multi-component setting affects the decision making process in both environments since aspects like multiplicity (one component can be obtained by the disassembly of different product types) have to be incorporated. Another interesting topic that can be included in the analysis is a disposal option. This might be a worthwhile option if the fixed costs of remanufacturing is quite high and the yield realization is very small. In the proposed model context, at least one remanufacturing has to be set up in such a disassembly cycle. However, if there is a disposal option, the obtained components can be disposed of and the total customer demand will be satisfied by newly manufactured components, i.e. the optimal number of remanufacturing lots R can be 0. As a last extension, all heuristic approaches can be tested not only for proportional stochastic yields but also for non-proportional yields. In order to achieve this objective, the yield fraction distribution can not be modeled as a beta-distribution any more but needs to be modeled for instance with a binomial distribution.

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7 Appendix

A Calculation of TC^D

The total cost function TC^D minimizes the sum of all necessary fixed and holding costs per time unit which is optimal in the model setting presented in chapter 2. The decisions that need to be made in order to calculate the cost minimum consist of determining the disassembly cycle length T as well as the number of remanufacturing and manufacturing lots per disassembly cycle R and M, respectively. All three decision variables depend on both the fixed and the holding costs. The total cost function TC^D is presented subsequently as it has been formulated in chapter 3:

$$TC^D = \frac{F^D}{T} + \frac{\lambda T H^D}{2}$$

with $F^D = K_d + RK_r + MK_m$ and
$H^D = \alpha h_d + \frac{R-1}{R}\alpha^2\beta^2 h_r + (\frac{\alpha^2\beta^2}{R} + \frac{(1-\alpha\beta)^2}{M})h_s$.

The fixed cost term F^D contains all relevant fixed costs multiplied with the number of respective lots that are set up in a disassembly cycle. As defined in the model setting, only one disassembly lot is allowed per cycle which leads to $1 \cdot K_d$. However, the number of remanufacturing lots R and manufacturing lots M has to be determined. Consequently, the fixed costs for remanufacturing in a disassembly cycle are represented by $R \cdot K_r$ and the fixed costs for manufacturing in a disassembly cycle by $M \cdot K_m$. Afterwards, the sum of all fixed costs F^D needs to be divided by T in order to determine the fixed costs per time unit.

Regarding the holding costs, the three different stock levels shall be analyzed seperately. Beginning with the used product level, one disassembly cycle is presented in figure 8. As only one disassembly lot is allowed per cycle and the disassembly cycle length is a decision variable, this lot has the size of $\lambda\alpha T$.

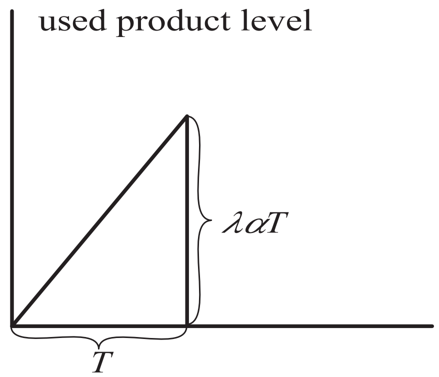

Figure 8: Used product level for one disassembly cycle

The holding costs at the used product level per disassembly cycle can be determined as:

$$\frac{1}{2}T \cdot \lambda\alpha T \cdot h_d$$

As the holding costs at the used product level need to be calculated per time unit, the formula simplifies to the following equation which represents the first part of H^D:

$$\frac{T \cdot \lambda\alpha T \cdot h_d}{2 \cdot T} = \frac{\lambda T}{2} \cdot \alpha h_d$$

After analyzing the holding costs at the used product level, it shall be focussed on the holding costs at the remanufacturables level in the following. Assuming equal remanufacturing lot sizes which is proven to be optimal in appendix B, each remanufacturing lot has the size of $\frac{\lambda\alpha\beta T}{R}$. By setting up such a remanufacturing lot one obtains $\frac{\lambda\alpha\beta T}{R}$ serviceable components which satisfy the customer demand for $\frac{\alpha\beta T}{R}$ periods. Subsequently, another remanufacturing lot or a manufacturing lot has to be set up since no backlogs are allowed in the model. As the number of remanufacturing lots per cycle R is a decision variable, the remanufacturables inventory depends on the value of R. This dependency is visualized in figures 9 and 10. While on the left hand side the remanufacturables inventory is displayed for $R = 2$, the right hand side presents the inventory for $R = 3$.

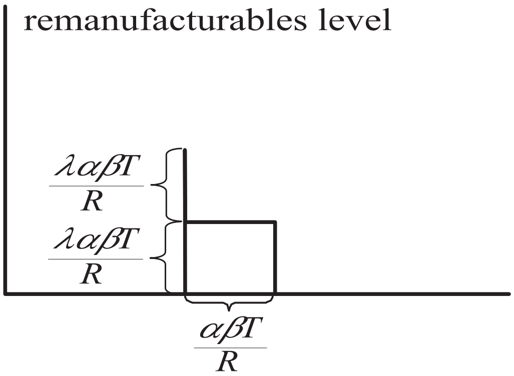

Figure 9: Remanufacturables inventory for R=2

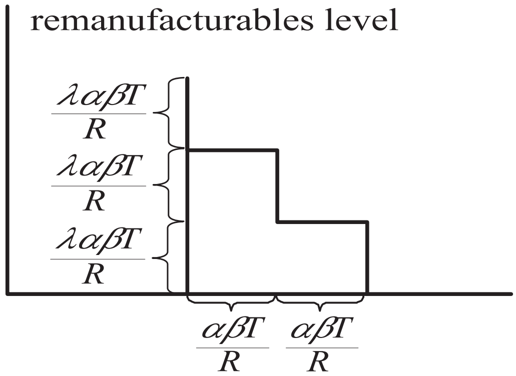

Figure 10: Remanufacturables inventory for R=3

Depending on the number of remanufacturing lots R, the remanufacturables inventory area that is used to calculate the inventory cost can be subdivided into several equally sized rectangles. For $R = 2$ as presented in figure 9 only one rectangle of the size $\frac{\lambda\alpha^2\beta^2T^2}{R^2}$ needs to be evaluated. If $R = 3$ on the other hand, three rectangles have to be considered. In general, the number of equally sized rectangles that has to be evaluated can be formulated as $\frac{R(R-1)}{2}$. Thus, the holding costs for the remanufacturables level per time unit can be formulated as:

$$\frac{R(R-1)}{2} \cdot \frac{\lambda\alpha^2\beta^2T^2}{R^2} \cdot h_r \cdot \frac{1}{T} = \frac{\lambda T}{2} \cdot \frac{R-1}{R}\alpha^2\beta^2 h_r$$

Finally, the holding costs at the serviceables level need to be evaluated. Following the R remanufacturing lots, M equally sized manufacturing lots are set up at this level. While satisfying $\alpha\beta$ % of the customer demand by remanufacturing, $(1 - \alpha\beta)$ % of this demand has to be satisfied by manufacturing new components. Hence, each manufacturing lot has the size of $\frac{\lambda(1-\alpha\beta)T}{M}$ units and lasts for $\frac{(1-\alpha\beta)T}{M}$ time units. Figure 11 presents the serviceables inventory for a disassembly cycle with two remanufacturing and two manufacturing lots.

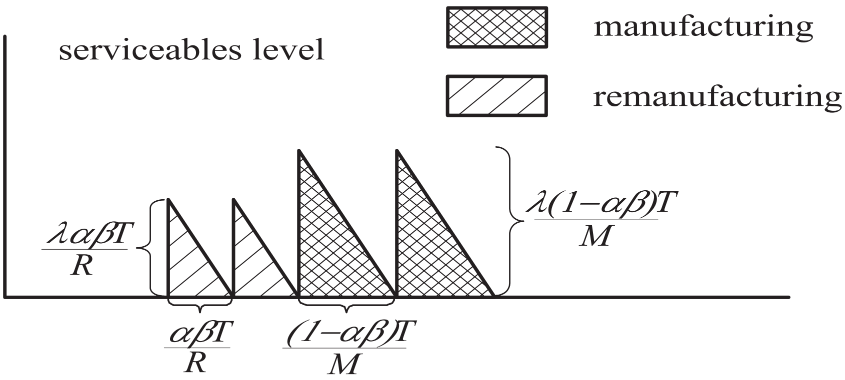

Figure 11: Serviceables level for $R = 2$ and $M = 2$

The holding costs for the serviceables level per time unit can be formulated as follows:

$$(R \cdot \frac{1}{2} \cdot \frac{\lambda\alpha^2\beta^2T^2}{R^2} \cdot h_s + M \cdot \frac{1}{2} \cdot \frac{\lambda(1-\alpha\beta)^2T^2}{M^2} \cdot h_s) \cdot \frac{1}{T} = \frac{\lambda T}{2} \cdot (\frac{\alpha^2\beta^2}{R} + \frac{(1-\alpha\beta)^2}{M})h_s.$$

By determining the holding costs of the serviceables level the holding cost term H^D is completed and can be used to derive the total cost function TC^D.

B Optimality of equal lot sizes in a disassembly cycle

B.1 Optimality of equal remanufacturing lots

For a given cycle length T the total number of remanufactured components is given in the deterministic setting by $\lambda\alpha\beta T$. As a decision variable, the number of remanufacturing lots needs to be determined. Let q_i denote the lot size of remanufacturing lot i with $i = 1, .., R$. If all lot sizes are equal each q_i contains $\frac{\lambda\alpha\beta T}{R}$ units. This analysis shall prove that unequal remanufacturing lot sizes result in higher total cost than equal ones. Therefore, the remanufacturing lot sizes in a more general setting shall be described as:

$$q_i = \frac{\lambda\alpha\beta T}{R} + \Delta_i \qquad \forall i = 1, .., R-1 \tag{10}$$

$$q_R = \frac{\lambda\alpha\beta T}{R} - \sum_{i=1}^{R-1} \Delta_i \tag{11}$$

The distortion from the equal lot sizes which is denoted for each remanufacturing lot by Δ_i lies within the range of $-\frac{\lambda\alpha\beta T}{R} \leq \Delta_i \leq \frac{(R-1)\cdot\lambda\alpha\beta T}{R}$ as all lot sizes have to be non-negative and cannot exceed the value of $\lambda\alpha\beta T$. In formula (11), the lot size q_R has been simplified using the fact that the sum of all distortions has to be zero, i.e. $\sum_{i=1}^{R} \Delta_i = 0$. As an illustration figure 12 presents the remanufacturables and the serviceables inventory for three equally sized remanufacturing lots.

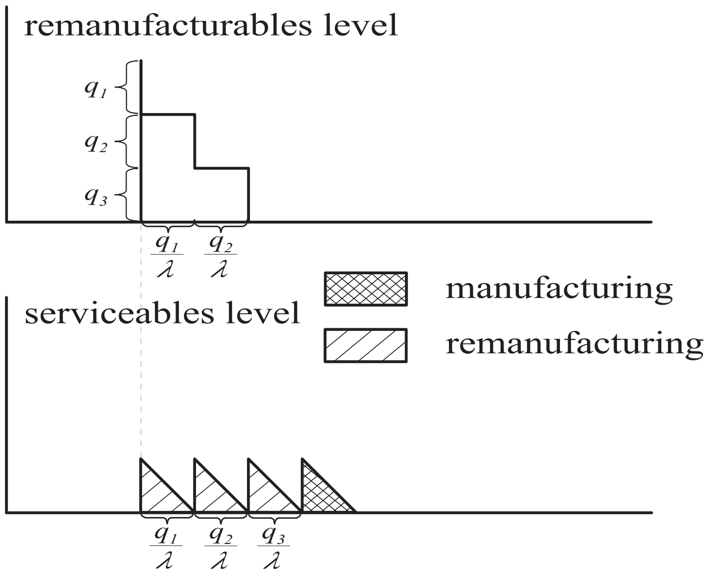

Figure 12: Remanufacturables and serviceables inventory with three equally sized remanufacturing lots

As the remanufacturing lot sizes affect the remanufacturables as well as the serviceables holding costs, these holding costs shall be analyzed subsequently. Starting with the remanufacturables level, let HC_{rem} denote the holding costs per time unit for the remanufacturables inventory. It can be determined by:

$$HC_{rem} = \frac{h_r}{\lambda T} \cdot (\sum_{i=1}^{R-1} q_i \cdot \sum_{j=i+1}^{R} q_j).$$

By $(\sum_{i=1}^{R-1} q_i \cdot \sum_{j=i+1}^{R} q_j)/\lambda$ the total remanufacturables inventory per disassembly cycle is determined. While the first sum in this formula represents the width of the rectangles the second sum represents the corresponding heights. Using this formula for the example in figure 12 leads to $q_1 \cdot (q_2 + q_3) + q_2 \cdot q_3$. This expression can be simplified using equations (10) and (11) and replacing the term $\lambda\alpha\beta T/R$ by X:

$$\begin{aligned} HC_{rem} &= \frac{h_r}{\lambda T} \cdot \sum_{i=1}^{R-1} q_i \cdot \left[\sum_{j=i+1}^{R-1} q_j + q_R \right] \\ &= \frac{h_r}{\lambda T} \cdot \sum_{i=1}^{R-1} (X + \Delta_i) \cdot \left[\sum_{j=i+1}^{R-1} (X + \Delta_j) + X - \sum_{j=1}^{R-1} \Delta_j \right] \\ &= \frac{h_r}{\lambda T} \cdot \sum_{i=1}^{R-1} (X + \Delta_i) \cdot \left[(R - i) \cdot X - \sum_{j=1}^{i} \Delta_j \right] \qquad (12) \\ &= \frac{h_r}{\lambda T} \cdot \left[X^2 \cdot \sum_{i=1}^{R-1} (R - i) + X \cdot \sum_{i=1}^{R-1} \Delta_i \cdot (R - i) - X \cdot \sum_{i=1}^{R-1} \sum_{j=1}^{i} \Delta_j - \sum_{i=1}^{R-1} \Delta_i \cdot \sum_{j=1}^{i} \Delta_j \right]. \end{aligned}$$

It can be shown that $X \cdot \sum_{i=1}^{R-1} \Delta_i \cdot (R - i) - X \cdot \sum_{i=1}^{R-1} \sum_{j=1}^{i} \Delta_j$ equals to 0. Furthermore, the sum $\sum_{i=1}^{R-1} (R - i)$ can be simplified to $\sum_{i=1}^{R-1} i$. Consequently, the holding costs in the remanufacturables inventory can be formulated as:

$$HC_{rem} = \frac{h_r}{\lambda T} \cdot \left[X^2 \cdot \sum_{i=1}^{R-1} i - \sum_{i=1}^{R-1} \Delta_i \cdot \sum_{j=1}^{i} \Delta_j \right]. \tag{13}$$

Let HC_{rem}^0 denote the holding cost of the remanufacturables level with equally sized remanufacturing lots, i.e. $\Delta_i = 0 \quad \forall i = 1, .., R$. Therefore, the difference in holding costs for the remanufacturables level if the remanufacturing lots are not equally sized can be expressed by the difference between HC_{rem} and HC_{rem}^0. This difference can be formulated depending on the distortions Δ_i:

$$\begin{aligned} HC_{rem} - HC_{rem}^0 &= \frac{h_r}{\lambda T} \cdot \left[X^2 \cdot \sum_{i=1}^{R-1} i - \sum_{i=1}^{R-1} \Delta_i \cdot \sum_{j=1}^{i} \Delta_j - X^2 \cdot \sum_{i=1}^{R-1} i \right] \\ &= -\frac{h_r}{\lambda T} \cdot \sum_{i=1}^{R-1} \Delta_i \cdot \sum_{j=1}^{i} \Delta_j. \end{aligned} \tag{14}$$

It can be shown that this term is strictly negative if at least one Δ_i is not zero. Thus, the holding cost for the remanufacturables level always decrease whenever the remanufacturing lots are not equally sized. The analysis shall now be put forth for the serviceables level. Let HC_{serv} denote the holding cost for the serviceables inventory without considering the holding costs for manufacturing lots as these costs do not depend on changes of the remanufacturing lot sizes.

$$\begin{aligned} HC_{serv} &= \frac{h_s}{2\lambda T} \cdot \left[q_R^2 + \sum_{i=1}^{R-1} q_i^2 \right] \\ &= \frac{h_s}{2\lambda T} \cdot \left[(X - \sum_{i=1}^{R-1} \Delta_i)^2 + \sum_{i=1}^{R-1} (X + \Delta_i)^2 \right] \\ &= \frac{h_s}{2\lambda T} \cdot \left[X^2 - 2X \sum_{i=1}^{R-1} \Delta_i + 2 \sum_{i=1}^{R-1} \Delta_i \sum_{j=1}^{i} \Delta_j - \sum_{i=1}^{R-1} \Delta_i^2 + \sum_{i=1}^{R-1} (X^2 + 2X\Delta_i + \Delta_i^2) \right] \\ &= \frac{h_s}{2\lambda T} \cdot \left[R \cdot X^2 + 2 \sum_{i=1}^{R-1} \Delta_i \sum_{j=1}^{i} \Delta_j \right]. \end{aligned} \tag{15}$$

Consistent to the analysis of the remanufacturables level, HC_{serv}^0 denotes the holding cost of the serviceables inventory if all remanufacturing lots are equally sized. Hence, the cost effect of a possible distortion can be calculated by the difference of $HC_{serv} - HC_{serv}^0$ which is presented in the following formula:

$$\begin{aligned} HC_{serv} - HC_{serv}^0 &= \frac{h_s}{2\lambda T} \cdot \left[R \cdot X^2 + 2 \sum_{i=1}^{R-1} \Delta_i \sum_{j=1}^{i} \Delta_j - R \cdot X^2 \right] \\ &= \frac{h_s}{\lambda T} \cdot \sum_{i=1}^{R-1} \Delta_i \sum_{j=1}^{i} \Delta_j. \end{aligned} \tag{16}$$

This term proves that whenever the remanufacturing lots are not equally sized (which means that at least one Δ_i is not zero), the holding cost of the serviceables inventory increase. As the increase of holding cost at the serviceables level is always larger than the decrease of the holding cost at the remanufacturables level because of $h_s > h_r$ the total cost increase in any situation that is characterized by the fact that all remanufacturing lots in a disassembly cycle are not of equal size. This means as a conclusion that it is optimal to choose equal remanufacturing lot sizes in a disassembly cycle in this problem setting.

The differences to the results that were derived in the work of Minner and Lindner [4] can be explained twofold. On the one hand their problem setting does not consider that the recovery process can be separated into a disassembly and a remanufacturing process. Because of that, they compare a two stage inventory system with a continuous inflow of old products to the first stage and a continuous outflow from the second one. In our paper's problem setting the remanufacturing stage with discrete inflows is compared to the serviceables inventory with a continuous outflow. This fact can be used to explain the different results because the basic flow pattern of the analysis has changed. On the other hand a cycle is defined differently in the analysis of Minner and Lindner and in our work. While the former includes the possibility that more than one recovery lot can be set up on the first stage of the system (the one with a continuous inflow), the latter only allows for one disassembly lot per cycle. The results derived above that only equally sized remanufacturing lots are optimal depend on this assumption since different disassembly lot sizes would mean that the size of all remanufacturing lots can not be kept constant over the entire planning horizon.

B.2 Optimality of equal manufacturing lots

The analysis of manufacturing lots that are not equally sized is similar to the analysis of not equal remanufacturing lots on the serviceables inventory level. Thus, a distortion of the manufacturing lot sizes leads always to higher holding costs on the serviceables level. We omit the presentation of the mathematics behind that conclusion as the analysis that led to the derivation of formula (16) can be applied.

C Optimizing the relaxated total cost function TC^D

If the number of remanufacturing and manufacturing lots need not to be integer valued, the relaxated total cost function (1) can be solved to optimality by simple calculations. The partial derivatives of the total cost function with respect to all decision variables have to be obtained firstly:

$$\begin{aligned}
\frac{\partial TC^D}{\partial T} &= \frac{\lambda}{2}(\alpha h_d + \alpha^2\beta^2(h_r + \frac{h_s - h_r}{R}) + \frac{h_s}{M}(1-\alpha\beta)^2) - \frac{K_d + RK_r + MK_m}{T^2} = 0 \\
\frac{\partial TC^D}{\partial R} &= \frac{K_r}{T} - \lambda\alpha^2\beta^2 T \cdot \frac{h_s - h_r}{2R^2} = 0 \\
\frac{\partial TC^D}{\partial M} &= \frac{K_m}{T} - \lambda T h_s \cdot \frac{(1-\alpha\beta)^2}{2M^2} = 0.
\end{aligned}$$

The solution of this equation system with respect to all decision variables and assuming that these decision variables have to be positive results in the optimal values for T, R, and M is:

$$R^* = \alpha\beta T^* \cdot \sqrt{\frac{\lambda \cdot (h_s - h_r)}{2K_r}} \tag{17}$$

$$M^* = (1-\alpha\beta)T^* \cdot \sqrt{\frac{\lambda h_s}{2K_m}} \tag{18}$$

$$T^* = \sqrt{\frac{2(K_d + R^* K_r + M^* K_m)}{\lambda(\alpha h_d + \alpha^2\beta^2(h_r + \frac{h_s - h_r}{R^*}) + \frac{h_s}{M^*}(1-\alpha\beta)^2)}}. \tag{19}$$

As equation (19) contains both optimal values of the remanufacturing and manufacturing lot sizes, this equation can be further simplified by equations (17) and (18):

$$T^* = \sqrt{\frac{2(K_d + R^* K_r + M^* K_m)}{\lambda\left(\alpha h_d + \alpha^2\beta^2\left(h_r + \frac{h_s - h_r}{R^*}\right) + \frac{h_s}{M^*}(1-\alpha\beta)^2\right)}}$$

$$T^* = \sqrt{\frac{2\left(K_d + \alpha\beta T^* \sqrt{\frac{\lambda(h_s-h_r)K_r}{2}} + (1-\alpha\beta)\,T^*\sqrt{\frac{\lambda h_s K_m}{2}}\right)}{\lambda\left(\alpha h_d + \alpha^2\beta^2\left(h_r + \frac{1}{\alpha\beta T^*}\sqrt{\frac{2K_r(h_s-h_r)}{\lambda}}\right) + \frac{(1-\alpha\beta)}{T^*}\sqrt{\frac{2K_m h_s}{\lambda}}\right)}}$$

$$T^* = \sqrt{\frac{2K_d + T^*\sqrt{2\lambda}\left(\alpha\beta\left(\sqrt{K_r(h_s-h_r)} - \sqrt{K_m h_s}\right) + \sqrt{K_m h_s}\right)}{\lambda\alpha\left(h_d + \alpha\beta^2 h_r\right) + \frac{\sqrt{2\lambda}}{T^*}\left(\alpha\beta\left(\sqrt{K_r(h_s-h_r)} - \sqrt{K_m h_s}\right) + \sqrt{K_m h_s}\right)}}$$

$$(T^*)^2 = \frac{2K_d + T^*\sqrt{2\lambda}\Theta}{\lambda\alpha\left(h_d + \alpha\beta^2 h_r\right) + \frac{\sqrt{2\lambda}}{T^*}\Theta} \text{ with } \Theta = \alpha\beta\left(\sqrt{K_r(h_s-h_r)} - \sqrt{K_m h_s}\right) + \sqrt{K_m h_s}$$

$$T^* = \sqrt{\frac{2K_d}{\lambda\alpha\left(h_d + \alpha\beta^2 h_r\right)}} \tag{20}$$

Equation (20) can be used to calculate the optimal number of remanufacturing and manufacturing lot sizes by inserting it in equations (17) and (18):

$$R^* = \beta \cdot \sqrt{\frac{\alpha K_d(h_s - h_r)}{K_r(h_d + \alpha\beta^2 h_r)}} \tag{21}$$

$$M^* = (1-\alpha\beta) \cdot \sqrt{\frac{h_s K_d}{K_m \alpha(h_d + \alpha\beta^2 h_r)}} \tag{22}$$

By inserting the optimal values of T^*, R^*, and M^* into the respective second derivatives it can be proven that the total cost function is in a minimum at this point.

D Convexity of the total cost function TC^D

In order to analyze the convexity of the total cost function it is necessary to show that the Hessian matrix of the total cost function TC^D is positive semidefinite. Therefore, the Hessian matrix H has to be set up and analyzed:

$$H = \begin{bmatrix} \frac{\partial^2 TC^D}{\partial T^2} & \frac{\partial^2 TC^D}{\partial T \partial M} & \frac{\partial^2 TC^D}{\partial T \partial R} \\ \frac{\partial^2 TC^D}{\partial M \partial T} & \frac{\partial^2 TC^D}{\partial M^2} & \frac{\partial^2 TC^D}{\partial M \partial R} \\ \frac{\partial^2 TC^D}{\partial R \partial T} & \frac{\partial^2 TC^D}{\partial R \partial M} & \frac{\partial^2 TC^D}{\partial R^2} \end{bmatrix}.$$

By calculating the three eigenvalues of the Hessian matrix one can see that two of them are always positive for positive values of T, R, and M. However, the third eigenvalue becomes negative for certain parameter values. We omit the presentation of the eigenvalues as they are of a very complex nature. Therefore, the total cost function TC^D is not entirely convex in all variables in all cases.

E Determining the upper and lower bound for T for random yields

This section focusses on determining an upper and lower bound for the disassembly cycle length T in a stochastic yield scenario which shall be denoted by T_{min} and T_{max}, respectively. As the question on how to define these bounds exactly appears to be very complex, two different heuristic approaches will be presented subsequently. Common to both approaches is the presumption that a stochastic yield distribution can be regarded as a combination of several equivalent deterministic problems. Thus, the approaches assume that the solution to a stochastic yield problem could also be obtained by combining the solutions of a number of deterministic problems. Hence, the lower and upper bound for the stochastic problem T_{min} and T_{max}, respectively can be obtained by calculating the highest and lowest disassembly cycle length of all possible deterministic yield scenarios. While appendix E.1 omits the integrality constraint of the total cost function and studies the disassembly cycle lengths for all deterministic settings, appendix E.2 includes the fact that the number of remanufacturing and manufacturing lots have to be integer valued.

E.1 Analyzing the relaxated total cost function TC^D

In order to determine the upper and lower bound for the disassembly cycle length for the relaxated total cost function one needs to analyze the optimal disassembly cycle length T^* presented in appendix C (formula (20)) for each possible deterministic yield fraction β. One can see that the first derivative of T^* with respect to β is strictly negative for $0 < \beta < 1$ which means that the optimal disassembly cycle length decreases as larger the deterministic yield is. This can be mathematically proven by the following formula:

$$\frac{dT^*}{d\beta} = \frac{-\frac{1}{2} \cdot \sqrt{\frac{2\lambda\alpha K_d}{h_d+\alpha\beta^2 h_r}} \cdot 2\alpha\beta h_r}{\lambda\alpha(h_d + \alpha\beta^2 h_r)}$$
$$= -\sqrt{\frac{2\alpha K_d}{\lambda(h_d + \alpha\beta^2 h_r)}} \cdot \frac{\beta h_r}{h_d + \alpha\beta^2 h_r} < 0 \quad (23)$$

Therefore, the upper bound of the disassembly cycle length for the relaxated total cost function T_{max}^{rel} can be observed for the lowest possible yield fraction, i.e. $\beta = 0$. On the other hand, the lower bound for the disassembly cycle length T_{min}^{rel} can be observed for the largest possible yield fraction, i.e. $\beta = 1$. By inserting $\beta = 0$ and $\beta = 1$ into equation (20) the values of T_{min}^{rel} and T_{max}^{rel} can be calculated by the following formulae:

$$T_{min}^{rel} = \sqrt{\frac{2K_d}{\lambda\alpha h_d}} \quad (24)$$
$$T_{max}^{rel} = \sqrt{\frac{2K_d}{\lambda\alpha(h_d + \alpha h_r)}} \quad (25)$$

However, these results were derived under the assumption that the number of remanufacturing and manufacturing lots in a disassembly cycle need not to be integer valued. This fact shows the heuristic character of this procedure as for example, if $\beta = \epsilon$ (with ϵ being a very small positive number) the optimal number of remanufacturing lots reveals only a very small but positive amount. This is of course not possible in the model context presented above as the number of remanufacturing lots must be a positive integer number. Hence, the smallest possible value for R is one which leads to the fact that the bounds T_{min}^{int} and T_{max}^{int} must be calculated in a different way. The next section focusses on this topic.

E.2 Analyzing the mixed-integer non-linear problem

If the number of remanufacturing and manufacturing lots have to be integer valued, the simple procedure of appendix E.1 needs to be adjusted in order to cope with this change in the problem setting. Yet, the general approach of the first heuristic is applied to the second heuristic as well. This means that the disassembly cycle length is evaluated for a certain number of possible deterministic yield fractions and the minimum and maximum value become the lower and the upper bound for the cycle length. However, if the number of remanufacturing and manufacturing lots need to be integer valued, the cycle length T can not be formulated as a continuous function with respect to the yield β as a switch in either R or M results in a discontinuity of the function. By analyzing these discontinuities as well as the function $T(\beta)$ between these discontinuities, heuristic values for the lower and upper bound of the disassembly cycle length can be calculated.

However, the formulation of an algorithm that can handle this specific problem in an efficient manner is very complex. Therefore, this paper presents a simpler approach on how to deal with this problem setting that needs not to achieve the solution quality of the complex algorithm discussed before. This approach solves q deterministic problems between the smallest and largest yield fraction using formula (2). It shall be mentioned that the interval between two consecutively examined yield fractions is always $1/(q-1)$, as a yield distribution is generally defined between 0 and 1. The following pseudocode can be applied in order to obtain the upper and lower bound for the disassembly cycle length:

For i **= 1 to** q

$\beta = (i-1)/(q-1)$

calculate T_i by using the deterministic model from chapter 3

Next i

$T_{min}^{int} = \min_i(T_i), \quad T_{max}^{int} = \max_i(T_i)$

The numerical study conducted in chapter 5 provides a data set of 1000 instances. Both heuristic approaches from the preceding subsection (which will be referred to as the relaxated TC (total cost) approach) as well as from this subsection (which will be referred to as the integer TC approach) are tested for these instances in order to evaluate their performance. Therefore, the actual minimum and maximum cycle lengths (denoted by T_{min}^{*} and T_{max}^{*}) for each instance were obtained by applying policy III to all tested yield distributions for all instances. The actually observed minimum values are afterwards compared to the lower bounds T_{min}^{rel} and T_{min}^{int} that were calculated by both heuristic approaches. The left hand side of table 2 summarizes the results of these experiments. The right hand side of this table illustrates the comparison of the calculated upper bounds T_{max}^{rel} and T_{max}^{int} with the actually observed ones.

Table 2: Performance of the relaxated and integer total cost approach regarding their estimations of the minimum and maximum disassembly cycle length

	percentage of instances with $T_{min}^{rel} < T_{min}^{*}$	percentage of instances with $T_{max}^{rel} > T_{max}^{*}$
relaxated TC approach	100 %	13.6 %
	percentage of instances with $T_{min}^{int} < T_{min}^{*}$	percentage of instances with $T_{max}^{int} > T_{max}^{*}$
integer TC approach (q=10)	90.7 %	100 %
integer TC approach (q=20)	96.9 %	100 %
integer TC approach (q=50)	98.4 %	100 %
integer TC approach (q=100)	98.8 %	100 %
integer TC approach (q=10000)	99.1 %	100 %

It can be seen that the performance of the relaxated TC approach introduced in appendix E.1 is ambivalent. While the minimum cycle length is always estimated correctly by formula (24), the maximum cycle length T_{max} is frequently underestimated by formula (25). By incorporating the fact that the number of remanufacturing and manufacturing lots must be integer valued, the performance of the heuristic approach presented in this subsection can be described as very good. The actually observed upper bounds T_{max} have never been underestimated even for a very small number of calculations. The lower bounds T_{min}, on the other hand, seem to benefit from an increasing number of calculations q. However, the performance of only 10 calculations has been already very good (90.7 % of all estimations were correct). Although the general performance of the integer TC heuristic of this subsection seems to improve with an increasing number of calculations, the general heuristic approach can be observed by the fact that even if q is very large not all lower bounds will be estimated correctly. Yet, the percentage error that is made by the false estimation of the integer TC approach is rather small. The average error over the 9 instances for which the bounds could not be calculated correctly is around 0.463% with a maximum deviation of 0.6 % (for q=10000). To conclude this section, it can be said, that the best method for calculating the lower bound of the disassembly cycle length the relaxated TC approach should be applied (formula (24)). For estimating the upper bound of the disassembly cycle length, on the other hand, the integer TC approach from this subsection should be used as it (even for low q values) always estimates the actually observed upper bound of all instances correctly.