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The Relevance of Irrelevant Alternatives: An experimental investigation of risky choices

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Abstract

Experimental economists have discovered various violations of expected utility theory and offered alternative models that can explain laboratory results. This study discovers a new violation in risky choices that cannot be explained by theories like Prospect Theory, Disappointment or Regret Theory. In an experimental setting using a between-subject design, the influence of a dominated alternative on certainty equivalents is shown. One group of subjects was offered a series of choices between a lottery ticket with a 50-50 chance of winning and a sure payoff. A second group was offered the same choice plus a third alternative, that as it turned out was not chosen by any participant. As a result, the average chosen sure payoff in the second group was higher than in the first group. That means, by adding a dominated alternative to a choice set, the certainty equivalent of a lottery is increased.

1 Introduction

Since the beginning experimental economists provided results that show violations of Expected Utility Theory triggering the development of adaptations such as Prospect Theory and new more intuitive models such as Disappointment or Regret Theory. Nonetheless, in real world business settings most of the violations of the axiomatic structure Expected Utility Theory are not considered very relevant and decision support systems still use tools based on that theory. This paper reports experimental results that violate the very core of the known theories by showing that adding a dominated alternative to choice set can change decisions. From the experiment reported in this paper one can argue that people do not have consistent underlying preferences.

Therefore, this violation does not only challenge existing theory, but does have implications for most real world business settings.

The theory of rational choice claims that individuals have underlying preferences. However, when eliciting those preferences participants in laboratory experiments show violations of the axiomatic structure of expected utility theory. The theory of rational choice is based on two fundamental axioms, which are independence and transitivity. So far, a variety of violations of these axioms is reported in the literature and diverse models are offered to explain laboratory findings. Following a recent suggestion of modelling utility function with reference to the task at hand (Köszegi & Rabin, 2007), this paper explores basic decisions between prospects. Experimental results are reported that raise doubt about whether individuals have a consistent set of underlying preferences.

In the following passage a variety of such violations are discussed and possible solutions provided in economic literature mentioned. A known violation of the independence axiom is the common ratio effect (Allais, 1952), showing that the choice between two lotteries does depend on the scaling of the problem. Solutions to this violation are provided by the Machina triangle (Machina, 1982) by introducing fanning out utility functions and prospect theory (Kahneman & Tversky, 1979) introducing a weighting function over probabilities, stating an overestimation of low probabilities.

The axiom of transitivity is also found to be violated as described by the Allais Paradox (Allais, 1952). The violation describes the change of preferences after probabilities were lowered while relation between payoffs and probabilities of winnings remain the same. This effect is also accommodated by the weighting of probabilities in prospect theory (Kahneman & Tversky, 1979).

A different violation of transitivity has been observed in a form that is known as preference reversals (Lichtenstein & Slovic, 1971). The violation was interpreted by differences in mental processes of choosing between alternatives and the valuation (Slovic & Lichtenstein, 1983). Further observations of the phenomenon (Loomes *et al.*, 1991) were not accommodated by this explanation, could, however, be explained by regret theory (Loomes & Sugden, 1982). The most recent attempt to explain preference reversals was by imprecision (Loomes & Butler, 2007), however, the explanation fails to accommodate the violation discovered in this paper.

Another form of violation of the axioms of utility theory is the change of risk preferences for negative prospects, which means, while people tend to be risk averse for positive payoffs, they are risk seeking for negative payoffs (Fishburn & Kochenberger, 1979). This violation has also been accommodated by prospect theory (Kahneman & Tversky, 1979). To avoid the influence of

this effect, only prospects with positive payoffs were analysed in this paper.

Additionally inconsistencies in preferences occur when components of alternatives are disregarded by decision-makers because they focus on components that distinguish alternatives (Tversky, 1972). Prospect theory claims that the inconsistencies are caused by an editing phase in the decision, where problems are processed in a person's mind before alternatives are evaluated. All the models mentioned above have one assumption in common, which is that individuals have consistent underlying preferences (Cubitt *et al.*, 2001). While there is growing evidence of violations of transitivity, this paper challenges the assumption of the existence of such underlying preferences. With consistent underlying preferences, one has to assume that adding a third alternative to a set of choices that is dominated by the two previous choices could not have an effect on the certainty equivalent of another lottery. That means, if $A \succ B \succ C$ the CE of A has to be the same whether C is offered or not. This paper will show that adding an alternative C to a set of choices alters the CE of A.

In order to design laws to protect individuals from poor choices in the result of false consultation by financial agents and to provide financial institutions with instruments to discover their clients' needs one has to understand the processes of choice in the human mind. The question therefore is, what the point of consumer protection is if preferences are altered by introducing irrelevant alternatives.

2 Experiment

The group of participants consisted of 186 students from the Otto-von-Guericke University Magdeburg matched randomly to five different groups. The experiment was conducted in a laboratory environment.

Session 1 (18 participants) consisted of two steps. First the participants were shown a table with two lottery tickets on it, Lottery A {0.5, 1'000; 0.5, 0} and Lottery B {p, 5'000; 1-p, 0}. The participants were asked to determine a p for which it did not matter to them, which of the two lotteries they would receive. In order to pay this choice the experimenter drew a ball from a bingo cage labelled with probabilities 1%, 2%, 3%, ..., 100%. If the ball indicated a probability smaller than the p chosen by the participants Lottery A was paid. If the ball indicated a probability equal to the p chosen by the participant the toss of a coin determined which lottery was played and if the ball indicated a probability larger than the p chosen by the participant, Lottery B was played with the p being the probability drawn from the bingo cage.

In the second step they were shown another table with the two lottery tickets A and B (with p being the probability chosen by the participants in the step before) and a sure payoff X. Only one of the lotteries could be received but the participants did not know which one. They were asked to determine a sure payoff X for which it did not matter to them whether they would receive the sure payoff X or one of the lotteries. Although the first step ensured that the participants were indifferent between the lotteries, they still had the chance to indicate two values for X in the case this value depended on which lottery would be laying next to it. However, none of the participants chose to give two values for X. For the payment of this choice the experimenter drew a ball from a bingo cage with balls labelled with money values 1 Euro, 2 Euro, 3 Euro, ..., 5'000 Euro. If the ball indicated a money value smaller than the X chosen by the participants one of the lotteries was paid. If the ball indicated a money value equal to the X chosen by the participant the toss of a coin determined whether one of the lotteries was played or the participant received the money value X, and if the ball indicated a money value larger than the X chosen by the participant, the sure money value on the ball was paid.

Session 2 (20 participants) was similar, but this group only performed step 2 and only Lottery A and the sure payoff X was offered. That means, the choice was reduced by the Lottery B, which is a second lottery which the participants did not prefer over Lottery A due to the procedure in step 1. The payment procedure was the same as in step 2 of session 1.

Session 3 (29 participants) consisted of 13 choices between three alternatives, Lottery A {0.5, 1'000; 0.5, 0}, Lottery B {0.1, 5'000; 0.9, 0}, and a sure payoff S (with S = 200...440). The procedure is similar to (Holt & Laury, 2002). The participants indicated for each choice their preferred alternative by crossing the alternative they preferred. Also, they could indicate indifference between two alternatives by crossing the two alternatives they preferred over the third and indifference between all alternatives by crossing all of them.

	3 Alternatives	2 Alternatives
Table	**Session 1** *Step 1* Lottery A: {.5,1'000,.5,0} Lottery B: {p,5'000,(1-p),0} Task: Determine a p, for which it does not matter to you, which of the alternatives you receive *Step 2* Lottery: A or B Money value: X Task: Determine an X for which it does not matter to you whether you receive one of the lotteries or the sure money	**Session 2** Lottery: {.5,1'000,.5,0} Money value: X Task: Determine an X, for which it does not matter to you, whether you receive the lottery or the sure money
Paired Choices	**Session 3** *Step 1* Lottery A: {.5,1'000,.5,0} Lottery B: {.1,5'000,.9,0} Money Value: S (with $S = 200...400$) Task: 13 choices between A, B, and S	**Session 4** Lottery: {.5,1'000,.5,0} Money value: X Task: 13 choices between A and S

Table (small)	**Session 5** *Step 1* Lottery A: {.5,10,.5,0} Lottery B: {p,50,(1-p),0} Task: Determine a p, for which it does not matter to you, which of the alternatives you receive *Step 2* Lottery: A or B Money value: X Task: Determine an X for which it does not matter to you, whether you receive one of the lotteries or the sure money	**Session 6** Lottery: {.5,10,.5,0} Money value: X Task: Determine an X, for which it does not matter to you, whether you receive the lottery or the sure money
Paired Choices (small)	**Session 7** *Step 1* Lottery A: {.5,20,.5,0} Lottery B: {.1,100,.9,0} Money Value: S (with $S = 2.00...8.80$) Task: 13 choices between A, B, and S **Session 9** Lottery A: {.5,20,.5,0} Lottery B: {.1,4,.9,0} Money Value: S (with $S = 2.00..8.8$) Task: 13 choices between A, B, and S	**Session 8** Lottery: {.5,20,.5,0} Money value: X Task: 13 choices between A and S

Table 1: Treatments

Session 4 (19 participants) consisted of the same mechanism as session 3, but without Lottery B leaving only the lottery {0.5, 1'000; 0.5, 0} and a sure payoff S (with S = 200...440).

For all participants in all sessions the decisions were about real money

depending on a condition. For each participant the experimenter placed a bet of 5 Euro on the number 19 of an American-Roulette-Table at the Casino in Magdeburg. If that bet won it paid 35 to 1 and all the winnings were placed on the number 23. If both bets won it paid 35 x 35 x 5 Euro = 6'125 Euro. In this case one of the choices of that participant became real. For participants in session 1 the toss of a coin determined which one of the choices was paid and for participants in sessions 3 and 4 the experimenter drew a ball from a bingo cage containing 13 balls numbered from 1 to 13. The number of the ball determined which choice was paid.

In sessions 5 (17 participants) and 6 (19 participants) the choices of sessions 1 and 2 were repeated, but with smaller sums of money. The lotteries offered had possible payoffs of 50 and 10 Euro. For each participant the one choice was paid at the end of the session without the dependence on winning in the Casino. Furthermore, tasks from sessions 3 and 4 were repeated in sessions 7 (14 participants) and 8 (19 participants). This time with lotteries {0.1,100;0.9,0} and {0.5,20;0.5,0}.

The final session 9 (29 participants) was similar to session 7, but alternative B was exchanged for a different lottery. In that session we offered lottery {0.1, 4, 0.9, 0}, which is clearly dominated by both of the other alternatives since both payoff and probability of receiving a positive payoff are lower.

3 Results

In sessions 1, 2, 5 and 6 the CE was elicited by giving the participants a choice between two lotteries and asking for a probability of winning for which it did not matter for them which of the lotteries they would receive. Following the procedure applied, the participants were indifferent between the lotteries for the probability of winning in alternative B. Since none of the participants chose to provide different money values it can be concluded that they were indifferent between the lotteries. However, the median of the CE was higher in the group with three alternative, 775 compared to 325 in the group with two alternatives. The CE is significantly larger in the group with three alternatives on the 1%-level (Wilcoxon-Test). 5 participants in group 1 even chose CEs larger than 1.000 Euros. The result does not change, however, if we exclude these 5 persons from the analysis.

Five participants in session 5 provided CEs larger than 10 Euro. These choices seem rather extreme and were excluded from the analysis. Furthermore it has to be noted, that three participants provided larger values for X if compared to lottery B than for lottery A. For purposes of this study these participants were excluded from the analysis since the added alterna-

Lotteries	Median CE (2 Alternatives)	Median CE (3Alternatives)	Significance (Wilcoxon-Test)
Paired Choices	300	400	1%-Level
Paired Choices (small)	5.00	7.00	5%-Level
		7.60	5%-Level
Table	325	775	1%-Level
Table (small)	6.20	7.80	1%-Level

Table 2: Test on differences in median CE

tive was not dominated. Values for X were significantly higher (1%-level, Wilcoxon-Test) in session 5 than in session 6.

For sessions 3 and 4 a different method of eliciting the CE was applied. The median CE for the group with three alternatives is 400 and larger than the median in the group with two alternatives with a median of 300. In group 3 most of the participants never chose alternative B, which means that these participants faced basically the same choice as participants in group 4. However, the CE in group 3 is significantly larger than in group 4. 4 participants in group 3 preferred alternative B for small money values in alternative C. Furthermore, 7 participants in group 4 indicated indifference between the offered lotteries for small values in alternative C. The analysis shows that the CE in group 3 for people who never choose the lottery 0.1, 5'000; 0.9, 0 is significantly larger than in group 4 on the 1%-level (Wilcoxon-Test). Including the participants who indicate indifference between lotteries A and B and those who preferred lottery B in some cases does not change this result.

Session 9 represents a rather extreme case since the added alternative was dominated by both alternatives. The risky choice offered a larger payoff with higher probability and the sure payoff was larger than the positive outcome of the lottery in alternative B. As expected none of the participants chose this alternative. The result shows that by adding a third alternative that is as clearly dominated as in this case, the median CE for the lottery in alternative A is significantly larger than in the group with two alternatives (Wilcoxon test, 5%-level).

4 Conclusion

The results reported show that introducing an alternative, that is dominated and therefore not chosen by the participants can increase the CE of the dominating alternative. For that reason, a range of choices between the domi-

nating alternative and a sum of money participants reverse their preference if the dominated alternative is added to the set of choices.

This violation of existing theory has implication in many business settings. For example, when consultants present managers with options, it might not always be useful to eliminate dominated alternatives before presenting the decision maker with the facts. Furthermore, when financial consultants advise private investors on how to invest money into pension funds, thought has to be given on what options should be provided to the investor. If adding a dominated alternative to the choice set alters the decision of the investor, the risk structure of the portfolio changes once the alternative is added. Therefore, the consultant cannot identify the real preference structure of the investor by offering different choice sets. The implication of the violation of independence of dominated alternatives has been discussed already in (Arrow, 1951). Consequences have to be discussed for eliciting social preferences, bargaining and choices under uncertainty.

For the elicitation of social preferences, a variety of methods have been discussed in economic literature. Attention has been drawn on the possibility of strategic voting (Myatt, 2007), where true preferences are not reflected, to alter the overall outcome to one's favour. The result of this paper adds another point to that discussion. Following these results the alternatives that are presented do the decision makers are subject to strategy as whether a dominated alternative is offered can affect the preferences of the individuals. The same is true for bargaining situations. One can think of situations, where offering the opponent something one knows the other side does not want can alter the valuation of other items.

Prospect Theory is a powerful tool to explain a number of violations of utility theory and is seemingly more promising to pursue in the future than redefining utility theory (Camerer, 1998). The violation discussed in this paper, however, cannot be explained by either theory. The weighting of probabilities cannot explain this phenomenon since it does not explain the evaluation of one prospect dependent on other alternatives. A simpler and more intuitive model was proposed in form of regret theory (Bell, 1982) (Loomes & Sugden, 1982) which became an axiomatic structure (Sugden, 1993). Although regret theory is applicable to decisions where prospects are involved and explain phenomena like the reluctance of people to exchange lottery tickets (Bar-Hillel & Neter, 1996). However, regret theory implies independence of irrelevant alternatives and cannot explain the violation found in this paper.

The case-based decision theory (Gilboa & Schmeidler, 1995) suggests that the context in which a decision is made determines how preferences are formed by individuals. However, it still does not explain the phenomenon dis-

covered in this paper. According to that theory the third alternative might alter the way how participants perceive the situation, but a difference of preferences between the dominating alternatives are not accommodated.

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