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Capacity Reservation under Spot Market Price Uncertainty

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Abstract: The traditional way of procurement, using long-term contract and capacity reservation, is competing with the escalating global spot market. Considering the variability of the spot prices, the flexibility of combined sourcing can be used to benefit from occasional low short-term spot price levels while the long-term contract is a means to hedge the risk of high spot market price incidents. This contribution focuses on the cost-effective management of the combined use of the above two procurement options. The structure of the optimal combined purchasing policy is complex. In this paper we consider the capacity reservation - base stock policy to provide a simple implementation and comparison to single sourcing options. Our analysis shows that in case of large spot market price variability the combined sourcing is superior over spot market sourcing even in case of low average spot market price and also superior over long-term sourcing even in case of high average spot market price.

Key words: Capacity reservation; spot market; purchasing policy; supply chain contracts; stochastic inventory control

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1. Introduction

With the growing importance of electronic commerce and global sourcing on the spot market, new procurement opportunities have evolved which are competing with the traditional contract based procurement. Spot market sourcing has a large flexibility but also a high risk of price increase. Long-term procurement price is, on average, lower than the price of short-term supply. In order to profit from this cost advantage often a long-term capacity reservation contract has to be negotiated. We consider contracts that entail the delivery of any desired portion of a reserved fixed capacity in exchange for a guaranteed payment by the buyer. Long-term contracts provide price reliability but much less flexibility even with the option that the reserved quantity is not required exercising. In this environment of multiple procurement options with differences in costs and flexibility, the process of coordinating the procurement activities has become more difficult including the decision about the alternatives of long-term procurement contracts or short-term procurement or if a combination of both alternatives should be used.

This contribution focuses on the cost-effective management of the combined use of these procurement options. In our case, the short-term option is given by a spot-market with a random spot-market price (which is independent of the quantity procured), whereas the long-term alternative is characterized by a wholesale price contract with a capacity reservation level. The planning situation we consider gains further complexity by the fact that in addition to the stochastic spot-market price also the randomness of demand for the procured goods is taken into account. Under these conditions, the managerial decision is to fix a long-term capacity reservation level and to decide period-by-period on how to combine the two supply options in order to profit from the cost savings of long-term procurement while still remaining flexible. Concerning the price variations on the spot market, this flexibility can be used to benefit from low short-term price levels while the long-term contract is a means to hedge the risk of high spot prices.

This paper is related to different research streams in the operations management literature. A topic of recent interest is the study of the supply chain procurement strategies combining spot market purchases with purchases made in advance from a specific long-term supplier. Henig et al. (1997) derived a three-parameter optimal policy but without the consideration of uncertainty that is a critical factor in practice. Bonser and Wu (2001) studied the fuel procurement problem for electric utilities in which the buyer can use a mix of long-term and spot purchases. Our problem was first defined and studied in the inventory literature by Serel et al. (2001). They considered the simple (*R, S*) capacity reservation – order up policy but they disregarded the spot market price uncertainty. Wu, Kleindorfer, and Zhang (2002) considered uncertainty in spot market prices and analyzed the contracts for non-storable goods involving options executable at a predetermined price. Using a similar single-period model, Spinler and Huchzermeier (2006) show that, mainly due to the decrease in the supplier's production costs when an options contract is used, the combination of an options contract and a spot market is Pareto improving with respect to the other alternative market structures. Seifert et al. (2004) also analyzed a single-period problem from the buyer's standpoint with changing levels of buyer's risk preferences. Kleindorfer and Wu (2003) linked this literature to evolving B2B exchanges on the Internet. In Sethi et al. (2004) a situation with both demand and price uncertainty is taken into consideration and a quantity flexibility contract is employed, but no capacity reservation takes place. Serel (2007) considered spot market uncertainty but not in price but in available quantity.

Various types of supply contracts involving advance capacity purchases have been investigated, generally based on a single-period framework. Erkoc and Wu (2005) model the negotiations between a manufacturer and a supplier when the supplier has to make a costly investment in additional production capacity. Jin and Wu (2001) analyzed capacity reservation contracts between a single supplier and multiple buyers with reservation fees deductible from the purchase price paid in delivery. Deng and Yano (2002) studied the contracts between a component supplier and a manufacturer involving a fixed wholesale price for advance purchases, and a spot price determined and charged for purchases after the demand is realized. Burnetas and Ritchken (2005) considered the impact of option contracts on the wholesale and retail prices under price-dependent demand in a manufacturer–retailer chain. Other papers dealing with the use of options in supply chains include Kamrad and Siddike (2004).

In our paper, we consider both demand and spot market price uncertainty in a multi-period framework. The optimal combined purchasing policy structure is a complex three-parameter decision policy with a price-dependent order-up-to level for short-term procurement. In this paper we consider only the combined reservation and base stock policy to provide a simple implementation and comparison tool to single sourcing options. In Section 2, the optimal capacity reservation and base stock policy is derived. In Section 3, managerial and numerical results are presented which give also an impression on the performance of the two-parameter optimal base stock policy developed for this inventory management and capacity reservation problem. In Section 4, we compare this policy with the optimal single sourcing policy of short-term procurement only and also with the sourcing using long-term procurement only. We discuss under which conditions it is better to use the combination of both alternatives and what is the expected monetary gain compared to the single sourcing options. We close the paper with future extension plans.

2. Simple base stock policy combining capacity reservation and spot market procurement

We assume that for the random stationary demand, ξ, per period and random spot market price, π, per period we know the following characteristics:

$F(x)$, μ_x, σ_x	distribution function, expected value and standard deviation of **demand** and
$G(p)$, μ_p, σ_p	the same distribution characteristics for the **spot market price**.

Both demand and price are assumed to be independent and identically distributed.

We consider a periodic decision process involving different level of knowledge in time. The **first decision** is on

R the capacity reservation quantity

that must be fixed for a longer time horizon based on the random demand and spot market price distribution and the following stationary cost factors:

c the unit purchase price charged by the long-term supplier,

r the capacity reservation price per period for a unit of capacity reserved,

h the inventory holding cost per unit and period,

v the shortage cost per unit and period.

The **next decision** is at the beginning of each time period about

Q_L order quantity from the long-term supplier, and/or

Q_K order quantity from the spot market

at the beginning of each period, t, knowing

I_t inventory level at the beginning of the period and

p_t the realized spot market price

but without knowing the realized demand for the period. The shipments are assumed to arrive before the period demand is realized. The final cost and finishing inventory level is only known after the realization of the demand at the end of the period.

The optimal policy structure for the above combined ordering decision process, is an ($R, S_L, S_K(p)$) policy, characterized by the constant capacity reservation quantity, R, and constant base stock level S_L for long-term supplier and a price dependent base stock S_K(p) for spot market. However we can analytically prove the optimality of the above policy (Inderfurth and Kelle, 2008), the optimal parameters generally can only be calculated by elaborate numerical methods. Thus, for practical applicability, we have the two main options: either to provide a simple *heuristic approximation* for the policy parameters or consider a *simpler policy structure* where the optimal parameters can be derived analytically. In this paper the second option, a simplified policy is considered.

For practitioners, a simple decision policy is often appealing even if it is not providing the possible lowest cost. In this paper we consider the combined reservation and base stock policy to provide a tool for simple implementation and comparison to single sourcing options. A periodic order-up policy is considered where the base stock, S, determination is integrated with the decision on the capacity reservation quantity, R. Determining the optimal (R, S) parameters we evaluate the combined long-term and spot market procurement. The ordering policy is a simplified version of the optimal ordering policy in the sense that there is only one static order up to level, S, replacing both S_L and $S_K(p)$.

Proposition 1: In backorder case the optimal capacity reservation quantity, R, and base stock, S, of the combined ordering policy are

$$R^* = F^{-1}\left(\frac{\Delta - r}{\Delta}\right) \tag{1}$$

$$S^* = F^{-1}\left(\frac{v}{h+v}\right) \tag{2}$$

with the cost parameters, r, h, and v defined above and the *conditional expected gain*, Δ , of having the fixed price, c, in case of higher spot price (p > c) that can be expressed by

$$\Delta = E[\pi - c \mid \pi > c] = \int_{c}^{\infty} (p-c)g(p)dp \tag{3}$$

further, $F^{-1}(\,)$ denotes the inverse of the cumulative distribution function of demand.

Proof: see in Appendix A.

3. Managerial evaluation of the combined sourcing

The simplicity of the above combined sourcing policy makes it easy to apply and the parameters as the percentiles of the demand distribution are easy to calculate. For regular demand distributions the inverse $F^{-1}(\,)$ is a monotone increasing function, thus S* increases with shortage cost increase and decreases with holding cost increase as we expect. The optimal base stock level, S*, does not depend on the procurement price data, it is always positive if the shortage cost is positive.

The combined sourcing has the advantage of using long-term contract to hedge against high spot price. The capacity reservation quantity, R, determines the amount of hedge. The optimal R* does not depend on the holding and shortage cost it is a monotone increasing function of the relative net gain, $(\Delta\text{-r})/\Delta$, that can be achieved by having the fixed price hedge in case of higher spot price ($p > c$). It is always economic to reserve capacity if the capacity reservation price, r, is less than the expected gain, Δ, but $R^* = 0$ if $r > \Delta$.

From equation (1) directly follows that the optimal quantity of the long term contract, R*, is decreasing with the increase of the *capacity reservation price, r*. From equation (3), we can also see that Δ decreases with the increase of the *contract price, c*, thus R* is also decreasing function of c. On the other hand, from equation (3) we can derive that with higher *average spot price*, μ_p, the long-term contract is getting more and more attractive as the relative gain, Δ, increases. As we may expect the optimal contract quantity, R*, also increases with increasing *expected demand*, μ_x.

It is interesting to examine the effect of *spot price and demand uncertainty*. In order to examine a wide range of spot price and demand variability, we used the versatile gamma distribution for the demand and price distribution (F and G). With fixed expected values $\mu_x = 100$, $\mu_p = 100$ the different σ_x and σ_p parameters represent the coefficient of variation in percentage as a measure of uncertainty.

With larger σ_p the relative expected gain, $(\Delta\text{-r})/\Delta$, is increasing and the effect is an increasing R* and the increase rate is getting larger for larger demand variability as it is illustrated in Table 1. This shows the important message that *with increasing spot price uncertainty* the *reservation quantity should be increased more and more as demand uncertainty is increasing*.

σ_x	σ_p=	20	40	60	80	100
	$(\Delta\text{-r})/\Delta$ =	0.53	0.63	0.70	0.74	0.78
20	R*=	100.17	105.40	109.45	111.99	114.80
40		97.67	108.17	116.52	121.89	127.89
60		92.55	108.11	120.86	129.21	138.68
80		85.03	105.16	122.19	133.57	146.65
100		75.50	99.43	120.40	134.71	151.41
120		64.58	91.24	115.58	132.58	152.76

Table 1. The optimal R* for combined sourcing for the case of $\mu_x = 100$, $\mu_p = 100$, r = 10, c = 80, h = 20, v = 50 and gamma distributed price and demand with different variability.

It is more challenging to examine the effect of demand uncertainty, σ_x, on R*. As we indicated earlier, R* = 0 if $r \geq \Delta$, that is the capacity reservation price, r, is larger than the expected gain of having the **fixed** price. As the capacity reservation is getting economic, with decreasing capacity reservation price, for demand with symmetric distribution like normal or uniform demand there are two different ranges as the capacity reservation price decreases. First R* is a *decreasing* function of σ_x, if $\Delta/2 \leq r < \Delta$. Further decreasing the capacity reservation price, for $r < \Delta/2$, R* will become an *increasing* function of σ_x, The formal proof of the above statement for normal and uniform distributed demand is in Appendix B. This ambiguous influence of σ_x is due to the fact that for sufficiently small r or for sufficiently large Δ (i.e. $r < \Delta/2$) reservation level R* is higher than mean demand μ_x, while the opposite (i.e. $R^* < \mu_x$) holds for high reservation price $r > \Delta/2$. Obviously, for $R^* > \mu_x$ the reservation level R* is further increasing with an increase in σ_x while this impact is reverse in case of $R^* < \mu_x$. Thus, the change of Δ caused by increasing σ_p has an impact on the direction in which σ_x affects the reservation level R*.

However, for general demand distribution there is no such general monotonous effect described above. We see different effects of demand variability depending also on the price variability. We summarize next the general tendencies based on numerical experiments.

- For large $(\Delta\text{-r})/\Delta$ relative gain the optimal R* is a monotone *increasing* function of the demand variability, σ_x.

- For smaller $(\Delta\text{-r})/\Delta$ values the optimal R* is monotone *increasing* function of σ_x for smaller spot price variability σ_p , then it becomes a monotone *decreasing* function of σ_x for larger spot price variability σ_p

- Further decreasing $(\Delta\text{-r})/\Delta$ the optimal R* becomes a monotone *decreasing* function of σ_x.

This non-monotonous impact of demand variability is typically observed if we consider a non-symmetric demand distribution like gamma distribution. For a skewed triangular distribution, as an example, such an effect can be shown analytically. In general, we find that monotony is restricted because a variation of σ_x for a given μ_x level also changes the skewness and shape of the distribution. Thus, the actual ranges of $(\Delta\text{-r})/\Delta$ and σ_x depend on several factors, including the shape of the distribution and it requires further investigations.

In a particular period, t, the optimal *combined sourcing* policy can result in single sourcing (with *spot market only* or with *long-term supplier only*) or *dual sourcing*, depending on the actual spot market price and initial inventory. In a period, t, with $p_t < c$ apply *only spot* purchase ordering up to the base stock level, S*. In a period with $p_t \geq c$ use *only long-term* supplier if the reserved capacity is sufficient to order up to S* otherwise apply *combined sourcing*, ordering the reserved quantity from the long-term supplier and the rest from the spot market for the higher price. This way the relatively simple base stock policy provides large flexibility and cost advantages over single sourcing as we show it next.

4. Comparison of combined sourcing with using single sourcing (spot market or long-term)

If the **spot market** is the only purchasing source, the optimal base stock level, S*, is identical to expression (2) of combined sourcing. Thus the inventory holding and shortage cost is the same as for the optimal combined sourcing policy. The expected purchase cost using the spot market only is the product of the expected demand and spot market price

$$E_{spot}(PC) = \mu_x * \mu_p. \tag{4}$$

The expected purchase cost using combined sourcing can be expressed with our previous notation as

$$E_{comb}(PC) = \mu_x(\mu_p - p^+) + c[1 - G(c)]\left[\int_0^R xf(x)dx + R(1 - F(R))\right] + p^+ \int_R^\infty (x - R) f(x)dx \tag{5}$$

using the additional notation

$$p^+ = \int_c^\infty pg(p)dp \tag{6}$$

Thus the cost saving of using combined sourcing compared to single spot market sourcing is

$$CS_1 = E_{spot}(PC) - [E_{comb}(PC) + r\,R] \tag{7}$$

including the capacity reservation cost into the comparison. This cost saving monotonously increases with the increase of the price variability and with the decrease of demand variability, contract price and reservation price. The managerial evaluation is discussed next.

Intuitively, the spot market seems to be the best option if the expected spot price is less than the sum of the contract price and reservation price, $\mu_p < c + r$. However, this statement is valid only if the spot price variability, measured by the coefficient of variation ($CV_p = \sigma_p / \mu_p$), is small. In this case $r > \Delta$ and the optimal $R^* = 0$, and no reservation is the optimum. Though, as the variability of the spot price is increasing the conditional expected gain, Δ, of having the fixed price, is getting larger and the combined sourcing ($R^* > 0$) provides cost improvements that gets larger with increasing spot price uncertainty. Using gamma distributed spot price, we illustrate in Table 2 that even in an inferior case for combined sourcing when the expected spot price is considerable less than the sum of the contract price and reservation price, the combined sourcing is still more economic. The condition is that the coefficient of variation for the spot price is above a specified limit, CV_p^{min} , illustrated in Table 2.. With increasing spot price variability the optimal reservation quantity and also the monetary gain of combined sourcing increases.

μ_p=100, r = 10								
c =	90	100	110	120	130	140	150	160
CV_p^{min}=	5.5	26.1	34.8	43.4	52.1	58.1	63.3	69.2

Table 2. Minimal value of the *coefficient of variation for the spot price*, CV_p^{min} *, that provides cost advantage for combined sourcing* with contract price, c, and reservation price, r , expressed as the percentage of average spot price, μ_p.

The monetary gain of using combined sourcing depends also on the demand variability. The larger the demand coefficient of variation ($CV_x = \sigma_x / \mu_x$) the smaller is the monetary gain of combined sourcing over using spot market sourcing only. In Table 3 we illustrate the joint effect of price and demand variability on the relative monetary gain achieved by combined sourcing for the case when the expected spot price is the same as the sum of contract price and reservation price. Because of the wide range of variability, we used gamma distributed spot price and demand as earlier. The actual gain percentages depend on the cost parameters but the tendencies are the same as illustrated in Table 3.

σ_x	σ_p= 20	40	60	80	100
20	2.77	9.47	17.53	26.57	36.39
40	1.91	7.28	13.99	21.53	29.66
60	1.25	5.46	10.98	17.24	23.98
80	0.77	3.99	8.47	13.66	19.24
100	0.44	2.82	6.43	10.70	15.34
120	0.23	1.94	4.79	8.29	12.14
140	0.11	1.29	3.50	6.35	9.53
160	0.05	0.82	2.51	4.80	7.43

Table 3. The *expected percentage monetary gain* of combined sourcing compared to spot market sourcing only for the case of $\mu_x = 100$, $\mu_p = 100$, r = 10, c = 90, h = 20, v = 50 and gamma distributed price and demand with different variability.

The impact of increasing price variability σ_p on the relative monetary gain of dual sourcing is straight-forward. The loss in monetary gain that is observed for increasing demand variability σ_x is mainly due to the increasing expected holding and shortage costs which are the same for combined and single sourcing in this case.

If the **long-term supplier** is the only purchasing source, the inventory position cannot be increased to the base stock level if the reserved capacity is not sufficient. The capacity-reservation and base stock parameters depend on each other. This fact makes the policy optimization more complex. In Appendix C we derive the steady state distribution of the inventory position and based on it, the expected cost per period. Generally, the optimal policy parameters cannot be expressed in analytic form, thus numerical search technique and simulation was applied to find the optimal R and S parameters.

The optimal R and S parameters for the long-term only supply do not depend on the spot price. However, they are close to the optimal parameters of the combined sourcing policy calculated for small spot price and demand variability. The combined sourcing has smaller R and S parameters as the demand and/or price variability increases to provide more flexibility for spot market sourcing.

The cost savings of using combined sourcing compared to long-term sourcing only is monotonously increasing with the increase of the price variability and with the decrease of contract price and reservation price. The impact of demand variability σ_x is ambiguous. This reflects the ambiguity of the impact of σ_x on the optimal capacity reservation level R*.

The long-term single sourcing option has the same expected cost as the combined sourcing as long as the demand and stock market price variability are both very low. However, with increasing uncertainties the cost advantage of using the spot market is increasing. For higher *demand uncertainty* the cause of gain is the finite contract limit that may not be enough to order up to the base stock level increasing the chance of stockout and resulting high shortage costs. For higher spot market *price uncertainty* there is a higher chance to gain from occasional low spot prices. The monetary gain is a strictly increasing function of the spot price variability but as a function of the demand variability the strict increase is only valid for combinations of small price and sufficiently large demand variability.

In Table 4 we illustrate the joint effect of price and demand variability on the relative monetary gain achieved by combined sourcing for a case when the expected spot price is considerable higher than the sum of contract price and reservation price (spot market is inferior). As earlier, we used gamma distributed spot price and demand. The actual gain depends on the cost parameters but the tendencies are the same as illustrated in Table 4.

σ_x	σ_p = 20	40	60	80	100
20	0.68	4.98	9.38	18.85	28.08
40	0.93	4.36	9.59	17.01	23.49
60	2.78	4.87	10.04	16.03	21.98
80	4.45	7.95	10.12	15.41	20.10
100	5.80	8.21	11.11	15.62	18.79
120	6.50	8.46	11.96	16.41	19.30
140	8.78	9.03	14.07	17.40	17.01
160	14.47	17.02	16.43	22.56	22.78

Table 4. The *expected percentage monetary gain* of combined sourcing compared to long-term single sourcing option for the case of $\mu_x = 100$, $\mu_p = 100$, r = 10, c = 70, h = 20, v = 50 and gamma distributed price and demand with different variability.

5. Conclusions

Our analytic and numerical investigations provided plenty of evidence of the advantages of the capacity reservation and simple base stock policy allowing combined sourcing over the single sourcing options. They showed that using combined sourcing can be highly advantageous in many cases. Further research is necessary to reveal if these results hold in the same way if lost sales have to be considered instead of backorders. Furthermore, we don't know how far our simple base stock policy from the global optimum is. As we mentioned, the optimal policy structure for the above combined ordering decision process, is an ($R,S_L,S_K(p)$) policy, characterized by the constant capacity reservation quantity, R, and constant base stock level S_L for long-term supplier and a price dependent base stock $S_K(p)$ for spot market. Stochastic dynamic programming solutions should be calculated using numerical methods to provide a benchmark.

The elaborate numerical calculations of the stochastic dynamic programming solutions cannot serve as a practical alternative. However, for practical applicability we have another option: to provide a simple heuristic approximation for the policy parameters. It is an open research stream to consider this option.

Another research direction is to evaluate the impact of buyer's decision on long term supplier's contract parameterization. This can help to extend the analysis to supply chain coordination among long time supplier and buyer.

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Appendix A. Proof of Proposition 1.

The expected purchase cost per period, E(PC), for the combined sourcing with different prices c and π can be expressed in two different forms depending on the actual realization, p, of the spot market price π.

For p < c :

$$E_1(PC) = E[E(\pi Q_K \,|\, \pi < c)] = \mu_x \int_0^c pg(p)dp \qquad \text{(A1)}$$

and for p ≥ c and backorder case:

$$E_2(PC) = cE[E(Q_L|\pi \geq c)] + E[E(\pi Q_K \mid \pi \geq c)]$$
$$= c\left[1-G(c)\right]\left[\int_0^R xf(x)dx + R(1-F(R))\right] + \int_c^\infty pg(p)dp\int_R^\infty (x-R)f(x)dx \quad \text{(A2)}$$

thus the purchasing cost $E(PC) = E_1(PC) + E_2(PC)$ does not depend on S.

The expected total cost of purchasing, inventory holding/shortage plus capacity reservation in a period is

$$ETC(R,S) = E(PC) + L(S) + rR \quad \text{(A3)}$$

with expected inventory holding/shortage cost

$$L(S) = h\int_0^S (S-x)f(x)dx + v\int_S^\infty (x-S)f(x)dx \quad \text{(A4)}$$

Since E(PC) does not depend on S and L(S) does not depend on R, it is easy to show that ETC(R,S) is convex and the necessary optimality conditions of setting the partial derivatives equal to zero provide the global optimal parameters R and S specified in expressions (1) and (2) of Section 2.

Appendix B.

Since for normal distribution $R^* = \mu_x + \Phi^{-1}(1-r/\Delta)\cdot\sigma_x$ and for uniform distribution

$R^* = \mu_x + (1-2r/\Delta)\cdot\sqrt{3}\cdot\sigma_x$, the partial derivatives have the values

$$\Rightarrow \quad \frac{\partial R^*}{\partial \mu_x} > 0$$

$$\Rightarrow \quad \frac{\partial R^*}{\partial \sigma_x} > 0 \quad \text{if} \quad r < \Delta/2$$

$$\Rightarrow \quad \frac{\partial R^*}{\partial \sigma_x} \leq 0 \quad \text{if} \quad \Delta/2 \leq r < \Delta$$

Appendix C.

The ordering policy with long-term sourcing and capacity reservation R

$$Q_L = \begin{cases} S - I & \text{if} \quad S - I \le R \\ R & \text{if} \quad S - I > R \end{cases}$$

with I denoting the inventory at the beginning of a period (and also at the end of previous period)

The inventory process

$$I_1 = \begin{cases} S - x & \text{if} \quad I_0 \ge S - R \\ I_0 + R - x & \text{if} \quad I_0 < S - R \end{cases}$$

with I_0 / I_1 denoting the inventory at the beginning/end of a period and x denoting the demand of a period. The conditional distribution function of the inventory can be expressed

$$\text{Prob}\{I_1 \le y | I_0 = z\} = \begin{cases} \text{Prob}\{x \ge S - y\} & \text{if} \quad z \ge S - R \\ \text{Prob}\{x \ge z + R - y\} & \text{if} \quad z < S - R \end{cases}$$

To derive the steady state distribution function, H(y), of the inventory we have

$$H(y) = \int_{-\infty}^{S-R} [1 - F(z + R - y)] h(z) dz + \int_{S-R}^{S} [1 - F(S - y)] h(z) dz$$

or

$$H(y) = \int_{-\infty}^{S-R} [1 - F(z + R - y)] h(z) dz + [1 - F(S - y)][H(S) - H(S - R)]$$

We need to calculate *H(y)*, *h(y)* for given parameters *S* and *R* under a demand cdf F(.)

$$H(y) = H(y, R, S) \text{ and } h(y) = h(y, R, S)$$

The cost function can be expressed by

$$C(R,S) = h\int_{0}^{S} I \cdot h(I,R,S)dI - v\int_{-\infty}^{0} I \cdot h(I,R,S)dI + c\int_{-\infty}^{S-R} R \cdot h(I,R,S)dI + c\int_{S-R}^{S} (S-I) \cdot h(I,R,S)dI + rR$$

or after some algebraic manipulations (with μ_I as expected inventory)

$$C(R,S) = (h+v)\int_{0}^{S} I \cdot h(I,R,S)dI - v\mu_I(R,S) +$$

$$+c[S \cdot H(S,R,S) - (S-R) \cdot H(S-R,R,S) - c\int_{S-R}^{S} I \cdot h(I,S,R)dI] + r\,R$$

Generally there is no closed form solution of the above integral equation, thus a numerical grid search can be used over R and S for minimizing C(R, S) over the search range R* ≤ R ≤ S* + R* and S* ≤ S ≤ S* + R*, where R* and S* are from the combined -sourcing (R,S)-policy solution.