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Myopic behavior and overall utility maximization - A study of linked hawks and doves -

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Abstract

At present, in the domain of simultaneous action selection and network formation games, game-theoretic behavior and experimental observations are not consistent. While theory typically predicts inefficient outcomes for (anti-)co-ordination games, experiments show that subjects tend to play efficient (non-Nash) strategy profiles. One reason for this discrepancy is the tendency to model corresponding games as one-shot and derive predictions. In this paper, we calculate the equilibria for a finitely repeated version of the Hawk-Dove game with endogenous network formation and show that the repetition leads to additional sub-game perfect equilibria; namely, the efficient strategy profiles played by human subjects. However, efficiency crucially depends on the design of the game. This paper theoretically demonstrates that, although technically feasible, the efficient profiles are not sub-game perfect equilibria if actions are fixed after an initial period. We confirm this result using an experimental study that demonstrates how payoffs are higher if actions are never fixed.

Keywords:
Network; Hawk-Dove; Game theory; Behavioral experiment; Finitely repeated game

JEL Codes:
D85; C72; C73; C92

1. Introduction

In Network Hawk-Dove games, players decide with whom to interact by establishing links at a certain cost and play a Hawk-Dove game with all linked players. Existing studies have calculated the equilibria of the one-shot version of this game (Bramoullé et al., 2004; Berninghaus and Vogt, 2006) and existing research indicates that subjects do not resort to these equilibria (Berninghaus et al., 2012). This paper extends existing research in this area in two directions: (1) We formally show that, although repeated play typically tends to lead to more efficiency, in Network Hawk-Dove games, repetition alone does not yield efficient outcomes. It is the order in which players optimize their strategies; i.e., whether they form links or choose actions first that allows them to play efficient strategy profiles. (2) By conducting an experimental study, we demonstrate that subjects do, in fact, follow the theoretically predicted patterns.

Existing analyses on anti-coordination games, such as Hawk-Dove games that incorporate endogenous network formation, are rather scarce. Bramoullé et al. (2004) revealed that one-shot networks in which all players myopically maximize their payoffs against more aggressive players, i.e., hawks, than against defensive players, i.e., doves, do exist. However, as the overall payoff in the network increases, the more aggressive players switch to defensive strategies (Bramoullé et al., 2004; Berninghaus and Vogt, 2006). In behavioral experiments on finitely repeated versions of the game, subjects tend to choose hawk less often than the one-shot model predicts (Berninghaus et al., 2012). If subjects are limited to bilateral links, they seem to favor fair outcomes. That is, interactions only occur if both players agree on them. In corresponding experiments, subjects alternate between aggressive and defensive behavior in each period to gain equal payoffs for all participants (Tsvetkova and Buskens, 2013). However, the discrepancy between theoretically predicted aggression and empirically observed defensive behavior has not yet been resolved. Specifically, whether the high frequency of defensive behavior in laboratory experiments is the result of repeated play or a deviation from myopic payoff maximizing behavior remains unclear.

In this sense, research on coordination games, i.e. games in which both players resort to the same strategy in equilibrium as a result of endogenous network formation, is a step ahead. Here, players choose their action in a 2×2 coordination game and play this with all other players to whom they are linked (Hojman and Szeidl, 2006). Unilateral linking leads to complete networks in which all players resort to the same equilibrium of the one-shot game; namely, the payoff-dominant or risk-dominant equilibrium (Goyal and Vega-Redondo, 2005). This changes if bilateral linking is allowed; i.e., if both players have to agree on the establishment of the link. At this point, different network structures emerge and the strategies played depend on the network structures generated; for example, in circles only risk dominant actions are chosen, while in complete networks payoff and risk dominant equilibria are possible (Jackson and Watts, 2002). Existing theoretic analyses confirms that risk-dominant equilibria are more likely. In contrast, subjects in behavioral experiments (Corten and Buskens, 2010; Corbae and Duffy, 2008) play the payoff-dominant equilibrium more frequently. Hence, predictions for the one-shot version of the game are in line with experimental observations.

Aside from the discussion on overall payoffs and myopically best behavior, literature on coordination games exceeds the literature on anti-coordination games in terms of dynamics. Theoretical analyses show that all players who repeatedly play in a circle (Ellison, 1993; Jackson and Watts, 2000) or a lattice (Blume, 1993; Kosfeld, 2002) converge to the risk dominant equilibrium. In corresponding experiments (Berninghaus et al., 2002), subjects reach both the risk-dominant and the payoff-dominant equilibrium. However, the payoff-dominant equilibrium is more likely if the underlying network is a circle. The situation changes if players can choose both the network structure and their actions. According to existing theoretical models, populations converge towards the payoff-dominant equilibrium because players can remove their links to players who resort to the inefficient strategy (Ely, 2002; Bhaskar and Vega-Redondo, 2004). This result is in agreement with corresponding behavioral experiments (Corten and Buskens, 2010; Corbae and Duffy, 2008). In sum, existing literature indicates that the network structure influences the behavior of subjects in experimental studies to the extent that, in contrast to claims that repetition leads to efficiency, efficiency depends on the structure and not (solely) on repetition. Hence, it is important that formal analyses of networked (anti-)coordination games are conducted and compared to experimental results.

This paper aims to extend existing literature that examines Hawk-Dove games with endogenous network formation in both directions. First, we analyze Hawk-Dove games with endogenous network formation, hereafter called the Network Hawk-Dove game, as a finitely repeated, as opposed to one-shot, game. We show that by applying simple trigger strategies, the strategies we observe in our experiment are theoretically plausible. Second, we show that, by limiting the strategy set for some periods, the theoretical predictions change. Namely, we introduce three treatments: (1) In Treatment Basic, subjects are free to choose actions and links throughout the game; (2) In Treatment Fixed Action, the actions chosen in the initial period are fixed for the upcoming four periods; and (3) In Treatment Fixed Link, the links chosen in the initial period are fixed for the next four periods. Although identical outcomes are theoretically possible in all three treatments, i.e. in the initial period in all treatments links and actions are not fixed, the theoretical predictions differ. In Treatment Basic and Fixed Action efficient strategy profiles are sub-game perfect equilibria, while in Treatment Fixed Link only inefficient strategy profiles are sub-game perfect. We confirm this theoretical prediction through the use of a behavioral experiment. In Treatment Basic and Fixed Action, the behavior of subjects converges towards strategy profiles that are more efficient than those employed in Treatment Fixed Link.

2. The Network Hawk-Dove game

Each player $i \in \{1, \ldots, n\}$ in a Network Hawk-Dove game participates in a Hawk-Dove game with all players j she is linked to, using a Network game. We first introduce the Hawk-Dove game and the Network game separately before combining them into the Network Hawk-Dove game.

In the Network game $G^N := \{\Sigma^N, \Pi^N(\cdot)\}$, player i decides with whom she wants to interact.[1] Hence, each strategy in the Network game σ_i^N is a subset of all players $\sigma_i^N \subseteq \Sigma^N \setminus \{i\}$ with $\Sigma^N := \{1, \ldots, n\}$. Establishing links to other players is not without cost $k > 0$ for each established link. The payoff of the Network game is $\Pi^N(\sigma_i^N, \sigma_{-i}^N) := -k \cdot |\sigma_i^N|$. Each strategy profile $\sigma^N = (\sigma_1^N, \ldots, \sigma_n^N)$ implies a directed graph $g(\sigma^N) = (V(\sigma^N), E(\sigma^N))$ with edges $E(\sigma^N)$ and vertices $V(\sigma^N)$. Each vertex $v_i \in V(\sigma^N)$ that corresponds to 1 player $i \in \{1, \ldots, n\}$. For each link player i establishes to player j; i.e., if $j \in \sigma_i^N$, an edge (i, j) from vertex v_i to v_j exists in $E(\sigma^N)$. Each player i has a set of contacts $N_i(\sigma^N) := \sigma_i^N \cup \{j | (j, i) \in E(\sigma^N)\}$, consisting of the players to whom she establishes a link, σ_i^N, and those who establish a link to her, $\{j | (j, i) \in E(\sigma^N)\}$.

The Hawk-Dove game is a symmetric 2×2 normal form game $G^B := \{\Sigma^B, \Pi^B(\cdot)\}$. Every player i in the Hawk-Dove game can choose between two

[1] We model the Network game as a non-cooperative game (Bala and Goyal, 2000).

strategies: hawk (H) and dove (D); i.e., $\Sigma^B := \{H, D\}$. One can interpret hawk as an aggressive strategy and dove as a defensive strategy. If two hawks interact, their payoff is minimal. All other strategy profiles lead to pareto-optimal outcomes (Berninghaus et al., 2012); however, only (H,D) and (D,H) represent Nash equilibria. Table 1 summarizes the payoff matrix of $\Pi^B(\cdot)$.

Table 1: Payoff matrix Hawk-Dove game (with $a > b > c > d > 0$)

	Hawk (H)	Dove (D)
Hawk (H)	d,d	a,c
Dove (D)	c,a	b,b

The non-cooperative Network Hawk-Dove game $\Gamma := \{S; P\}$ is a combination of the Hawk-Dove game G^B and the Network game G^N. Hereafter, we refer to strategies in the Hawk-Dove game *actions* and strategies in the Network game *links* to distinguish them from the strategies in the Network Hawk-Dove game. In the Network Hawk-Dove game, each player i chooses her action σ_i^B and her links σ_i^N. Hence, the strategy set $S := \Sigma^B \times \Sigma^N$ in the Network Hawk-Dove game is a Cartesian product of the strategy sets in G^B and G^N. Notice that, although each player i chooses whether to establish a link to every other player j, she only chooses her action once. Each player i pays for her links and participates in a Hawk-Dove game G^B with every linked player. Clearly, the payoff of player i using strategy $s_i = (\sigma_i^B, \sigma_i^N) \in S$ depends on the number of contacts playing hawk $n_H^i(s) = \sum_{j \in N_i(\sigma^N)} 1_{\{\sigma_j^B = H\}}$ and dove $n_D^i(s) = \sum_{j \in N_i(\sigma^N)} 1_{\{\sigma_j^B = D\}}$. The number of contacts in the neighborhood is $n^i(s) := n_H^i(s) + n_D^i(s)$. Given the payoff function $P : S \to \mathcal{R}$, the payoff of player i is:

$$P_i(s_{-i}, \{\sigma_i^B = H; \sigma_i^N\}) := d \cdot n_H^i(s) + a \cdot n_D^i(s) - k\,|\sigma_i^N|$$

$$P_i(s_{-i}, \{\sigma_i^B = D; \sigma_i^N\}) := c \cdot n_H^i(s) + b \cdot n_D^i(s) - k\,|\sigma_i^N|$$

Example 1. *As a running example throughout the paper, we will employ the parameters summarized in Table 2. Specifically, in a network of $n = 6$ players, establishing links costs $k = 50$. A hawk that is linked to another hawk earns $d = 20$, while those linked to a dove earn $a = 80$. Doves earn*

$b = 60$ *for links to other doves and* $c = 40$ *for links to hawks. We also use these parameters in the behavioral experiment described in Section 5.1.*

Table 2: Running example of Network Hawk-Dove game with $k = 50$ and $n = 6$

	Hawk (H)	Dove (D)
Hawk (H)	$d = 20, d = 20$	$a = 80, c = 40$
Dove (D)	$c = 40, a = 80$	$b = 60, b = 60$

In the remainder, we limit our analysis to (1) $k \geq 2d$, (2) $a > b > k > c > d$, (3) $2c > k$ and (4) $n \geq 3$ (see the appendix for a justification).

3. Social dilemmas in one-shot Network Hawk-Dove games

Human interaction is often characterized by two goals: (1) myopic utility maximization; i.e. the Nash criteria, and (2) overall utility maximization; i.e. efficiency (Engelmann and Strobel, 2004). Social dilemmas occur if these two goals differ. In the remainder of this section, we investigate whether social dilemmas occur in the Network Hawk-Dove game. We focus on three aspects of the game: a link-centric game, an action-centric game and the integrated game representing the Network Hawk-Dove game.[2]

3.1. Link-centric game

We first assume that players can only choose with whom they interact, while the actions of the players are given. To reach an efficient strategy profile, all players have to establish links with the players that they benefit from. As the cost k exceeds $2d$ but is lower than a and b, all unilateral links increase the payoffs, with the exception of links between hawks.

Theorem 1 (Link efficiency). *A strategy profile is efficient with respect to links, i.e., it is link efficient, if all links are unilateral and, with the exception of links between hawks, all links are established.*

[2] In the remainder of the paper we present the intuition and the main results of our formal analysis, while all formal proofs are part of our appendix. Each main result references to the corresponding result and proof in the appendix.

However, not all efficient links represent myopically best replies. Namely, myopic players will drop any links that incur a higher cost than the benefit attained. Namely, doves will remove all links to hawks (as $k > c$). Corresponding links are then established by the involved hawks (as $a > k$).

Theorem 2 (Link balance). *A strategy profile is in equilibrium with respect to links, i.e., it is link balanced, if all links are link efficient and doves only pay for links to other doves.*

When comparing link efficient and link balanced strategy profiles, it becomes obvious that all link balanced strategy profiles are a subset of the link efficient strategy profiles. In addition, the intuition behind all link balanced strategy profiles clearly indicates that, for every link efficient strategy profile that emerges, a link balanced strategy profile with identical overall payoffs will exist. Hence, no social dilemma occurs in the Network game.

3.2. Action-centric game

The situation changes if we look at the played actions in a fixed network. Here, starting from a network that consists of hawks alone, the overall payoff increases the more hawks switch to dove; for both hawks and doves, the payoff from being linked to another dove is higher than that achieved for being linked to another hawk. However, no corresponding results can be found for an upper bound of the number of doves[3].

Theorem 3 (Action efficiency). *A strategy profile with at least one link is efficient with respect to actions, i.e., it is action efficient, if the number of hawks n'_H lies below a certain upper bound. For certain network structures, no upper bound for the number of doves exists.*

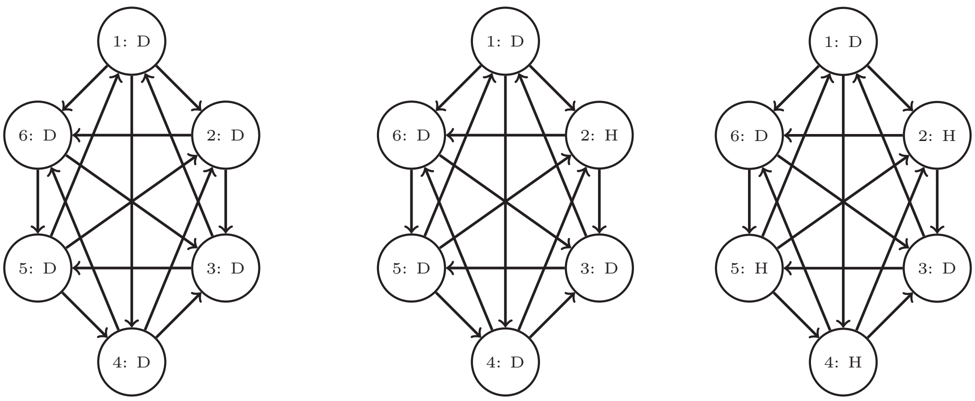
A myopic payoff-maximizing player will always choose the hawk action if his payoff for playing hawk exceeds his payoff for playing the dove action. As a consequence, for each neighborhood, choosing hawk is beneficial if the fraction of players in the neighborhood playing hawks lies below a certain threshold.

[3]This result is the most unspecific result throughout the paper. In the remainder, we will describe action efficient strategy profiles for certain network structures more precisely. However, we cannot find an overall simple result that captures all possible underlying network structures.

Theorem 4 (Action balance). *A strategy profile $s^* \in (S^B, \Sigma^N)$ is in equilibrium with respect to actions, i.e., is action balanced, if the following statement holds: Each player i uses the hawk action if, in his neighborhood, the following condition holds: $n_H^i(s^*) < \frac{a-b}{a-b+c-d} \cdot n^i(s^*)$, with $n_H^i(s^*)$ ($n^i(s^*)$) being the number of hawks (players) in her neighborhood. Otherwise, every player resorts to dove.*

Given a fixed network structure in an equilibrium, the number of doves always has an upper limit. As soon as two doves are linked, one can increase her payoff by switching to hawk, as the number of hawks in her neighborhood is $n_H^i(s^*) = 0$ and, therefore, below $\frac{a-b}{a-b+c-d} \cdot n^i(s^*)$. This is in contrast to the efficient strategy profile in which no such upper limit exists. Hence, for Hawk-Dove games in a fixed network, social dilemmas can occur.

Figure 1: Establishment of action balance



(a) Unbalanced network (b) Player 2 switches to hawk (c) Action balance

Example 2. *Let us return to the running example. To establish action balance, a player will switch to hawk if less than half the other players in her neighborhood resort to hawk (due to $n_H^i(s^*) < \frac{a-b}{a-b+c-d} \cdot n^i(s^*) = \frac{80-60}{80-60+20-20} \cdot n^i(s^*) = \frac{1}{2} \cdot n^i(s^*)$). Think of a network in which all of the $n = 6$ players are interlinked with all other participants (see Figure 1 (a)). In equilibrium, at least three players will resort to hawk if less than three hawks are in the network; each dove has below $\frac{2}{6} < \frac{1}{2}$ hawks in her neighborhood and* *will benefit from switching to hawk. However, the resulting network is not efficient (see Figure 1 (c)). The payoff of each dove player is* $240 = 2 \cdot 60 + 3 \cdot 40$ *and the hawks receive* $320 = 2 \cdot 40 + 3 \cdot 80$ *for their links, yielding an overall payoff of* $1680 = 3 \cdot 240 + 3 \cdot 320$*. If all players resort to dove, the payoff per dove is* $300 = 5 \cdot 60$ *and the overall payoff is* $1800 = 6 \cdot 300 > 1680$*.*

3.3. Integrated game

Let us now combine the action- and link-centric games to perform an integrated analysis of the Network Hawk-Dove game. As we have seen, when investigating the game with a focus on links only, links between hawks are not established in efficient strategy profiles. Hence, the neighborhood of doves consists of all other players, while the neighborhood of hawks consists of doves alone. Using this intuition, one can easily calculate the efficient payoff for a certain number of hawks and doves in the network. Maximizing this payoff results in the optimal number of hawks for a network:

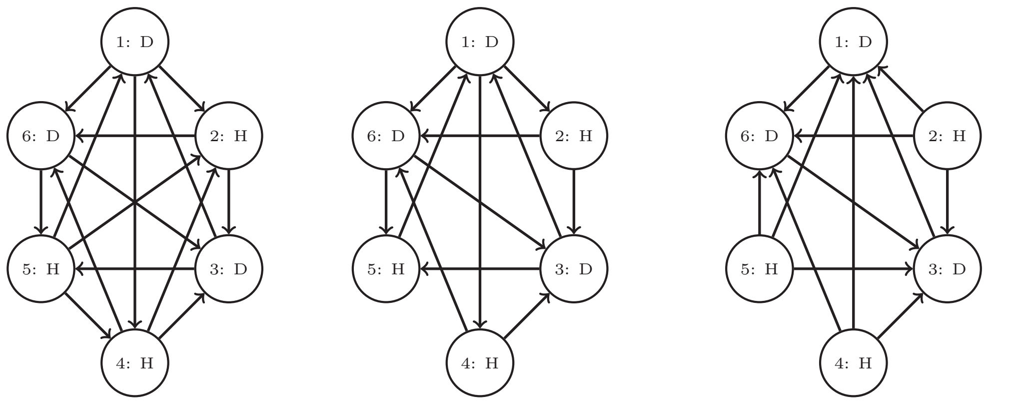
Theorem 5 (Efficient Network Hawk-Dove game). *In efficient strategy profiles of Network Hawk-Dove games, all links are link efficient and the number of hawks* $n_H^{'}$ *satisfies*
$n_H^{'} = \frac{1}{2}(n - \frac{b-0.5k}{a+c-b-0.5k}(n-1))$.

An analysis of the Nash equilibria in the Network Hawk-Dove game follows a similar intuition as the search for efficiency. Given a link balanced network, whether a player can increase her payoff by deviating unilaterally can be calculated for every hawk and every dove. A corresponding analysis yields a lower bound for the number of hawks n_H^* in the Nash equilibrium.

Theorem 6 (Equilibrium Network Hawk-Dove game). *In Nash equilibria of Network Hawk-Dove games, all links are link balanced and the number of hawks* n_H^* *satisfies*
$n_H^* \geq \frac{a-b}{a-b+c-d}(n-1) > 0$.

Again social dilemmas occur. On the one hand, Nash equilibria exceed the number of efficient strategy profiles, as Nash equilibria occur for a range of different numbers of hawks, while only one optimal number of hawks exists. On the other hand, the number of hawks in efficient strategy profiles might be below the minimum number of hawks desired for a Nash equilibrium.

Figure 2: Emergence of equilibrium



(a) Initial network (b) Without (hawk,hawk) links (c) Links to doves only

Example 3. *Think of a network that consists of three hawks and three doves (see Figure 2 (a)). To reach an equilibrium in a Network Hawk-Dove game, the links between hawks are dropped (see Figure 2 (b)) all other links are unilateral and always point to doves (see Figure 2 (c)). The overall payoff in the population is now* $240 = 2 \cdot 60 + 3 \cdot 40$ *for doves (Players 1, 3 and 6) and* $240 = 3 \cdot 80$ *for hawks (Players 2, 4 and 5) yielding an overall payoff of* $1440 = 3 \cdot 240 + 3 \cdot 240$. *To reach this payoff, all three hawks establish three links (one per dove) and each dove establishes exactly one link to one other dove, yielding costs of* $600 = (3 \cdot 3 + 3 \cdot 1) \cdot 50)$. *Hence the benefit after subtracting the cost is* 840. *However, by switching from hawk to dove and establishing links to the remaining doves, (resulting in a network as in Figure 1 (a)) the payoff could be increased. That is, in a system that consists of interlinked doves alone, the benefit from participation per dove is* $300 = 5 \cdot 60$, *while facing average costs of* $125 = 2.5 \cdot 50$. *Hence, the payoff per dove is* 175 *and, for the whole population,* $1050 = 6 \cdot 175 > 840$.

In other words, for the given set of parameters, Nash equilibria and efficient strategy profiles differ, and this difference is not limited to our running example. If $a+c \leq b+\frac{1}{2}k$, the number of hawks in an efficient strategy profile is $\overline{n_H} = 0$ (see Theorem 5). This always differs from the equilibrium prediction, which needs n_H^* to be greater than 0 (see Theorem 6). If $a+c > b+\frac{1}{2}k$, the upper bound for the number of hawks is $\overline{n_H} = \frac{n}{2} \geq \frac{1}{2}(n - \frac{b-0.5k}{a+c-b-0.5k}(n-1))$

in efficient strategy profiles, while the Nash equilibrium prescribes a lower bound for the number of hawks. Hence, this also allows for several parameter combinations in which social dilemmas occur.

4. Impact of finite repetition on social dilemmas

While we have shown that if players only choose links, all Nash equilibria are efficient strategy profiles, we see that Nash equilibria and efficient strategy profiles differ as soon as players can choose their actions. In the remainder of this paper, we analyze whether playing the games repeatedly can help us to align efficiency and Nash equilibria.

One trivial result from game theory is that finitely repeating a Nash equilibrium is a Nash equilibrium in itself. However, repeatedly playing the game, even for a finite number of periods, can help to reach additional equilibria. An approach to play the efficient strategy for at least a couple of periods represents trigger strategies. Here, the efficient strategy profile is played in the first period. Playing this strategy profile is continued unless one of the players deviates. The deviating player is then punished as all other players resort to the Nash equilibrium, which yields the lowest payoff for the deviator. If there is no deviation, all players switch to the Nash equilibrium and this yields the highest overall payoff for the last $\{t^* + 1, ..., T\}$ periods (Friedman, 1985; Benoit and Krishna, 1985). Solution 1 summarizes a trigger strategy (Friedman, 1985; Benoit and Krishna, 1985) to establish efficient strategy profiles that are no equilibria of the stage game.

Solution 1 (Trigger strategy for games with finite horizon). *To establish a desired strategy profile in a finitely repeated game, the following trigger strategy can be used:*

a) In $t = 1$ play the efficient strategy profile.

b) In $t \in \{2, ..., t^*\}$ keep playing the efficient strategy profile if no player deviated. Otherwise, resort to the Nash equilibrium of the stage game, yielding the lowest payoff for the deviator.

c) In $t \in \{t^* + 1, ..., T\}$ play the Nash equilibrium of the stage game, yielding the highest overall payoff if no player deviated. Otherwise, resort to the Nash equilibrium of the stage game, yielding the lowest payoff for the deviator.

Using this trigger strategy results in an additional sub-game perfect equilibria, in which players resort to a strategy profile that yields higher payoffs than the payoff maximal Nash equilibrium of the one-shot game, providing the following conditions for the game hold. First, players need to resort to a Nash equilibrium in the terminal periods $\{t^* + 1, ..., T\}$. Otherwise, at least one player could deviate from the strategy profile played. Second, during the terminal periods $\{t^* + 1, ..., T\}$, two Nash equilibria, one to punish deviators and one to reward non-deviators, are required; otherwise, a player who deviated in period t^* could not be punished in subsequent periods. Third, for the punishment to be effective, the sum of payoffs in the Nash equilibrium to reward, minus the payoffs in the Nash Equilibrium to punish in periods $\{t^* + 1, ..., T\}$, has to exceed the payoff gain for deviating from the desired strategy in period t^*. Otherwise, a player would deviate in t^*. Note, this condition is sufficient for periods $\{1, ..., t^* - 1\}$ to prevent any deviations because deviations in earlier periods would increase the number of periods other players could use to punish and, hence, would not lead to a payoff increase. As the described trigger strategy does not allow for beneficial deviations in any period, the resulting Nash equilibrium is sub-game perfect.

4.1. Repeated Network Hawk-Dove game

Our analysis of the one-shot game shows that efficient strategy profiles and Nash equilibria in the Network Hawk-Dove game differ in the number of hawks in the network. Hence, (1) the efficient strategy profile played in period $t = 1$ should ensure a certain number of hawks n_H. All links should be link balanced (see Theorem 2) to ensure that payoffs are equally distributed. (2) The best punishment for deviations from the efficient strategy profile is to play the Nash equilibrium (see Lemma 1), with $n_H = n$ hawks having no links. This ensures that all payoffs are 0. (3) As the desired strategy profile in periods 1 to t^* is no equilibrium in the one-shot game, players have an incentive to deviate. To overcome this, we choose to play the Nash equilibrium of the one-shot game yielding the highest payoff during the last periods. The following definition summarizes this trigger strategy:

Theorem 7 (Finitely repeated equilibrium). *Let the following inequality hold:* $t^* \leq T - \frac{n'_H \cdot (d-c) + (n - n'_H - 1) \cdot (a-b)}{min\{n^*_H \cdot c + (n - n^*_H - 1)(b-k); (n - n^*_H - 1)(a-k)\}}$. *To reach Nash equilibria that consists of efficient strategy profiles during the first periods, the stage game in the* T*-period repeated Network Hawk-Dove game* Γ*, the players have to resort to the following trigger strategy:*

a) In $t = 1$, play hawk or dove so that n'_H hawks are in the network and ensure that the links are link balanced.[4]

b) In $t \in \{2, ..., t^*\}$ keep playing your strategy as in (a) if no player deviated. Otherwise, play the hawk strategy and remove all links.

c) In $t \in \{t^*, ..., T\}$ play the Nash equilibrium of the one-shot game yielding the highest overall payoff; i.e., the Nash equilibrium, with the minimum number of hawks n^*_H, if no player deviated from the desired strategy in $t \leq t^*$. Otherwise, play the hawk strategy and remove all links.

Example 4. *Applied to our running example, the efficient number of hawks n'_H equals $0.5 = \frac{1}{2}(6 - \frac{60-0.5\cdot 50}{80+40-60-0.5\cdot 50}(6-1)) = \frac{1}{2}(n - \frac{b-0.5k}{a+c-b-0.5k}(n-1))$. As per period only an integer number of hawks is possible, efficiency is reached with either $n'_H = 0$ or $n'_H = 1$. In the remainder of this example, we focus on the latter. Hence, 1 player will resort to hawk and 5 players will resort to dove in the first period. The minimum number of hawks n^*_H has to satisfy $n^*_H \geq 2.5 = \frac{80-60}{80-60+40-20}(6-1) = \frac{a-b}{a-b+c-d}(n-1)$. Again, only integer numbers of hawks can be part of the network $n^*_H = 3$ holds. Hence, players resort to the efficient strategy profile for periods $t \in \{1, ..., t^*\}$ with $t^* \leq T - 1 = T - \frac{1\cdot(20-40)+(6-1-1)\cdot(80-60)}{min\{3\cdot 40+(6-3-1)(60-50);(6-3-1)(80-50)\}}$.*

As such, they only resort to the Nash equilibrium with $n^*_H = 3$ in the last period.

4.2. Repeated play with fixed actions

One disadvantage of the equilibrium reached when playing a trigger strategy is that a Nash equilibrium has to be played for the last period, resulting in an efficiency loss. In addition, our analysis of the Network Hawk-Dove game with a focus on action and links indicated that efficiency and equilibria are in line if only links can be chosen. Hence, we investigate whether limiting the set of actions / links after an initial period without any limitations can overcome the efficiency loss.

When limiting the set of actions, the intuition is as follows. During the periods with fixed actions, it is myopically best to establish the links to all

[4]Notice that finding a corresponding strategy profile is a coordination problem. However, we do not discuss how to find this strategy profile. Deviations can be punished, since, after observing the strategies in the previous period, all players know whether it was found or not.

other players who do not resort to the hawk strategy (see the definition of link balance). The corresponding equilibrium is also efficient. The payoff during the periods with fixed actions is higher if, initially, an efficient strategy profile was played than when the players resorted to a Nash equilibrium. Hence, the longer the actions are fixed, the more beneficial it is to play the efficient strategy profile in $t = 1$.

Theorem 8 (Finitely repeated equilibrium with fixed actions). *By limiting the strategy set to links only in $t > 1$, playing an efficient strategy profile with $n_H^{'}$ hawks in $t = 1$ and all subsequent periods $\{1, .., T\}$ is a Nash equilibrium, if*

a) All links are link balanced.

b1) If $T(a - b + c) \geq (T - 1)k - d$ holds, the number of hawks $n_H^{'}$ in the population has to satisfy the condition
$0 \leq n_H^{'} \leq \frac{T(a-b)}{T(a-b)+(T-1)c}n$.

b2) If $T(a - b + c) < (T - 1)k - d$ holds, the number of hawks $n_H^{'}$ in the population has to satisfy the condition
$\frac{T(a-b)-(T-1)k}{T(a-b+c)-(T-1)k-d}(n - 1) \leq n_H^{'} \leq \frac{T(a-b)}{T(a-b)+(T-1)c}n$.

Example 5. *Applied to our running example with the efficient number of hawks $n_H^{'} = 1$ condition, (a) and (b1) have to be fulfilled as $T(a - b + c) = T(80-60+40) \geq (T-1)50-20 = (T-1)k-d$ holds. $n_H^{'} \leq \frac{T(80-60)}{T(80-60)+(T-1)40}6 = \frac{T(a-b)}{T(a-b)+(T-1)c}n$. The condition is always fulfilled for $n_H^{'} = 1$.*

Hence, limiting the strategy set for at least one period results in playing the efficient strategy profile of the one-shot Network Hawk-Dove game for all T periods.

4.3. Repeated play with fixed links

Let us now focus on the impact of limiting the strategy space to choosing actions after the first period. As the description of action efficiency and action balance has already demonstrated, limiting the strategy set to choosing actions can result in a discrepancy between equilibrium and efficiency. This result is confirmed by applying a trigger strategy to the resulting game. As we show, limiting the strategy set to actions only ensures that, even by repetition, no additional equilibria are found, except for those in the one-shot Network Hawk-Dove game.

Theorem 9 (Finitely repeated equilibrium with fixed links). *By fixing actions after the first period ($t = 1$), applying a trigger strategy yields no additional sub-game perfect equilibria, with the exception of those played in the one-shot version of the Network Hawk-Dove game.*

5. Hypotheses

The one-shot Network Hawk-Dove game results in a potential social dilemma. This social dilemma is linked to the final payoffs in the game. From an efficiency-focused perspective, it is best for many players to resort to dove (see Theorem 5). The more doves that are in the network, the higher the overall payoff. From an myopically payoff-maximizing perspective, however, it is best for the players to resort to the hawk action (see Theorem 6). As long as all others keep playing their strategy, this increases their individual payoffs.

Finitely repeating the Network Hawk-Dove game (see Theorem 7) can resolve this issue. That is, by simply repeating the Network Hawk-Dove game, strategy profiles with higher numbers of doves are sub-game perfect.

Hypothesis 1 (Efficiency). *Strategy profiles in the finitely repeated version of the Network Hawk-Dove game are more efficient than those in the one-shot Nash equilibrium.*

Whether efficiency is sub-game perfect or not depends on the specific design of the repetition. If the played actions are fixed after the initial period, efficient outcomes still occur (see Theorem 8). However, efficiency does not occur if links are fixed (see Theorem 9). Hence, we expect similar payoffs in the finitely repeated Network Hawk-Dove game as well as the repeated game with fixed actions. However, payoffs should be lower if links are fixed.

Hypothesis 2 (Fixed Links). *Efficiency in a game with fixed links is lower*

(a) *than in a treatment with neither links nor actions fixed and*
(b) *than in a treatment with only actions fixed.*

5.1. Experimental design

To evaluate the hypotheses described, we conducted a laboratory experiment at the Karlsruhe Institute of Technology. Each experimental session lasted approximately 1.5 hours. For all sessions, we recruited a total of 162 subjects using ORSEE (Greiner, 2004) from a pool of students in Karlsruhe. At the beginning of each experimental session, we randomly split the subjects into groups of six. Each subject was given a set of written instructions that described the experimental setup. After all subjects had read the instructions, they played one treatment at a computer terminal, using zTree (Fischbacher, 2007). Finally, we paid the subjects in private in accordance with their success during the treatment.

The baseline treatment (Treatment Basic) is equivalent to the Network Hawk-Dove game and consisted of 50 periods. In every period, subjects could specify which subjects they wanted to establish links to and the action they wanted to play. At the end of each period, the experimental software calculated subjects' payoff (with the payoff function being identical to our running example in Table 2). For each link a subject established, he had to pay $k = 50$ points. All linked subjects then played the Hawk-Dove game. That is to say, if a subject played dove and his neighbor hawk, he received 40 points, while his neighbor received 80 points. If both subjects played hawk, they received 20 points each. They received 80 points when they coordinated in the dove action. After the calculation of the payoff, the computer terminal displayed the actions, links, and individual performance of each subject.

We conducted two modifications of Treatment Basic: (1) Treatment Fixed Link and (2) Treatment Fixed Action. Treatment Fixed Link was identical to 10 times playing the Network Hawk-Dove game, before fixing the links for four periods. In the Treatment Fixed Action, we fixed the actions for four periods after playing the one-shot Network Hawk-Dove game. That is to say, in both modifications, we limited the strategy sets of the subjects in periods $2, ..., 5, 7, ..., 10$ and allowed them to choose both, actions and links, in periods $1, 6, ...$ Each subject participated in one treatment only. Hence, 54 subjects participated in 9 groups per treatment.

Notice that in our formal analysis of the modifications, we only analyzed the Network Hawk-Dove game and the subsequent four periods. However, we repeated all modifications 10 times. Hence, in both Treatment Fixed Link and Treatment Fixed Action, subjects interacted for 50 periods. This design allowed us to identify whether the efficiency was the result of the specific design and not just the result of different adaptation speeds towards the efficient outcome in the three treatments.

All subjects received a show-up fee of 5.00 Euro. For $1,000$ points earned during the experiment, a subject received 1.00 Euro. On average, each subject earned 11.39 Euro.

6. Experimental results

The remainder of this section presents an investigation of our two hypotheses. We first compare the outcomes in Treatment Basic to the Nash equilibrium of the one-shot version of the game, before we analyze differences in a treatment with fixed link (Treatment Fixed Link) to treatments with variable links (Treatments Basic and Fixed Action).

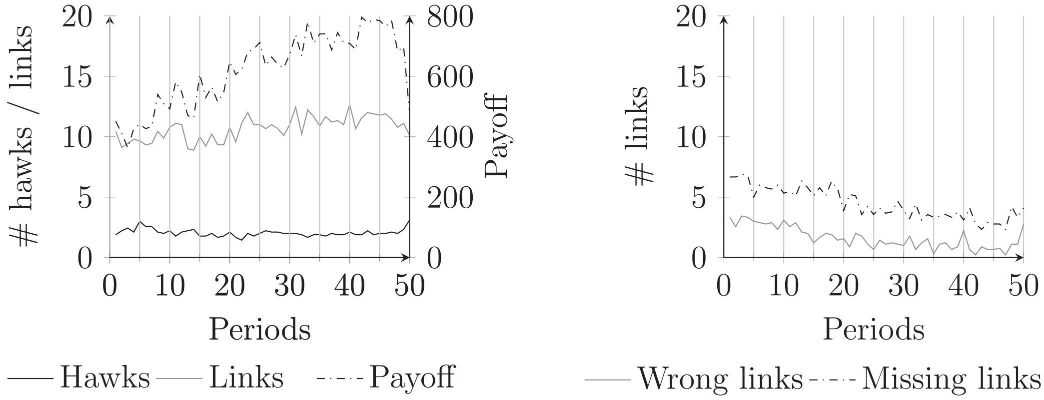
6.1. Hypothesis Efficiency

Remember, according to Theorem 6, the number of hawks in the Nash equlibrium of the one-shot game satisfies $n_H^* \geq \frac{a-b}{a-b+c-d}(n-1) = \frac{80-60}{80-60+40-20}(6-1)) = 2.5$. That is, if players resort to the Nash equilibrium of the one-shot game, three or more hawks interact with three or less doves. The number of links in a Nash equilibrium is maximal if the number of doves is maximal, as all players only link to doves in Nash equilibrium. Hence, in a corresponding network, the maximum number of links is 12 ($= 3 \cdot 3$ [links from three hawks to three doves] $+3$ [links between three doves]). As the payoff also increases in the number of doves, the maximum payoff per hawk is 90 ($= 3 \cdot 80 - 3 \cdot 50$) and it is, on average, per dove 190 ($= 3 \cdot 40 + 2 \cdot 60 - 1 \cdot 50$). Hence, the payoff of the group in the one-shot equilibrium is 840 ($= 3 \cdot 90 + 3 \cdot 190$).

We find that the overall average number of hawks in Treatment Basic significantly lies below the number of hawks predicted by the one-shot Nash equilibrium (signed rank test, two-sided, p = 0.039). However, the number of links does not significantly differ from the prediction (signed rank test, two-sided, p = 0.180) and, as a consequence, the payoffs are significantly lower than the maximum payoff achievable in a Nash equilibrium (signed rank test, two-sided, p = 0.004).

As Figure 3 (a) suggests, both the number of hawks (generalized mixed model, fixed effects, p = 0.364) and the number of links (generalized mixed model, fixed effects, p = 0.203), do not increase over time. The payoff significantly increases the longer the experiment lasts (generalized mixed model, fixed effects, p = 0.000). To understand why the payoff increases, despite the fact that neither the number of links or the number of hawks change, we take a closer look at the links established (see Figure 3 (b)). The number of missing links, i.e. unestablished links between doves or from hawks to doves, significantly decreases over time (generalized mixed model, fixed effects, p = 0.002). Similarly, the number of bad links, i.e. established links between hawks or redundant links, decreases as the periods commence (generalized

Figure 3: Behavior in Treatment Basic



(a) Properties of strategy profiles

(b) Link errors

mixed model, fixed effects, p = 0.003). In other words, the behavior of the subjects converges towards the sub-game perfect equilibrium over time. From period 25 on, less than 6 links per group are either bad or missing. That is, from period 25 onwards, each subject makes only one (or less) link error per period.

Hence, we observe that, from the initial period onwards, the number of hawks in Treatment Basic is lower than those predicted by the one-shot Nash equilibrium. Although the number of links and, as a consequence the payoffs, do not exceed the number of links and payoffs predicted by the one-shot Nash equilibrium, both figures increase over time. Hence, the subjects learn and play the efficient Nash equilibria of the repeated game, thus clearly confirming Hypothesis Efficiency.

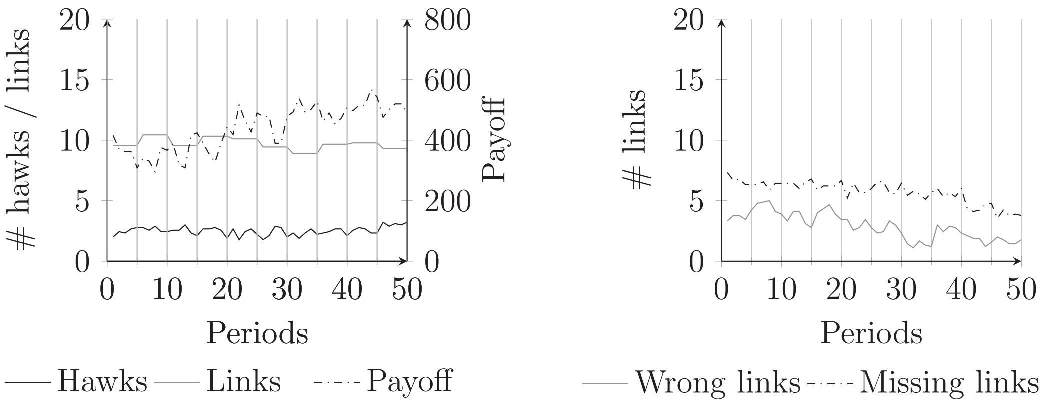
6.2. Hypothesis Fixed Links

We now compare the outcome of Treatment Fixed Links to the other two treatments with respect to efficiency (Hypothesis Fixed Links). In Treatment Fixed Links, the number of hawks does not significantly differ from the minimum number of hawks in the Nash equilibrium ($n_H^* = 2.5$, Wilcoxon test, two-sided, p = 0.944). Similar to Treatment Basic, the number of links (signed rank test, two-sided, p = 0.004) and the payoffs (signed rank test, two-sided, p = 0.004) are significantly lower than in the payoff maximal one-shot Nash equilibrium. These results indicate that fixing the links after an initial period leads to lower payoffs than a game in which subjects can choose their links during every period.

As expected, the number of hawks in Treatment Fixed Links lies below the number of hawks in Treatment Basic (Mann Whitney U test, one-sided, p=0.032). In addition, the number of links (Mann Whitney U test, one-sided, p=0.039) as well as the payoff (Mann Whitney U test, one-sided, p=0.002) is lower in Treatment Fixed Links than in Treatment Basic.

To ensure that the differences are not only the consequence of the subjects learning the equilibria slower in Treatment Fixed Links than in Treatment Basic, we analyze the behavior of the subjects during the ten repetitions (see Figure 4). As we already observed in Treatment Basic, neither the number of hawks (generalized mixed model, fixed effects, p = 0.069) or the number of links (generalized mixed model, fixed effects, p = 0.998) changes significantly. That is, we can confirm that subjects do not learn the efficient behavior over time, but keep playing the Nash equilibria initially chosen. Nevertheless, the subjects do optimize their behavior. Namely, payoffs increase throughout the game (generalized mixed model, fixed effects, p = 0.010) due to the subjects optimizing the quality of their links. Over time, both the number of missing links (generalized mixed model, fixed effects, p = 0.013) and the number of wrong links (generalized mixed model, fixed effects, p = 0.011) decreases.

Figure 4: Behavior in Treatment Fixed Links



(a) Properties of strategy profiles

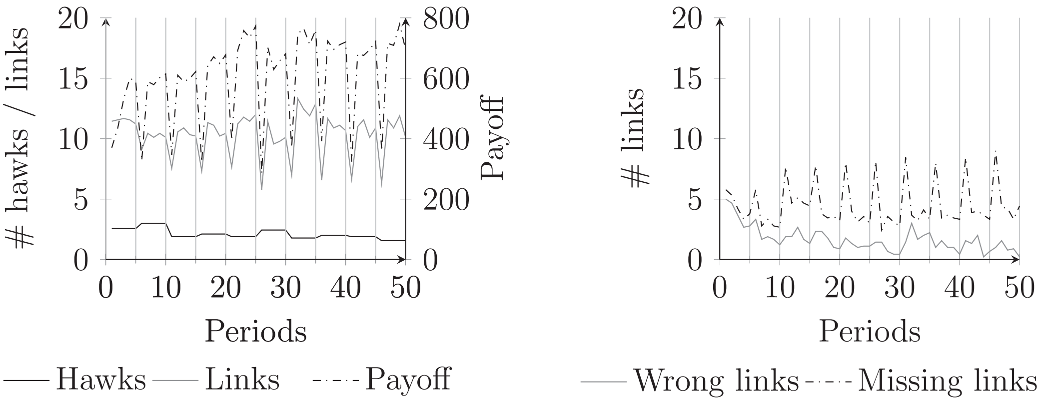
(b) Link errors

From the start of Treatment Fixed Links, we observe the number of hawks does not differ from the number of hawks in the one-shot Nash equilibrium. Although the subjects keep improving their links throughout the game, they never reach the levels of efficiency we observed in Treatment Basic, confirming Hypothesis Fixed Links (a).

Let us finally turn our focus to Treatment Fixed Action. Here, results are comparable to the results observed during Treatment Basic. Significantly less hawks participate per group than predicted in the one-shot Nash equilibrium (signed rank test, two-sided, p = 0.039). However, fewer links (signed rank test, two-sided, p = 0.004) and lower payoffs (signed rank test, two-sided, p = 0.004) than expected in the payoff maximal one-shot Nash equilibrium occur.

A comparison of Treatment Fixed Action with the Treatment Fixed Links confirms Hypothesis Fixed Links (b): Payoffs in Treatment Fixed Action are higher (Mann Whitney U test, one-sided, p=0.000) because the number of hawks is lower (Mann Whitney U test, one-sided, p=0.012). However, we find no significant difference in the number of established links (Mann Whitney U test, one-sided, p=0.081).

Figure 5: Behavior in Treatment Fixed Action



(a) Properties of strategy profiles

(b) Link errors

A closer look at the temporal patterns of behavior can justify the low number of links on average (see Figure 5). While payoffs increase (generalized mixed model, fixed effects, p = 0.000) and the number of hawks does not change (generalized mixed model, fixed effects, p = 0.110), as observed in both other treatments, the number of links increases (generalized mixed model, fixed effects, p = 0.002). In addition, the number of missing links (generalized mixed model, fixed effects, p = 0.001) and the bad links (generalized mixed model, fixed effects, p = 0.001) decrease.

We argue that learning how to play the sub game perfect equilibrium plays an even more important role in Treatment Fixed Action than in both other treatments. Whenever the subjects can choose actions and links simultaneously, they focus on choosing the correct action. It is only during the subsequent periods that they establish their links, ensuring that the number of wrong links is low throughout play. Let us now focus on the periods of Treatment Fixed Action in which actions are fixed. Here, subjects in Treatment Fixed Action adapt their links faster than in Treatment Basic. The number of links between hawks (Mann Whitney U, two-sided p=0.000) and the number of links from hawks to doves (Mann Whitney U, two-sided, p=0.014) is lower in Treatment Fixed Action than in Treatment Basic. Bilateral, i.e., inefficient, links between two hawks (Mann Whitney U, two-sided, p=0.006) as well as missing links from hawks to doves (Mann Whitney U, two-sided, p=0.000) are also less frequent in Treatment Fixed Action. Even links from doves to hawks do not occur as often (Mann Whitney U, two-sided, p=0.001).

7. Conclusion

This paper extended existing game-theoretic analyses of the one-shot Network Hawk-Dove game (Bramoullé et al., 2004; Berninghaus and Vogt, 2006; Schosser et al., 2012) to the finitely repeated version of the game. We showed that, although efficient strategy profiles and Nash equilibria of the one-shot game differ, efficient strategy profiles represent sub-game perfect equilibria in the finitely repeated version of the game. Our results are especially worthwhile because a small design change, i.e., limiting the strategy set to action selection in some periods, resulted in the inability of our game-theoretic to predict efficiency and the subjects failing to reach significantly lower payoffs in the lab.

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Appendix A. Formal analysis

Given the definition of the Network Hawk-Dove game (see Section 2), the cost per link k can be any real number. However, we limit our analysis in the remainder to (1) $k \geq 2d$, (2) $a > b > k > c > d$, (3) $2c > k$ and (4) $n \geq 3$. (1) $k > 2d$ follows the standard interpretation of Hawk-Dove games, where d represents injuries from aggressive behavior and typically is 0 (Neugebauer et al., 2008) or negative (Smith and Price, 1973). Hence, linking two hawks is not beneficial, even if both alternate in paying for the link. If $a > b > k > c > d$ holds (2), links from hawks to doves pay, while they do not pay between hawks, and links from doves to hawks do not pay, while they pay between doves, offering all players a variety of link decisions. This changes for all other parameter combinations: If $k > a$, no links are established. If $a > k > b$, only links from hawks to doves pay and each hawk will link to all doves. If $c > k > d$, only links between hawks do not pay. Hence, doves benefit from links to all others, turning their link decision trivial again. Due to our first restriction, $d > k$ cannot hold. (3) Next, we set $2c > k$. This is basically a simplification which ensures that for doves, who alternate in establishing their links, the link pays. (4) We assume that more than 2 players, i.e., $n \geq 3$, participate in our Network Hawk-Dove game to investigate real networks and not only Hawk-Dove games with an outside option.

Before describing the proofs of the paper in more detail, we first introduce the definition for Nash equilibria and efficiency in Network Hawk-Dove games. We use both definitions in the letter to characterize efficient strategy profiles and Nash equilibria in the Network Hawk-Dove game.

Definition 1 (Nash equilibrium). *Each strategy profile $s^* = (\sigma^{N^*}, \sigma^{B^*})$ in Γ is a Nash equilibrium if*

$$\forall i : \quad P_i(s^*_{-i}, s^*_i) \geq P_i(s^*_{-i}, s_i) \text{ for } s_i \in S_i.$$

Definition 2 (Efficiency). *Each strategy profile $s^{'} = (\sigma^{N^{'}}, \sigma^{B^{'}})$ in Γ is efficient if*

$$\forall \ s \neq s^{'} : \quad \sum_{\forall i} P_i(s^{'}_{-i}, s^{'}_i) \geq \sum_{\forall j} P_j(s_{-j}, s_j) \text{ for } \ s \in S.$$

A.1. Social dilemmas in one-shot Network Hawk-Dove games

The remainder of this and the subsequent sections follows the structure of the main paper. In this section, we characterize efficiency strategy profiles and Nash equilibria of the one-shot Network Hawk-Dove game using Definitions 1 and 2. We do so by first focusing on links and actions in isolation, before we discuss implications for the integrated game.

A.1.1. Link centric game

Theorem 1. *A strategy profile is efficient with respect to links, link efficient, if all links are unilateral and with the exception of links between hawks all links are established.*

Proof 1. *The theorem is equivalent to the following three conditions: (a) all links are unilateral, (b) all hawks are linked to all doves but not to any hawks, and (c) all doves are linked to all other doves. Condition (a) is a consequence of the game design. Players benefit from all other players they are linked to. Redundant links do not increase payoff. Conditions (b) and (c) follow from* $a > b > k$ *and* $k > 2d$.

Theorem 2. *A strategy profile is in equilibrium with respect to links, link balanced, if all links are link efficient and doves only pay for links to other doves.*

Proof 2. *The theorem is equivalent to the following three conditions: (a) all links are unilateral, (b) all hawks establish links to all doves but not to any hawks, and (c) all doves establish links to every other dove. Condition (a) and its proof are equivalent to Condition (a) of Theorem 1. Conditions (b) and (c) follow directly from* $a > b > k > c > d$.

A.1.2. Action centric game

Theorem 3. *A strategy profile with at least one link is efficient with respect to actions, action efficient, if the number of hawks* $n_H^{'}$ *lies below a certain upper bound. For certain networks structures no upper bound for the number of doves exists.*

Proof 3. *The payoff of a hawk is* $P_i(s_{-i}, \{\sigma_i^B = H; \sigma_i^N\}) := d \cdot n_H^i(s) + a \cdot n_D^i(s)$*, while it is* $P_i(s_{-i}, \{\sigma_i^B = D; \sigma_i^N\}) := c \cdot n_H^i(s) + b \cdot n_D^i(s)$ *for a dove. For both types of players the payoffs increase the more doves they have in* *their neighborhood as $n^i_H(s) = n^i(s) - n^i_D(s)$ and $a > b > c > d$. As $a > b$ network structures can exist in which the some hawks are beneficial for the whole population. However, as doves lose part of their payoff $(b - c)$, if a neighbor switches to the hawk strategy network structures can exist in which no hawk exist in efficient strategy profiles.*

Theorem 4. *A strategy profile $s^* \in (S^B, \Sigma^N)$ is in equilibrium with respect to actions, action balanced, if the following statement holds: (a) Each player i uses the hawk action if in his neighborhood the following condition holds $n^i_H(s^*) < \frac{a-b}{a-b+c-d} \cdot n^i(s^*)$, with $n^i_H(s^*)$ ($n^i(s^*)$) being the number of hawks (players) in her neighborhood. (b) Every player resorts to dove otherwise.*

Proof 4. *Conditions (a) and (b) follow from the payoff function $P(\cdot)$ of the Network Hawk-Dove game. The payoff of the hawk action exceeds that of the dove action if the following inequality holds: $P_i(s^*_{-i}, \{H; \overline{\sigma^N_i}\}) = d \cdot n^i_H(s) + a \cdot n^i_D(s) - k\,|\overline{\sigma^N_i}| > c \cdot n^i_H(s) + b \cdot n^i_D(s) - k\,|\overline{\sigma^N_i}| = P_i(s^*_{-i}, \{D; \overline{\sigma^N_i}\})$. With $n^i_H(s) = n^i(S) - n^i_D(s)$, this inequality simplifies to the inequality of condition (a). A player resorts to the dove action if the inequality does not hold (condition (b)).*

A.1.3. Integrated game

Theorem 5. *In efficient strategy profiles of Network Hawk-Dove games, all links are link efficient and the number of hawks $n^{'}_H$ satisfies*

$$n^{'}_H = \frac{1}{2}(n - \frac{b - 0.5k}{a + c - b - 0.5k}(n-1))$$

Proof 5. *Let n_H be the number of hawks and $n_D = n - n_H$ the number of doves in the network. In link efficient strategy profiles, the aggregated payoff of all players is*

$$P(\cdot) = n_H \cdot (n - n_H) \cdot (a + c - k) + 0.5(n - n_H) \cdot (n - n_H - 1) \cdot (2b - k)$$

with $n_H \cdot (n - n_H)$ being the number of links between hawks and doves yielding $a+c-k$ per link and $0.5(n-n_H)\cdot(n-n_H-1)$ being the number of links between doves yielding $2b-k$. To identify the maximum of $P(\cdot)$, we differentiate $P(\cdot)$ twice:[5]

$$\partial P(\cdot)/\partial n_H = (n - 2n_H) \cdot (a + c - k) + (2n_H - 2n + 1) \cdot (b - 0.5k)$$

[5] Notice that our analysis is a simplification as we search the optimum of $P(\cdot)$ under the condition that $0 \leq n_h \leq n$. However, we ignore this condition when differentiating and manually check for the borders of the condition later.

and

$$\partial^2 P(\cdot)/\partial n_H^2 = -2(a + c - b - 0.5k).$$

The optimum lies at $\partial P(\cdot)/\partial n_H = 0$ *being equivalent to* $n_H^{'} = \frac{1}{2}(n - \frac{b-0.5k}{a+c-b-0.5k} \cdot (n-1))$. *As we assume that* $2c > k$ *and* $a > b$ *(see preconditions of the game),* $a+c-b-0.5k$ *is always positive and therefore,* $\partial^2 P(\cdot)/\partial n_H^2 < 0$ *holds. Hence, the optimum is a maximum.*

Theorem 6. *In Nash equilibria of Network Hawk-Dove games, all links are link balanced and the number of hawks* n_H^* *satisfies*

$$n_H^* \geq \frac{a-b}{a-b+c-d}(n-1) > 0.$$

Proof 6. *In link balanced strategy profiles, unilateral deviation from hawk to dove is beneficial if the following inequality holds:* $P_i(s_{-i}^*, \{H; \sigma_i^{N^*}\}) = n_D^* a - k|\sigma_i^{N^*}| < n_D^* b - k|\sigma_i^{N^*}| = P_i(s_{-i}^*, \{D; \sigma_i^{N^*}\})$. *This inequality is always false as* $a > b$ *holds. Each dove player deviates from dove to hawk if* $P_i(s_{-i}^*, \{D; \sigma_i^{N^*}\}) = n_H^* c + (n_D^* - 1)b - k|\sigma_i^{N^*}| < n_H^* d + (n_D^* - 1)a - k|\sigma_i^{N^*}| = P_i(s_{-i}^*, \{H; \sigma_i^{N^*}\})$ *holds. This inequality is false if the inequality of the theorem is met.*

Notice, Definition 1, Theorem 6, and the corresponding proof follow the ideas introduced by Berninghaus and Vogt (2006) and are repeated here to simplify understanding of subsequent proofs. Although the arguments in these proofs differ from Bramoullé et al. (2004), the predictions of Theorem 6 are equivalent to their results.

In addition to the theorems above, we will rely on one additional lemma in the remainder, describing the properties of the payoff minimal Nash equilibrium in a Network Hawk-Dove game:

Lemma 1. *The Nash equilibrium of the Network Hawk-Dove game, yielding the lowest payoff for all players, is characterized by only hawk players* ($n_H = n$) *and no links.*

Proof 7. *According to Theorem 6, the described strategy is a Nash equilibrium as all links are link balanced and* $n = n_H^* \geq n - 1 \geq \frac{a-b}{a-b+c-d}(n-1)$ *holds if* $c > d$, *which is a property of the Hawk-Dove game. As no links exist, the payoff in this equilibrium is* 0. *Any other Nash equilibrium yields higher payoffs for at least two players and no lower payoffs for any of the players* *since in all other Nash equilibria at least one player resorts to dove and at least one link from another player to this dove exists, yielding positive payoffs for both players, given the links are link balanced.*

A.2. Impact of finite repetition on social dilemmas

We now investigate the impact of repetition on the played equilibria. We first investigate the game where in each period $t \in \{1, ..., T\}$ all players play the one-shot version of the Network Hawk-Dove game, before we limit the strategy set in $t > 1$ to links and actions only. Before discussing all proof, we first introduce another lemma, we will resort to in the remainder.

Lemma 2. *Playing a Nash equilibrium of the Network Hawk-Dove game in period $t = 1$ and all subsequent periods is a Nash equilibrium of the T times repeated game.*

Proof 8. *The proof is obvious. The Nash equilibrium of the one-shot Network Hawk-Dove game in period $t = 1$ is a Nash equilibrium in all subsequent stage games. Hence, an equilibrium can exist in which every period the Nash equilibrium of the one-shot game is played.*

A.2.1. Repeated Network Hawk-Dove game

Definition 3 (Trigger strategy for the Network Hawk-Dove game). *Let the trigger strategy in the Network Hawk-Dove game be as follows:*

a) In $t = 1$ play hawk or dove so that n_H hawks are in the network and ensure that the links are link balanced.

b) In $t \in \{2, ..., t^*\}$ keep playing your strategy as in (a) if no player deviated. Otherwise, play the hawk strategy and remove all links.

c) In $t \in \{t^*, ..., T\}$ play the Nash equilibrium of the one-shot game yielding the highest overall payoff with n_H^* hawks if no player deviated from the desired strategy in $t \leq t^*$. Otherwise, play the hawk strategy and remove all links.

Theorem 7. *To reach Nash equilibria consisting of efficient strategy profiles, the stage game in the T-period repeated Network Hawk-Dove game Γ, the trigger strategy for the Network Hawk-Dove game (Definition 3) has to be parameterized as follows:*

a) $n_H = n_H^{'}$

b) $n_H^* > n_H^{'}$ *and* n_H^* *is the minimum number of hawks in a Nash equilibrium*

c) $t^* \leq T - \frac{n_H^{'} \cdot (d-c)+(n-n_H^{'}-1)\cdot(a-b)}{min\{n_H^* \cdot c+(n-n_H^*-1)(b-k);(n-n_H^*-1)(a-k)\}}$.

Proof 9. *Condition (a) follows directly from the observation that the desired strategy profiles are non Nash equilibria of the stage game. Condition (b) characterizes the equilibrium played in periods* $t^*+1, ..., T$ *if no player deviated. In the equilibrium, all links are link balanced and the number of hawks is chosen such that the highest overall payoff for all players is reached. We assume that the efficient strategy profile is no Nash equilibrium (otherwise no trigger strategy would be needed to reach an equilibrium; see Lemma 2). Therefore* $n_H^{'} < n_H^*$ *holds. The Nash equilibrium with the highest overall payoff is characterized by* n_H^* *being the lowest possible number of hawks as the payoff in the one-shot game is monotonically decreasing if the number of hawks increases (see the proof of Theorem 5). Condition (c) characterizes the number of periods to resort to the payoff maximal equilibrium. If one player deviated in* $t \leq t^*$*, the payoff of all players is* 0 *in all subsequent periods. Therefore each player not deviating earns a minimum of*

$$min\{n_H^* \cdot c + (n - n_H^* - 1)(b - k); (n - n_H^* - 1)(a - k)\}$$

for each of the $T - t^*$ *last periods. For this payoff we expect that all links are link balanced (which they are in equilibrium). Further, the left argument* $(n_H^* \cdot c + (n - n_H^* - 1)(b - k))$ *is the minimal payoff for resorting to dove, i.e., the payoff if paying for all links, and the right argument* $((n - n_H^* - 1)(a - k))$ *is the payoff for resorting to hawk. The gain for not deviating has to be at least as high as the gain from deviating in* $t = t^*$*. A player being a hawk in* t^* *will not deviate as, given the links are link balanced, she is linked to doves only, and switching from hawk to dove will reduce her payoff. A dove, on the other hand, might switch to hawk in* t^* *to increase her payoff by*

$$n_H^{'} \cdot (d - c) + (n - n_H^{'} - 1) \cdot (a - b).$$

For the trigger strategy to reach an equilibrium $(T - t^*) \cdot min\{n_H^* \cdot c + (n - n_H^* - 1)(b - k); (n - n_H^* - 1)(a - k)\} \geq n_H^{'} \cdot (d - c) + (n - n_H^{'} - 1) \cdot (a - b)$ *has to hold, which simplifies to condition (c).*

A.2.2. Repeated play with fixed actions

Theorem 8. *By limiting the strategy set to links only in $t > 1$ playing an efficient strategy profile with $n_H^{'}$ hawks in $t = 1$ and all subsequent periods $\{1, .., T\}$ is a Nash equilibrium, if*

a) All links are link balanced.

b1) If $T(a - b + c) \geq (T - 1)k - d$ holds, the number of hawks $n_H^{'}$ in the population has to satisfy the condition

$$0 \leq n_H^{'} \leq \frac{T(a-b)}{T(a-b) + (T-1)c} n.$$

b2) If $T(a - b + c) < (T - 1)k - d$ holds, the number of hawks $n_H^{'}$ in the population has to satisfy the condition

$$\frac{T(a-b) - (T-1)k}{T(a-b+c) - (T-1)k - d}(n-1) \leq n_H^{'} \leq \frac{T(a-b)}{T(a-b) + (T-1)c} n.$$

Proof 10. *Condition (a) follows directly from the conditions for link balance (see Theorem 2). Condition (b1) follows from the gain a player, who is hawk in $t = 1$, receives by switching to dove in $t = 1$. Her payoff is $T(n - n_H^{'})(a - k)$ for playing hawk. Switching to dove yields $T(n - n_H^{'})(b - k) + (T - 1)n_H^{'} \cdot c$ because she will still link to all doves, but in periods $t \in \{2, .., n\}$ all hawks will link to her to be in equilibrium. Playing hawk pays if*

$$T(n - n_H^{'})(a - k) > T(n - n_H^{'})(b - k) + (T - 1)n_H^{'} \cdot c,$$

which simplifies to $n_H^{'} < \frac{T \cdot (a-b)}{T \cdot (a-b+c) - c} n$, condition (b1). Condition (b2) follows from the gain a player, who is dove in $t = 1$, makes by switching to hawk in $t = 1$. As dove, she receives $(n_H^{'} \cdot c + (n - n_H^{'} - 1)b)T - \sum_{t=1}^{T} |\sigma_{i,t}^N| \cdot k$. When deviating to hawk in $t = 1$, she receives $n_H^{'} \cdot d + (n - n_H^{'} - 1)a - |\sigma_{i,1}^N| \cdot k + (T - 1)(n - n_H^{'} - 1)(a - k)$. Simplified, the payoff for a dove exceeds that for deviating to hawk if

$$n_H^{'} \cdot (T(a-b+c) - (T-1)k - d) < (n-1) \cdot (T(a-b) - (T-1)k) + k \sum_{t=2}^{T} |\sigma_{i,t}^N|.$$

This inequality corresponds to two conditions: (1) If $T(a-b+c)-(T-1)k-d < 0$, $n_H^{'} > ((n-1)(T(a-b)-(T-1)k)+k\sum_{t=2}^{T} |\sigma_{i,t}^N|)/(T(a-b+c)-(T-$

$1)k-d)$ has to hold, which is always fulfilled as $k\sum_{t=2}^{T}|\sigma_{i,t}^{N}| \leq k(T-1)(n-1)$ and $n_H^{'} \geq 0$ and corresponds to (b1). (2) If $T(a-b+c)-(T-1)k-d>0$, $n_H^{'} < ((n-1)(T(a-b)-(T-1)k)+k\sum_{t=2}^{T}|\sigma_{i,t}^{N}|)/(T(a-b+c)-(T-1)k-d)$. Here the right-hand side of the inequality is minimal if $k\sum_{t=2}^{T}|\sigma_{i,t}^{N}| = 0$. This corresponds to condition (b2). The network is in equilibrium in periods 2 to T, while all conditions described here ensure that the overall game is in equilibrium. Hence, the described equilibrium is also sub game perfect.

A.2.3. Repeated play with fixed links

Lemma 3. *Every link efficient strategy profile (σ^B, σ^N) of the Network Hawk-Dove game Γ which is no Nash equilibrium implies two Nash equilibria (s^B, σ^N) in a game with fixed links $\Gamma^{\overline{N}}$:*

a) For all i with $\sigma_i^B = H$, $s_i^B = H$ and

$$n_H + \delta_H - 1 < \frac{a-b}{a-b+c-d} \cdot (n-1) < n_H + \delta_H$$

holds, or

b) For all i with $\sigma_i^B = H$, $s_i^B = D$ and

$$\delta_H - 1 < \frac{a-b}{a-b+c-d} \cdot (n-1) < \delta_H$$

holds,

with n_H being the number of players with $\sigma_i^B = H$ and δ_H being the number of players with $\sigma_j^B = D$, $s_j^B = H$.

Proof 11. *For all link efficient strategy profiles which are no Nash equilibria in Γ $n_H < \frac{a-b}{a-b+c-d}(n-1)$ holds (see Theorem 6). All n_H players i with $\sigma_i^B = H$ have identical contacts as they are linked to all doves and no hawks. Hence, they all resort to the same action s_i^B. All $n_D = n - n_H$ other players either resort to $s_j^B = H$ or $s_k^B = D$. Condition (a) analyzes the case of $s_i^B = H$. Whether an equilibrium is reached, depends on the fraction of players j with $s_j^B = H$ and $\sigma_j^B = D$ to players k with $s_k^B = D$ and $\sigma_k^B = D$. Let the number of players i be n_H, the number of players j be δ_H, and the number of players k be $n_D - \delta_H$ with $n = n_H + n_D$. In the neighborhood of each player i $n_H^i(s) < \frac{a-b}{a-b+c-d} \cdot n^i(s)$ has to hold for player i to resort to hawk (see*

Theorem 4). For players i this is equivalent to $\delta_H < \frac{a-b}{a-b+c-d} \cdot (\delta_H + n_D - \delta_H)$ as they are linked to players j and k only. All players j and k are linked to all players in the network. Hence, for them $n_H + \delta_H - 1 < \frac{a-b}{a-b+c-d} \cdot (n-1)$ and $n_H + \delta_H > \frac{a-b}{a-b+c-d} \cdot (n-1)$, respectively, have to be fulfilled. The three inequalities are equivalent to

$$\delta_H < \frac{a-b}{a-b+c-d} \cdot (n - n_H) \tag{A.1}$$

and

$$n_H + \delta_H - 1 < \frac{a-b}{a-b+c-d} \cdot (n-1) < n_H + \delta_H. \tag{A.2}$$

Inequality (1) is fulfilled if the left inequality of (2) is fulfilled. Condition (b) focuses on all n_H players i resorting to $s_i^B = D$. Let the number of players j be δ_H with $s_j^B = H$ and the number of players k be $n_D - \delta_H$ with $s_k^B = D$. In the corresponding neighborhoods, the following inequalities have to hold: players i, $\delta_H > \frac{a-b}{a-b+c-d} \cdot (n - n_H)$; players j, $\delta_H - 1 < \frac{a-b}{a-b+c-d} \cdot (n-1)$, and players k, $\delta_H > \frac{a-b}{a-b+c-d} \cdot (n-1)$. The three inequalities are equivalent to

$$\delta_H > \frac{a-b}{a-b+c-d} \cdot (n_D) \tag{A.3}$$

and

$$\delta_H - 1 < \frac{a-b}{a-b+c-d} \cdot (n-1) < \delta_H. \tag{A.4}$$

Inequality (3) is contained in the right-hand part of inequality (4) if $n_H \geq 1$, which is fulfilled for all non Nash equilibria of the one-shot Network Hawk-Dove game (see Theorem 6).

Definition 4. *In a game limiting the strategy set to actions only in $t > 1$ and to the Network Hawk-Dove game in $t = 1$, let the trigger strategy be:*

a) In $t = 1$ play hawk or dove so that n_H hawks are in the network. If you are a hawk, link to all doves; if you are a dove, ensure that you are unilaterally linked to all other doves.

b) In $t \in \{2, ..., t^*\}$ keep playing your strategy as in (a) if no player deviated. Otherwise, play the equilibrium yielding the lowest payoff for the deviating player.

c) In $t \in \{t^*, ..., T\}$ keep playing your strategy if no player deviated from playing the equilibrium of the one-shot game yielding the highest overall payoff. Otherwise, play the equilibrium with the lowest payoff for the deviating player.

Lemma 4. *For a link efficient strategy profiles (σ^B, σ^N) in a Network Hawk-Dove Γ which are no Nash equilibria, the following two observations hold:*

a) Players with $\sigma^B = H$ reach higher payoffs with $s^B = H$ if $\delta_H < \frac{a-b}{a-b+c-d}(n - n_H)$ and with $s^B = D$ otherwise.

b) Players with $\sigma^B = D$ reach higher payoffs if $s^B = H$ than if $s^B = D$.

Proof 12. *Let n_H players i be the players with $\sigma^B = H$, δ_H players j be the players with $\sigma^B = D$, $s^B = H$, and $n_D - \delta_H$ players k be the players with $\sigma^B = D$, $s^B = D$.*

Condition (a) follows from link balance (see Theorem 4). Condition (b) follows from considering the payoffs of the players with $\sigma^B = D$. We consider two cases: (1) all players i choose $s^B = H$ and (2) all players i choose $s^B = D$. In case (1), each player j earns $(n_H + \delta_H - 1)d + (n_D - \delta_H)a$ and each player k earns $(n_H + \delta_H)c + (n_D - \delta_H - 1)b$. Hence, for $s^B = H$ yielding higher payoffs: $(n_H + \delta_H - 1)d + (n_D - \delta_H)a > (n_H + \delta_H)c + (n_D - \delta_H - 1)b$ has to hold. With $n_D = n - n_H$, this simplifies to

$$n_H + \delta_H - \frac{a-d}{a-b+c-d} < \frac{a-b}{a-b+c-d}(n-1). \tag{A.5}$$

As $\frac{a-d}{a-b+c-d} > 1$, this inequality is fulfilled for all link efficient strategy profiles that are no Nash equilbria (see Lemma 3, condition (a)). In case (2), the condition to resort to hawk changes to $(\delta_H - 1)d + (n_H + n_D - \delta_H)a > (\delta_H)c + (n_H + n_D - \delta_H - 1)b$, which simplifies to

$$\delta_H - \frac{a-d}{a-b+c-d} < \frac{a-b}{a-b+c-d}(n-1). \tag{A.6}$$

As $\frac{a-d}{a-b+c-d} > 1$, this inequality is fulfilled for all link efficient strategy profiles that are no Nash equilbria (see Lemma 3, condition (b)).

Theorem 9. *By fixing actions after the first period ($t = 1$), applying a trigger strategy yields no additional sub game perfect equilibria except for the ones played in the one-shot version of the Network Hawk-Dove game.*

Proof 13. *We now consider whether the trigger strategy can yield additional equilibria in which more than* $n_H = \frac{a-b}{a-b+c-d}(n-1)$ *hawks persist. A dove following the trigger strategy in the worst case earns* $t^*(n_H \cdot c + (n - n_H - 1)b) + (T - t^*) \cdot ((n_H + \delta_H) \cdot c + (n - (n_H + \delta_H) - 1)b)$, *with* δ_H *being the number of additional hawks necessary to reach an equilibrium. Doves who switch to hawk in periods* t^* *to* T *earn more during these periods according to Lemma 4. Let us assume the dove deviates to hawk in* t^* *to ensure minimal punishment. Now her payoff is* $(t^* - 1)(n_H \cdot c + (n - n_H - 1)b) + n_H d + (n - n_H - 1)a + (T - t^*) \cdot ((n_H + \delta_H) \cdot c + (n - (n_H + \delta_H) - 1)b)$. *To establish an equilibrium,* $t^*(n_H \cdot c + (n - n_H - 1)b) + (T - t^*) \cdot (n_H \cdot c + (n - (n_H + \delta_H) - 1)b) > (t^* - 1)(n_H \cdot c + (n - n_H - 1)b) + n_H d + (n - n_H - 1)a + (T - t^*) \cdot ((n_H + \delta_H) \cdot c + (n - (n_H + \delta_H) - 1)b)$ *has to hold. This simplifies to* $n_H > \frac{a-b}{a-b+c-d}(n-1)$, *which is equivalent to condition (c) of Lemma 6, i.e., the Lemma describing the sub game perfect equilibria in the Network Hawk-Dove Game. As doves are always connected to all other players in the network, we can conclude that the Nash equilibria in the Fixed Link Game are a subset of the equilibria in the Network Hawk-Dove Game.*

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