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Setup Cost Reduction and Supply Chain Coordination in Case of Asymmetric Information

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Abstract

Screening contracts are a common approach to solve supply chain coordination problems under asymmetric information. Previous research in this area shows that asymmetric information leads to supply chain coordination deficits. We extend the standard framework of lotsizing decisions under asymmetric information by allowing investments in setup cost reduction. We find that asymmetric information leads to an overinvestment in setup cost reduction. Yet, the overall effect on supply chain performance is ambiguous. We show that these results holds for a wide variety of investment functions.

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1 Introduction

The model utilized in this paper captures a supply chain planning problem, in which the buyer asks the supplier to switch the delivery mode to Just-in-Time (JiT). We characterize the JiT mode with low order sizes.

The buyer faces several advantages from a JiT-delivery, such as lower opportunity costs (mainly for capital lockup), less inventory handling, storage room and handling equipment as well as less rework and scrap. Additionally, JiT allows for a more even workload (and therefore less idle time and more efficient production and material handling) and for less complexity for production planning and control (see Schonberger and Schniederjans [25]). However, we abstract from these multidimensional advantages and aggregate them to the buyer's holding costs. Hence, if the buyer faces high holding costs she is supposed to have high advantages from a JiT-delivery, and vice versa.

On the other hand, smaller order sizes can cause an increase of the suppliers setup, holding and distribution costs (see Fandel and Reese [8]). In our modelling approach, the supplier's setup costs per period reflect this disadvantages.

Yet, it is well known that small order sizes are not sufficient for a successful implementation of the JiT concept. Setup cost reduction, thus, is regarded to be one main facilitator for JiT to be efficient. Our model depicts the need for accompanying process improvements by the supplier's option to invest in setup cost reduction. Porteus [23] initially analyses the potential benefits of setup cost reduction in the economic order quantity-framework. Consecutive research often concentrates on the specific form of the investment function in either the economic order or economic production quantity model (e.g. van Beek and van Putten [28], Hahn et al. [11] , Kim [14]). Leschke [15] reviews this stream of research and conducts an empirical study to investigate a realistic shape of the investment function. Other authors extend Porteus initial work by considering stochastic lead times (and demand) or backorders (see Paknejad and Affisco [21], Keller and Noori [13] and Nasri et al. [20]). An extension to the case of two stage systems and full information availability was made by Paknejad, Nasri and Affisco [22]. This framework also was utilized to incorporate quality aspects in the suppliers' decision problem (see Affisco et al. 2002 [1], Liu and Cetinkaya [17]). Our paper contributes to this stream of research by analysing the impact of asymmetric information on the investment in setup cost reduction in a two stage supply chain.

From a supply chain perspective, an implementation of a JiT strategy is only profitable, if the buyer's cost advantages exceed the supplier's cost increase. Yet, previous research states that this is not always the case.[1] The supplier, thus, may have a strong incentive to convince the buyer to abandon the JiT strategy. However, as the buyer is supposed to be in a strong bargaining position (as it is, for instance, often the case in the automotive industry), he will not be convinced unless he is offered a compensation for the disadvantages of not implementing the JiT strategy. Yet, as long as there is a lack of coordination (i.e. as long as pareto improvements are possible by reducing the total supply chain's cost), the supplier can compensate the buyer while improving his own performance.

[1] Myer [19] finds that a buyer's single-handed JiT implementation causes a supply chain cost increase of 25% to 30% in the food industry (basically due to transportation, warehousing, unnecessary one-to-one communication on the sales side, as well as ineffective promotion and advertising on the marketing side), and a cost increase in the range of 10% to 20% in other consumer goods fields. In these cases, the buyer's isolated JiT decision obviously leads to a lack of supply chain coordination due to the supplier's cost increase.

Nonetheless, the above-mentioned advantages of a JiT strategy are multidimensional and contain to a major extent private information of the buyer. Thus, they can certainly not be easily observed and valued by the supplier. Of course, the supplier may ask the buyer about the disadvantages of abandoning the JiT strategy (or as the case may be how much compensation he demands to abandon the JiT strategy). However, the buyer will apparently claim that switching towards higher order sizes causes substantial costs and that a high compensation is required.

Assuming the strategic use of private information, it is in the supplier's best interest to offer a menu of contracts (i.e. screening contract) (see [Corbett and de Groote [5],Ha [10], Corbett [4],Corbett and Tang [6],Corbett et al. [7], and Sucky [27]]). Basically, this menu of contracts aligns the incentives of the supply chain members such that a buyer with low advantages of a JiT delivery will agree upon higher order sizes than a buyer with high advantages of this supply mode. One main result which stems from the screening literature is that asymmetric information leads to inefficiencies, because the resulting order sizes are too low compared to the supply chain's optimal solution. Starting from this insight, our main focus in this study is to analyse the impact of investments in setup cost reduction on this lack of coordination. Specifically, we are interested in investigating if the supplier's option to reduce his setup cost and, thereby decrease his lotsize, might lead to an improvement in supply chain coordination. If a complete cut of setup costs could be achieved at no (or very minor) cost, the supplier would offer an JiT contract and perfect coordination would take place. However, the impact of costly setup cost reduction on supply chain coordination is not clear at all.

Summing up, there are basically two streams of research (namely the inefficiencies due to asymmetric information and the optimal set-up cost reduction in an integrated lot-sizing decision) this paper combines.

2 Outline of the model

Figure 1: Supply Chain

This paper analyses a simplified Joint-Economic-Lotsizing model [see Goyal [9],Monahan [18]and Banerjee [2]]. In a dyadic relationship, composed of a buyer (B) and a supplier (S), the buyer decides upon the order lotsize (Q) from her supplier. Let f denote the setup costs for each delivery incurred by the supplier. This setup costs are a decision variable for the supplier's decision problem. The cost for reducing the setup costs from its original level f_{max} by $f_{max} - f, \forall f \geq f_{min} \geq 0$ are captured by the investment function $k(f)$. The investment $k(f)$ leads to a setup cost reduction over the whole (infinite) planning horizon. Hence, the supplier faces costs of $r \cdot k(f)$ in each period, where r denotes the company specific interest rate.[2] The buyer faces holding costs h per item and period. The demand is, without loss of generality, standardized to one unit per period. Hence period costs equal unit costs. Lot-for-Lot production is assumed on the supplier's side. The following situation is considered, with the buyer as the focal player in the supply chain. She asks the supplier to deliver a product Just-in-Time (JiT). As the supplier is in a weak bargaining position, he either delivers JiT or he offers a contract which makes the buyer as well off

[2]If the time horizon is finite, r can be interpreted as the annuity factor. In this case, a constant order size Q is (ceteris paribus) still optimal [see Brimberg and Hurley [3]].

as she would be by sourcing from an alternative JiT supplier. Obviously, both options can be accompanied by an investment in setup cost reduction. We show in the following how the supplier can optimally convince the buyer to accept higher order sizes even if the buyer's advantages of the JiT delivery can only be estimated.

Full Information If the supplier knows the buyer's holding costs h, he offers the following contract, consisting of order size Q, side payment T and the corresponding setup cost f, to minimize his period costs K_S. The buyer has the outside option to buy from an alternative supplier (AS) who delivers in a JiT mode. The cost of buying from the alternative supplier is C_{AS} per unit. To induce higher order sizes while ensuring that the buyer does not choose her outside option, the supplier must compensate the buyer for the additional holding cost with a side payment T per unit (e.g. by introducing a quantity discount on the wholesale price). The supplier's optimal contract is the outcome of the following optimization problem:

Problem FI

$$\begin{aligned}
\min K_S(Q,T,f) &= \frac{f}{Q} + T + r \cdot k(f) \\
s.t. & \\
\frac{h}{2}Q - T &\leq C_{AS} \quad \text{(PC)} \\
f_{min} &\leq f \\
f &\leq f_{max}
\end{aligned}$$

It is easy to see that the participation constraint (PC) needs to be binding for an optimal solution. Substituting $T = \frac{h}{2}Q - C_{AS}$ in the objective function, and setting up the

Lagrange-function gives:

$$\begin{aligned} L(Q,f,\alpha,\gamma) &= \frac{f}{Q}+\frac{h}{2}Q-C_{AS}+r\cdot k(f) \\ &+ \alpha(f_{min}-f)+\gamma(f-f_{max}). \end{aligned}$$

The solution of $\min\limits_{Q,f,\alpha,\gamma} L(Q,f,\alpha,\gamma)$ gives the (supply chain optimal) contract parameters $Q(SC)^*$ and $T(SC)^*$, the optimal investment level $f(SC)^*$ and the optimal lagrange parameters $\alpha(SC)^*, \gamma(SC)^*$, i.e. the order quantity $Q(SC)^*$ causes the lowest possible cost for the overall supply chain. The supply chain optimal order size is the well known economic order quantity (EOQ) with respect to the reduced setup costs, i.e. $Q(SC)^* = \sqrt{\frac{2\cdot f(SC)^*}{h}}$. Kim et al. [14] show that the optimal setup cost level $f(SC)^*$ of problem FI is dependent on the shape of the total cost function. They also give an optimization procedure for any investment function $k(f)$. These results can easily be integrated in our framework. Particularly, they show that for a concave and a linear investment function the optimal investment level $f(SC)^*$ is either f_{min} ($\Rightarrow \alpha(SC)^* \neq 0$) or f_{max} ($\Rightarrow \gamma(SC)^* \neq 0$). For a convex investment function the optimal investment level is either an interior solution (i.e. $\alpha(SC)^* = 0$ and $\gamma(SC)^* = 0$) or a corner solution (i.e. $\alpha(SC)^* \neq 0$ or $\gamma(SC)^* \neq 0$). In the following, we extend the framework to the case of asymmetric information and analyse the influence on supply chain coordination.

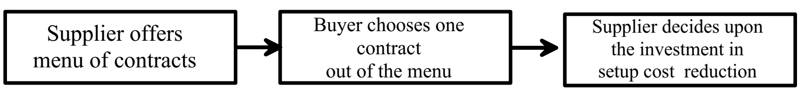

Figure 2: Decision sequence

Asymmetric Information As mentioned in the introduction, the buyer's several advantages from a JiT delivery are captured by an aggregated measure, namely her holding costs. As full information about all these JiT related advantages is a very critical assumption we study a situation where the buyer is only able to estimate these advantages. We formalize this estimation with a probability distribution $p_i(h_i)$ $(i = 1, ..., n)$ over all possible holding cost realizations h_i. Common knowledge of the probability distribution $p_i(h_i)$ is assumed.

One feasible solution to this problem with respect to the buyer's participation constraint (PC) can be obtained by solving it as if under full information (FI) with $h = h_n$. However, this solution is not optimal for the supplier as he can increase his expected profits by offering a menu of contracts Q_i, T_i $(i = 1, ..., n)$, which the buyer can choose from. Figure 2 illustrates the decision sequence. The optimal menu of contracts $(Q_i^*, T_i^* \quad \forall i = 1, ..., n)$ is the solution to the following optimization problem :

Problem AI

$$\begin{aligned}
\min E(K_S(f_i, Q_i, T_i)) &= \sum_{i=1}^{n} p_i \left(\frac{f_i}{Q_i} + T_i + r \cdot k(f_i) \right) \quad \forall i = 1, ..., n \\
s.t. & \\
\frac{h_i}{2} Q_i - T_i &\leq C_{AS} \quad \text{(PC)} \quad \forall i = 1, ..., n \\
\frac{h_i}{2} Q_i - T_i &\leq \frac{h_i}{2} Q_j - T_j \quad \text{(IC)} \quad \forall i \neq j; \quad i, j = 1, ..., n \\
f_i &\leq f_{max} \quad \forall i = 1, ..., n \\
f_{min} &\leq f_i \quad \forall i = 1, ..., n
\end{aligned}$$

The incentive constraint (IC) ensures that the buyer with holding costs h_i incurs the lowest possible cost per period when choosing the order size Q_i. Hence, the cost minimizing buyer reveals her holding cost through the contract choice. The participation constraint (PC) ensures that no buyer, regardless of her holding costs, will choose the alternative supplier (AS).

Setting up the Lagrange-function gives:

$$
\begin{aligned}
L(Q_i, T_i, f_i, \lambda_{ij}, \mu_i, \gamma_i, \alpha_i \quad | \quad i,j=1,...,n \quad \text{and} \quad i \neq j) \quad &= \\
\sum_{i=1}^{n} p_i \left(\frac{f_i}{Q_i} + T_i + r \cdot k(f_i) \right) \quad &+ \quad \sum_{i=1}^{n} \mu_i \left(\frac{h_i}{2} Q_i - C_{AS} - T_i \right) \\
+ \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} \lambda_{ij} \left(\frac{h_i}{2} Q_i - T_i - \frac{h_i}{2} Q_j + T_j \right) \quad &+ \quad \sum_{i=1}^{n} \alpha_i (f_{min} - f_i) \\
&+ \quad \sum_{i=1}^{n} \gamma_i (f_i - f_{max})
\end{aligned}
$$

The interested reader can find the Karush-Kuhn-Tucker (KKT) conditions for an optimal solution in Appendix 6.1. The optimal solution to the problem AI gives the optimal menu of contracts Q_i^*, T_i^*, the optimal investment level f_i^* and the optimal lagrange parameters μ_i^*, α_i^*, γ_i^* and λ_{ij}^*. The corresponding supply chain optimal order size (i.e. the order size that solves problem FI with $h = h_i$) is denoted by $Q_i^*(SC)$.

In the optimal screening contract, the order sizes increase with decreasing holding costs (i.e. $Q_{i-1}^* \geq Q_i^*, \quad \forall i = 2, ..., n$) and the side payments increase respectively (i.e. $T_{i-1}^* \geq T_i^*, \quad \forall i = 2, ..., n$). Please note that for certain combinations of probabilties $p_i(h_i), i = 1, ..., n$ and holding costs $h_i, i = 1, ..., n$ the order size relation $Q_{i-1} = Q_i$ may hold in the optimal menu of contracts. In this case, there is no information revelation through the buyer's contract choice (as the supplier cannot distinguish the buyer facing holding cost

h_{i-1} from the buyer facing holding cost h_i). Yet, in the screening literature it is common to rule out this cases by imposing a restriction on the probabilty distribution.[3] We follow this approach and ,thus, restrict our analysis of the settings in which information relevation is prevalent, as this is the main feature of screening models. We exclude the cases of no information relevation by assuming that the following condition between probabilities and holding costs hold:

$$h_i + \frac{\sum_{t=0}^{i-1} p_t}{p_i}(h_i - h_{i-1}) \quad < \quad h_j + \frac{\sum_{t=0}^{j-1} p_t}{p_j}(h_j - h_{j-1}) \tag{1}$$
$$\forall \qquad i = 1, ..., n-1; j = 2, ..., n; j > i \text{ and } p_0, h_0 = 0$$

This condition is always satisfied if there are only two possible holding cost realizations because $p_0 = 0, p_1 > 0$ and $h_1 < h_2$ holds. However, if the supplier faces more than two possible holding cost realization, there exist combinations of p_i and h_i for which (1) does not hold and information revelation would not be observable for every holding cost parameter h_i. The optimality conditions for the optimal menu of contracts in these cases are specified in Spence [26]. However, assumption (1) has no impact on the main results in this study but simplifies the derviation of the suppliers optimal decision.

Furthermore, the resulting order sizes are too low compared to the supply chain optimum (i.e. $Q_i^* < Q_i(SC)^*$ for all $i = 2, ..., n$) except the ordersize Q_1^*. Hence, there is a lack of supply coordination except for the buyer facing the lowest possible holding costs.

Finally, the buyer with holding costs h_i is indifferent between the contracts Q_i^*, T_i^* and Q_{i+1}^*, T_{i+1}^* ($\forall i = 1, ..., n-1$), and the buyer facing the highest holding cost h_n is indifferent

[3] In models with a continuous distribution over the private information a monotone hazard rate of the probabilty distribution is normally assumed.

between the contract Q_n^*, T_n^* and the alternative supplier.[4] Hence, information revelation requires that the buyer facing holding costs h_i chooses the order size Q_i even though the order size Q_{i+1} has the same cost impact for her. Thus, it is a common assumption within the screening literature that the indifferent buyer chooses the order size which is in her supplier's best interest. We refer to Inderfurth et al. [12] for a discussion of this behavioral assumption in screening models.

3 Impact on Supply Chain Coordination and Performance

In the following, we present the supplier's optimal menu of contracts Q_i^*, T_i^* and the corresponding setup cost level f_i^*. The optimal menu of contracts is derived from the KKT-conditions (see Appendix 6.2, (36)-(40)):

$$Q_i^* = \sqrt{\frac{2 \cdot f_i^*}{h_i + \phi_i}} \quad \forall i = 1, ..., n \tag{2}$$

$$\text{where} \quad \phi_i = \frac{\sum_{t=0}^{i-1} p_t}{p_i}(h_i - h_{i-1}) \quad \forall \quad i = 1, ..., n \quad \text{and} \quad p_0, h_0 = 0 \tag{3}$$

$$T_n^* = \frac{h_n}{2} Q_n^* - C_{AS} \tag{4}$$

$$T_i^* = \frac{h_i}{2}(Q_i^* - Q_{i+1}^*) + T_{i+1}^* \quad \forall i < n \tag{5}$$

The optimal setup costs f_i^* results from solving (see Appendix 6.2, (42))

$$\sqrt{\frac{h_i + \phi_i}{2 \cdot f_i^*}} = -r \cdot \left.\frac{dk(f_i)}{df_i}\right|_{f_i = f_i^*} \tag{6}$$

[4] For a derivation and a broader discussion of the optimal menu of contracts' properties we refer to Sappington [24] or Spence [26].

as long as the optimal setup cost level is an interior solution. Otherwise, the optimal setup cost level is a corner solution (i.e. f_{min} or f_{max}).

Obviously, the supplier takes the option of setup cost reduction into consideration when offering the menu of contracts. Hence, his decision in the third stage (see figure 2) is already considered in the first stage. Otherwise, he would offer suboptimally high order sizes as (2) is dependent on f_i^*, and f_i^* is computed from (6).

Distortionary effects of asymmetric information

As already mentioned in section 2, asymmetric information leads to supply chain inefficiencies caused through a downward distortion in order sizes. One can easily see from (2) that ϕ_i makes the difference compared to the EOQ-formula (i.e. the supply chain optimal solution). Hence, analysing the distortion caused by asymmetric information reduces to analysing ϕ_i. Hence, the higher ϕ_i the higher $Q_i(SC)^* - Q_i^*$, i.e. the distortion increases with increasing ϕ_i.

The distortion, thus, increases with an increasing ratio $\frac{\sum_{t=0}^{i-1} p_t}{p_i}$. The higher this ratio, the higher the probability that the buyer will choose an order size $Q_k, k < i$ due to the screening. Hence, the expected cost minimizing supplier will decrease (the less likely choosen order size) Q_i as he can decrease the side payments T_k for all $k \leq i$ (see (4),(5)) as well. The order size deviation from the supply chain optimum $(Q_i(SC)^* - Q_i^*)$ therefore increases.

Furthermore, the distortion depends on the distance between h_i and h_{i-1}. Intuitively, there are no asymmetric information if the difference of the respective holding costs is zero. Thus, the asymmetry of information (and therefore the distortion) increases the more different the respective holding costs.

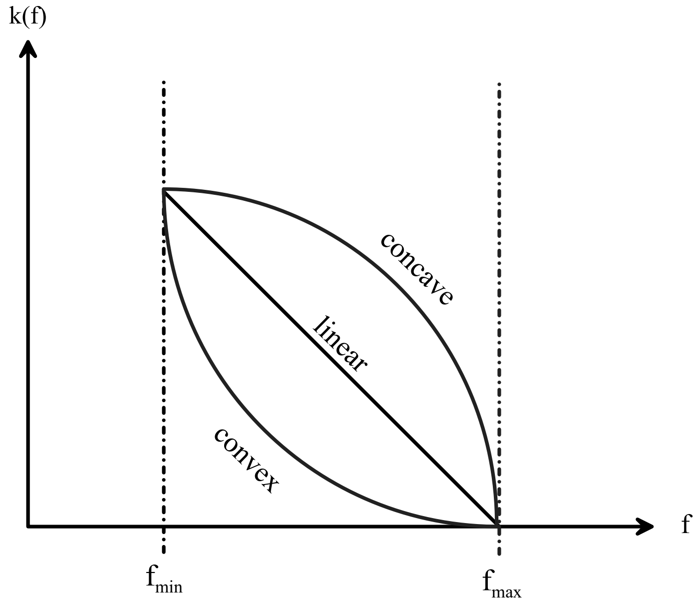

Figure 3: Progressive shapes of the analysed investment functions

In the following, we will analyse whether asymmetric information leads to a distortion of the buyer's investments in setup cost cost reduction. The analysis is carried out for convex, concave and linear investment functions. Figure 3 depicts the shape of the analysed investment functions.[5] We will utilize the KKT-conditions and therefore a marginal approach to derive our results. This approach seems reasonable since an intutive graphical representation of the analysed problem is possible. However, the interested reader finds another intuitive approach for the problem under full information in Kim et al [14].

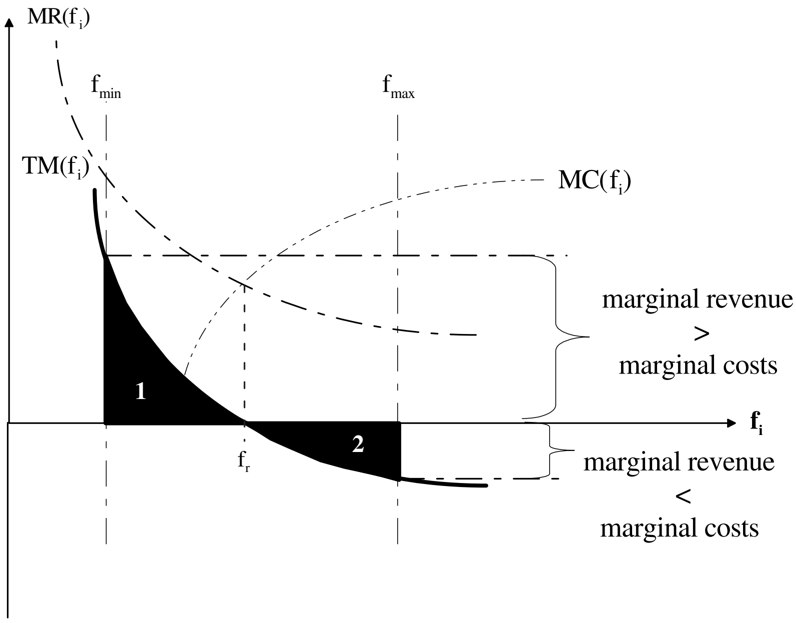

Figure 4: Optimal investment level given a concave investment function

3.1 Concave and linear investment function

From the evaluation of the KKT-condition in (6) we get $\sqrt{\frac{h_i+\phi_i}{2 \cdot f_i^*}} = -r \cdot \frac{dk(f_i)}{df_i}\Big|_{f_i=f_i^*}$, as long as $\alpha_i^* = 0, \gamma_i^* = 0$, i.e. as long as the optimal setup cost level is an interior solution. Hence, only a setup cost level f_i which satiesfies this condition is an interior solution. Next we will show, however, that this solution is a cost maximum instead a cost minimum. Thus, a corner solution will result in the cost minimum.

Let $MR(f_i) = \sqrt{\frac{h_i+\phi_i}{2 \cdot f_i}}$ denote the marginal revenues and $MC(f_i) = -r \cdot \frac{dk(f_i)}{df_i}$ the marginal costs for reducing the setup cost level f_i. Figure 4 depicts $MC(f_i)$, $MR(f_i)$ and $TM(f_i) = MR(f_i) - MC(f_i)$.

As the MC-curve is monotonically increasing for a concave investment function ($\frac{d^2k(f)}{d^2f} <$

[5] We refer to Leschke and Weiss [16] for a review of commonly assumed investment functions. The convex followed by the the linear investment function is most commonly assumed.

$0 \rightarrow -\frac{d^2k(f)}{d^2f} > 0$), the TM-curve is strictly monotonically decreasing (and there is at most one interior solution). Yet, we need to consider the integral over $TM(f_i)$ to evaluate the profitability of a setup cost reduction. Let f_r denote the root of $TM(f_i)$ (i.e. the interior solution to problem AI). If $f_r \in (f_{min}, f_{max})$, a reduction of setup costs to f_r causes a loss of $\int_{f_r}^{f_{max}} TM(f_i)df_i$ (i.e. area 2).[6] Hence, f_r is a local *cost maximum.* As $TM(f_i)$ is strictly monotonically decreasing, a setup cost reduction beyond f_r leads to profits of $\int_{f_{min}}^{f_r} TM(f_i)df_i$ (i.e. area 1). Hence, if TM intersects the abscissa between f_{min} or f_{max}, the optimal investment level depends on the ratio of the areas 1 and 2. If 1 is bigger than 2, then the optimal setup level is f_{min}, otherwise it is f_{max}. Therefore, $\alpha_i^* > 0$ or $\gamma_i^* > 0$ holds. Figure 4 depicts the case, in which the optimal setup cost level is f_{min}.

The same argument holds for a linear investment function (see figure 5). As $MC(f_i)$ is constant, $TM(f_i)$ is strictly monotonically decreasing. Hence, there is at most one intersection with the abscissa. Therefore, the optimal setup cost level is a corner solution as well (i.e. f_{min} or f_{max}). Therefore, $\alpha_i^* > 0$ or $\gamma_i^* > 0$ holds. Figure 5 depicts the case, in which area 2 is bigger than area 1. Hence, no setup cost reduction at all is profitable.

These results are in line with the findings in Kim et al [14]. Under full information, thus, this would be the optimal solution. Now, however, we consider the impact of asymmetric information on the investment decision. In (6) we see that the marginal revenue increases with increasing ϕ_i.

From $\phi_i = \frac{\sum_{t=0}^{i-1} p_t}{p_i}(h_i - h_{i-1}) \geq 0$ follows that the MR-curve under full information is beneath the MR-curve under asymmetric information. Let $TM^{FI}(f_i)$ denote the difference

[6]Please note that figure 4 depicts the setup cost level f_i instead of the setup cost reduction $f_{max} - f_i$. Hence, the higher the setup cost reduction the lower the setup cost level (i.e. f_i is closer to the point of origin).

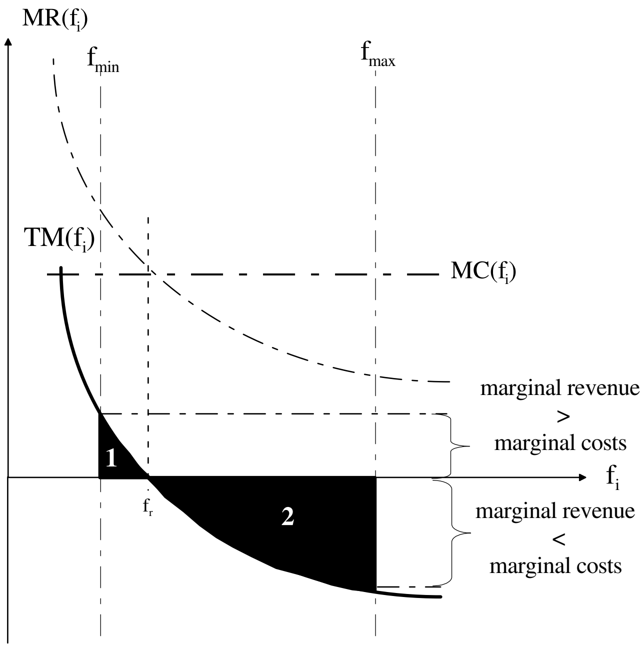

Figure 5: Optimal investment level given a linear investment function

between marginal revenue and marginal cost under full information and $TM^{AI}(f_i)$ under asymmetric information respectively. It follows directly that $TM^{FI}(f_i) \leq TM^{AI}(f_i), \ \forall \ f_i$. Hence, area 1 increases and area 2 decreases in size due to asymmetric information. If the supply chain optimal investment level is $f_i^*(SC) = f_{max}$ (i.e. area 1 < area 2), and the optimal investment level under asymmetric information is $f_i^* = f_{min}$ (i.e. area 1 > area 2) there is an overinvestment in setup cost reduction. Figure 6 depicts this case. The same arguments holds for a linear investment function. Hence, asymmetric information can lead to an overinvestment in setup cost reduction.

3.2 Convex investment function

Now we consider the distortionary effect of asymmetric information on the setup cost investment level in case of a convex investment function. Yet, as both $MR(f_i)$ and $MC(f_i)$

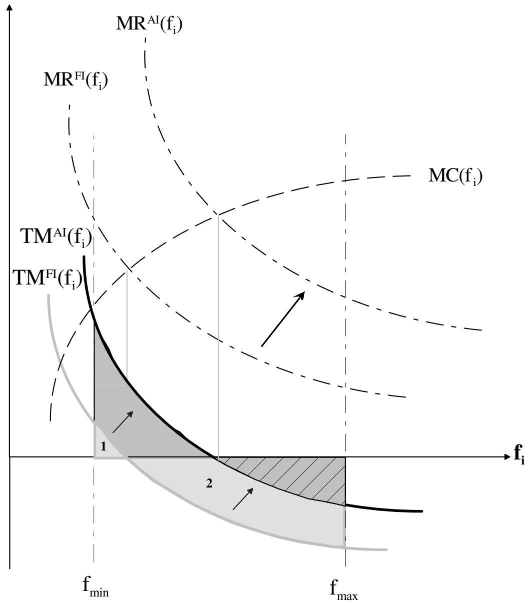

Figure 6: Overinvestment in case of a concave investment function

are monotonically decreasing ($\frac{d^2k(f)}{d^2f} > 0$), it is not clear whether $TM(f_i)$ is monotonic at all. Figure 7 shows the case of a) a monotonically increasing TM-curve and b) a monotonically decreasing TM-curve. If the TM-curve is not monotonic at all, there are multiple interior solutions. However, the argumentation in this situation can either be reduced to case a) or b).

For case b) the same argument as for the concave and linear investment function holds (i.e. there is at most one cost maximum). Hence, the optimal setup cost level is either f_{min} or f_{max}, and there is also the possibility of an overinvestment due to asymmetric information.

Yet, as long as $TM(f_i)$ is strictly monotonically increasing (case a), the optimal investment is

$$f_i^* = \begin{pmatrix} f_r & , & \text{if} \quad f_r \in [f_{min}, f_{max}] \\ f_{min} & , & \text{if} \quad f_r < f_{min} \\ f_{max} & , & \text{else} \end{pmatrix}. \tag{7}$$

This is because all setup cost reductions beyond f_r are not profitable. Hence, in contrast to a concave or linear investment function, f_r results in a cost minimum instead a cost maximum.

Next, we consider the effect of asymmetric information. As long as the order sizes change due to a screening (i.e. $MR^{AI}(f_i) > MR^{FI}(f_i)$) and the optimal setup cost level is not a corner solution there is an overinvestment in setup cost reduction. Figure 8 illustrates this case.

Yet, if $TM(f_i)$ is not monotonic at all, the same arguments as for the separated cases a) and b) in figure 7 hold. Hence, also in this situation, an overinvestment is possible if the investment decision changes due to an upward-shift of $TM(f)$ (i.e. due to asymmetric information).

As such, one can summarize that there is always the possibility of an overinvestment in setup reduction, regardless of the actual shape of the investment function. This result is basically driven by the fact that the supplier's screening regarding the buyer's private information leads to order sizes which are smaller than the supply chain optimal order sizes (i.e. $Q_i^* < Q_i^*(SC), \forall i = 2, ..., n$, see section 2). This leads to higher marginal revenues from the investment in setup cost reduction. Yet, it is not obvious whether the coordination deficit (i.e. the performance gap between supply chain optimum and screening contract) is increasing or decreasing due to an investment in setup cost reduction, as there are two contrary effects:

Overinvestment effect. As stated in section 3, an overinvestment in setup cost reduction due to asymmetric information is likely. Yet, from a supply chain perspective, this overinvestment causes a coordination deficit.

Setup cost effect. As shown in section 2, the coordination deficit due to asymmetric information is essentially caused by a deviation from the supply chain optimal order quantity (i.e. $Q_i^* \leq Q_i^*(SC)$). Therefore, there is no supply chain optimal trade-off between holding costs and setup costs per period.[7] The setup costs per period are suboptimally high and the holding costs suboptimally low. Yet, as there is the option to invest in setup cost reduction (and even an overinvestment is possible), this unbalanced trade-off carries less weight. The setup cost effect, thus, measures the isolated coordination gains from reducing the setup cost while disregarding the investment costs.

In the following section, we conduct a numerical study for two possible holding cost realization to analyse the impact of the overinvestment- and setup cost effect on the coordination deficit.

4 The Power Cost Function Case

In the following, we illustrate the previous analysis for Porteus “Power Cost Function Case“ [23] as an example for a convex investment function. If this investment function

[7] In the classical economic lotsizing model, the fixed- and holding costs per period are equal in the optimum. Yet, lower order sizes due to asymmetric information lead to fixed cost per period that are higher than the holding costs per period.

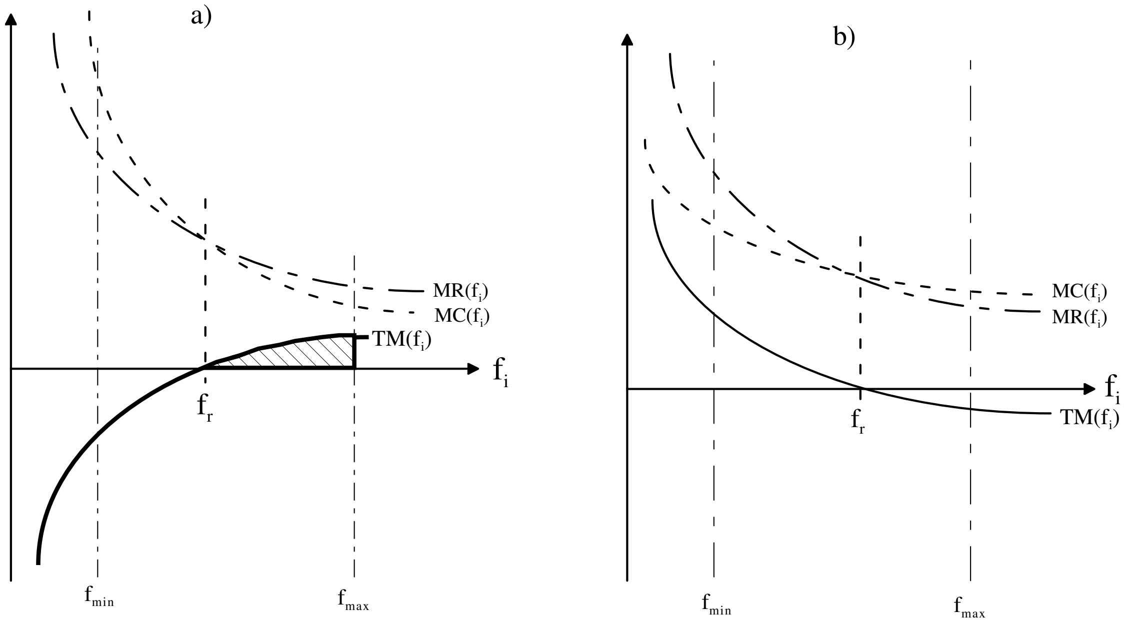

Figure 7: Optimal investment level in case of a convex investment function

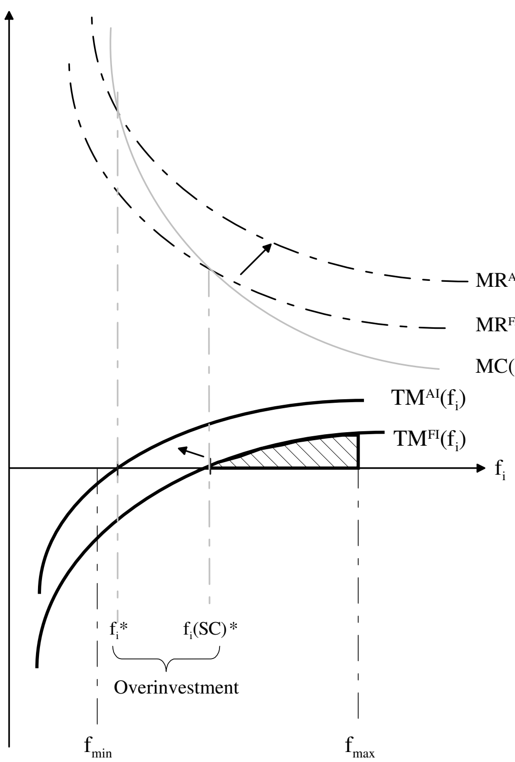

Figure 8: Overinvestment in case of a concave investment function

is utilized, decreasing marginal percentage returns are presumed. Therefore, the supplier faces investment costs in the amount of $k_i(f) = a \cdot f^{-b} - d \quad \forall a, b, d, f > 0$ if he reduces the setup costs from the initial value of f_{max} to f, where $f > f_{min} = 0$. Let c denote the costs of reducing the setup cost f_{max} by $\pi\%$, then an additional reduction by $\pi\%$ results in an investment of $(1 + \beta) \cdot c$. Then, the investment function has the following shape: $k_i(f_i) = a \cdot f_i^{-b} - d$ with $b = \frac{-ln(1+\beta)}{ln(1-0{,}01\pi)}$,$a = \frac{c \cdot f_{max}^b}{(1-0{,}01\pi)^{-b}-1}$ and $d = a \cdot f_{max}^{-b}$.

Full Information:

In the following we restrict our analysis on $n = 2$ (i.e. $h \in [h_1, h_2]$). Then, the optimal contract parameters under full information are

$$Q^*(SC) = \min\left(\sqrt{\frac{2f_{max}}{h}}, \left(\frac{2}{h}\right)^{\frac{b}{2b+1}} \left(\frac{2abr}{h}\right)^{\frac{1}{2b+1}}\right) \tag{8}$$

$$f^*(SC) = \min\left(f_{max}, \left(\frac{2(abr)^2}{h}\right)^{\frac{1}{2b+1}}\right) \tag{9}$$

[see Porteus [23]].

Asymmetric Information

The optimal contract parameters for two possible holding cost realizations h_1 and h_2 are (see Appendix 6.3):

$$f_2^* = \left(\begin{array}{lll} f_r & , & \text{if} \quad f_r \in [f_{min}, f_{max}] \\ f_{min} & , & \text{if} \quad f_r < f_{min} \\ f_{max} & , & \text{else} \end{array}\right) \tag{10}$$

$$\text{where} \quad f_r = \left(\sqrt{\frac{2}{h_i + \phi_2}} \cdot r \cdot a \cdot b\right)^{\left(\frac{1}{b+0.5}\right)} \tag{11}$$

$$\tag{12}$$

$$Q_2^* = \sqrt{\frac{2 \cdot f_2^*}{h_2 + \phi_2}} \tag{13}$$

$$T_2^* = \frac{h_2}{2} Q_2^* - C_{AS} \tag{14}$$

$$\text{where} \quad \phi_2 = \frac{p_1}{p_2}(h_2 - h_1) \tag{15}$$

$$f_1^* = f_1(SC)^* = \min \left(f_{max}, \left(\frac{2(abr)^2}{h_1} \right)^{\frac{1}{2b+1}} \right) \tag{16}$$

$$Q_1^* = Q_1(SC)^* = \min \left(\sqrt{\frac{2 \cdot f_{max}}{h_1}}, \left(\frac{2}{h_1} \right)^{\frac{b}{2b+1}} \left(\frac{2abr}{h_1} \right)^{\frac{1}{2b+1}} \right) \tag{17}$$

$$T_1^* = \frac{h_1}{2} \left(Q_1^* - Q_2^* \right) + \frac{h_2}{2} Q_2^* - C_{AS} \tag{18}$$

4.1 Numerical example

Suppose $\beta = 0.01, \pi = 10[\%](\Rightarrow b = 0.094), c = 25, f_{max} = 800 \quad (\Rightarrow a = 4700, d = 2500, k_i(f_i) = 4700 \cdot f^{-0{,}094} - 2500), C_{AS} = 2.5, f_{min} = 0, h_1 = 1, h_2 = 5, p_1 = 0.5$ and $p_2 = 0.5$. Let $(Q_i^{f_{max}}, T_i^{f_{max}} \quad , i = 1, 2)$ denote the optimal menu of contracts with no setup cost reduction possible. In this case, $Q_i^{f_{max}}(SC) \quad , i = 1, 2$ denotes the supply chain optimal order quantity. Furthermore, $K_i(SC)(Q_i, f_i) = \frac{f_i}{Q_i} + \frac{h_i}{2} Q_i + r \left(af^{-b} - d \right), i = 1, 2$ denote the supply chain costs that result if the buyer faces holding costs h_i. Finally, $E(K(SC)) = \sum_{i=1}^{2} p_i \cdot K_i(SC)$ denote the expected supply chain costs. In the following we concentrate our analysis on the overall supply chain. Hence, we do not list the side payments T_i that correspond to the respective order size as these payments have no impact on the overall supply chain.

If no setup cost reduction is possible, the optimal order sizes and respective supply chain costs are

$$Q_1^{f_{max}} = 40 \qquad Q_1^{f_{max}}(SC) = 40$$

$$K_1(SC)(Q_1^{f_{max}}, f_{max}) = 40 \qquad K_1(SC)(Q_1^{f_{max}}(SC), f_{max}) = 40$$

$$Q_2^{f_{max}} = 13.33 \qquad Q_2^{f_{max}}(SC) = 17.89$$

$$K_2(SC)(Q_2^{f_{max}}, f_{max}) = 93.34 \qquad K_2(SC)(Q_2^{f_{max}}(SC), f_{max}) = 89.44$$

$$E(K(SC)(Q_i^{f_{max}}, f_{max}) = 66.67$$

Yet, if setup cost reduction is feasible, the following optimal contract parameters and setup costs result:

$$Q_1^* = 40 \qquad Q_1^*(SC) = 40$$

$$f_1^* = 800 \qquad f_1^*(SC) = 800$$

$$K_1(SC)(Q_1^*, f_1^*) = 40 \qquad K_1(SC)(Q_1^*(SC), f_1^*) = 40$$

$$Q_2^* = 6.08 \qquad Q_2^*(SC) = 10.45$$

$$f_2^* = 166.6 \qquad f_2^*(SC) = 273.15$$

$$K_2(SC)(Q_2^*, f_2^*) = 82.52 \qquad K_2(SC)(Q_2^*(SC), f_2^*) = 78.97$$

$$E(K(SC)(Q_i^*, f_i^*)) = 61.26$$

Figure 11 in Appendix 6.4 depicts the marginal analysis from section 3 for the numerical example. In the following, we analyse the impact of the option to invest in setup cost reduction on the coordination deficit that arises due to asymmetric information. Additionally, the effect of setup cost reduction on the overall supply chain performance is analysed.

4.2 Coordination deficit and setup cost reduction

Let $CD = K_2(SC)(Q_2^*(SC), f_2^*(SC)) - K_2(SC)(Q_2^*, f_2^*)$ denote the coordination deficit with the option to invest in setup cost reduction and

$CD^{f_{max}} = K_2(SC)(Q_2^{f_{max}}(SC), f_{max}) - K_2(SC)(Q_2^{f_{max}}, f_{max})$ the coordination deficit without the option to reduce setup costs.[8] In the numerical example, the coordination deficit decreases from $CD^{f^{max}} = 3.9$ to $CD = 3.56$. The changes in the coordination deficit $\Delta CD = CD - CD^{f^{max}}$ can be split into the overinvestment effect (OE) and the setup cost effect (SE), i.e. $\Delta CD = OE - SE$ (please refer to Appendix 6.5 for a mathematical formulation of $\Delta CD, SE$ and OE). When $\Delta CD > 0$, the coordination deficit increases due to an investment in setup cost reduction, and vice versa. The overinvestment effect amounts to $OE = 13.23$ and the setup cost effect amounts to $SE = 13.56$. Hence, the coordination deficit changes by $\Delta CD = 3.56 - 3.9 = 13.22 - 13.56 = -0.34$ resulting in a decrease of the coordination deficit due to setup cost reduction. Note that this reduction completely benefits the supplier. In contrast, a positive ΔCD would completely increase the supplier's cost. The participation constraint in problem AI ensures that the buyer with holding cost realization $h_n = h_2$ yields the same cost for all parameter values of the problem. Hence, a change in the coordination deficit by ΔCD only affect the supplier's costs.

To investigate whether this result is robust against changing parameter values, we conduct a comparative static analysis for all possible parameter values r. Please notice that even if $\Delta CD < 0$, there would still be a coordination deficit. This deficit is always prevalent when there is a deviation from the optimal order quantity due to screening. ΔCD only depicts the effect of investments in setup cost reduction compared to no setup cost reduction.

Figure 9 depicts the changes of ΔCD in dependence of the interest rate r. The overinvestment effect carries less impact, when the investment in setup cost reduction is inexpensive

[8]As there is no coordination deficit, if the buyer faces holding costs h_1 we restrict the analysis to the cases in which the buyer faces the holding costs h_2.

(i.e. if the interest rate r is low). Yet, if r increases, the impact of the overinvestment effect becomes predominant and the supply chain performance deteriorates. The impact of the investment on supply chain performance is worst if r takes a value so that $f_2^*(SC) = f_{max}$ and $\gamma(SC)^* = 0$ holds, i.e. if f_{max} is the interior solution to problem FI ($r \approx 0,19$).[9] This is the situation where overinvestment reaches its maximum. Beyond this interest rate r, the total overinvestment $f_{max} - f_2^*$ decreases. Hence, the overinvestment as well as the setup cost effect decreases. Nonetheless, the overall effect on the supply chain performance is negative. Please note that the coordination deficit vanishes if the setup cost reduction is costless (i.e. $r = 0$) because the supplier will reduce the setup cost to the maximum extend. Figure 14 in Appendix 6.6 shows that this coordination deficit reduction is accompanied by low order sizes $Q_2^*(SC)$ and Q_2^*. The supply chain, thus, tends to the JiT mode if setup cost reduction is inexpensive. Furthermore, this figure points out that the downward distortion of order sizes (i.e. $Q_2^* \leq Q_2^*(SC)$), which is well known from other screening models, continues to be responsible for the inefficiencies within the supply chain. The interested reader can find more comparative static analyses for the parameters f_{max}, h_2 and π in Appendix 6.6, figure 12. Yet, the main result does not change for this analysis: whether the investment in setup cost reduction reduces the coordination deficit or not depends on the parameter values. As stated earlier, ΔCD directly affects the supplier's pay-offs. Yet, the coordination analysis is only conducted for the buyer with holding cost h_2. However, even if $\Delta CD > 0$, the menu of contract Q_i^*, T_i^* and the corresponding f_i^* is optimal for a supplier with risk neutral preferences.

[9] This value results from solving (6) with $f^* = f_{max} = 800$ and $\phi = 0$ for r

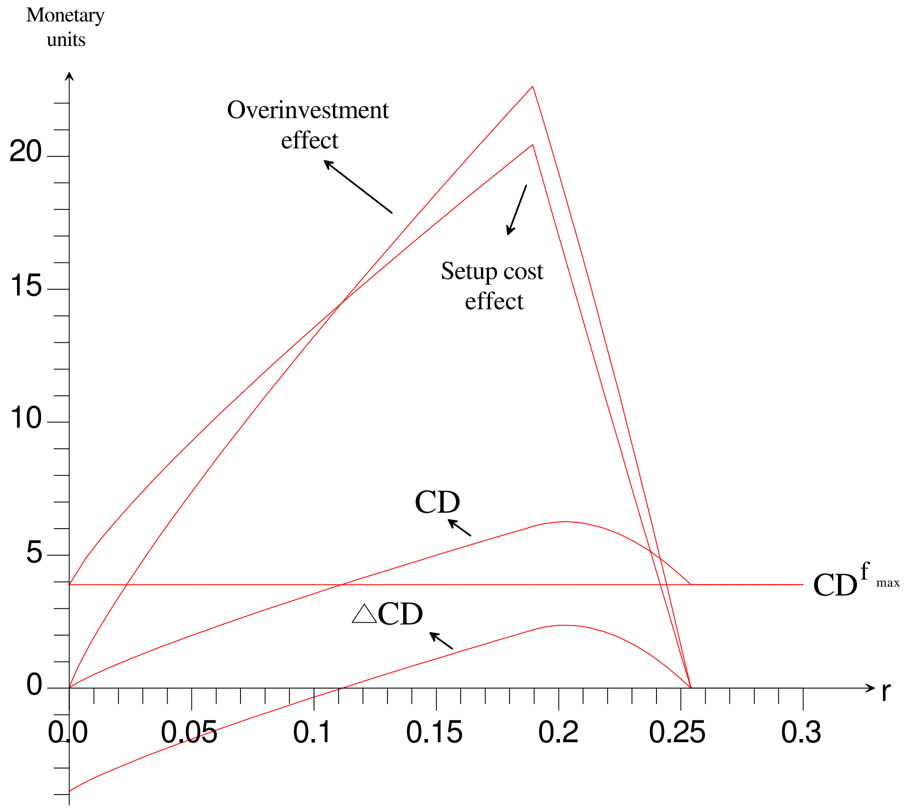

Figure 9: Comparative static analysis w.r.t. r

4.3 Supply Chain performance and setup cost reduction

So far, we mainly concentrated on the coordination deficit and therefore on the absolute inefficiencies that arise due to asymmetric information. However, an increase of the supply chain deficit does not necessarily lead to a deterioration of supply chain performance. To analyse the effect on the overall supply chain performance we compute the expected change in supply chain costs that result if setup cost reduction is possible, i.e. $\Delta P = E(K(SC)(Q_i^{f_{max}}, f_{max}) - E(K(SC)(Q_i^*, f_i^*))$. Hence, if $\Delta P < 0$, the supply chain performance deteriorates in the presence of setup cost reduction option due to asymmetric information. Please note, that the supply chain performance will never decrease under full information as no setup cost reduction is a feasible solution. In the numerical example the expected supply chain performance increases by $\Delta P = 66.67 - 61.26 = 5.41$. Hence, the expected supply chain performance increases if the supplier reduces his setup costs. We test again the robustness of this result for changing parameter values r.

Figure 10 illustrates the changes in supply chain performance in dependence from r. In contrast to the results under full information the overall supply chain performance deteriorates due to setup cost reduction in some regions. The parameter values for which the expected supply chain performance deteriorates are a subset of the parameter values for which $\Delta CD > 0$ holds. This is not suprising, as the supply chain performance can only improve if the buyer faces holding costs h_1. Nonetheless, the analysis of the changes in expected costs is important to evaluate the overall effect on the supply chain performance.[10] The evaluation of ΔCD and ΔP, thus, implies different interpretation for the overall supply chain. If the parameter values are such that $\Delta P < 0$, then the option of setup cost reduction harms the overall supply chain performance. In contrast, if $\Delta CD > 0$ and $\Delta P > 0$ holds, then the option of setup cost reduction improve the supply chain performance, but the supply chain managers should be aware that cooperative and truthful information sharing would have an even greater impact on improving supply performance as if no setup cost reduction is possible. Finally, if $CD < 0$, then the coordination problem due to asymmetric information lessens or even vanishes.

[10] The interested reader can find more comparative static performance analyses for the parameters f_{max}, h_2 and π in Appendix 6.6, figure 13. Again, the main result does not change for this analysis: whether the investment in setup cost reduction reduces the supply chain performance or not depends on the parameter values.

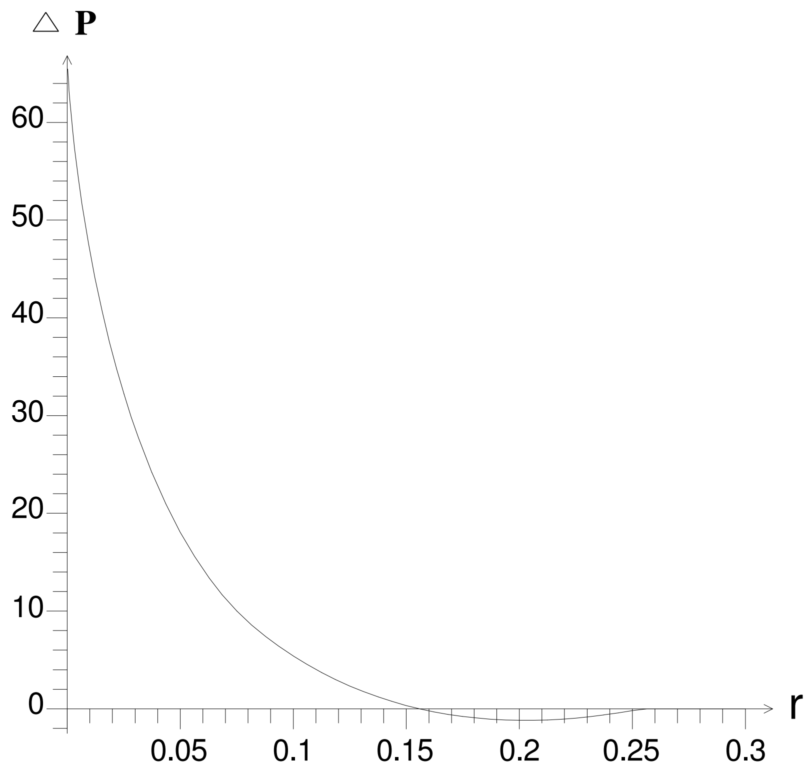

Figure 10: Changes in supply chain performance in dependence from interest rate r.

5 Conclusions

JiT delivery has received ever-increasing attention in the recent past. Typically, the implementation of JiT strategies is accompanied by setup cost reductions. If a weak bargaining position of the supplier is presumed, the supplier's optimal reaction is offering a pareto improving menu of contracts if he only posseses imperfect information about the buyer's cost position. The analysis of setup cost reduction under asymmetric information shows that the suppliers should take the option of setup cost reduction into account while offering a menu of contracts. Obviously, the suppliers will not be worse off in terms of expected profits, as the status quo (i.e. f_{max}) is still feasible. Yet, the effect on supply chain coordination and performance is ambiguous, as there are two contrary effects: the overinvestment and the setup cost effect. The screening of the buyer's private information leads to suboptimal order sizes (except for the lowest possible holding cost realization h_1). In turn, this can lead to suboptimally high investments in setup cost reduction. This analysis is robust for a wide variety of investment functions.

To obtain more differentiated insights in the impact of setup cost reduction on the coordination deficit as well as on the overall supply chain performance, the case of two possible holding cost realizations was analysed. Closed form solutions were computed, and a comparative static analysis was conducted. The analysis reveals that supply chain performance is particularly vulnerable, if the costs of setup cost reduction is relatively high.

This paper assumes a strong buyer's position. Hence, a buyer should account for the overinvestment effect when carrying out negotiations with a strategic partner in the supply chain. As the setup cost reduction is assumed to hold over the whole planning horizon, the overinvestment effect adversely impacts the supply chain performance even in the long run. One of the main assumptions in this article is that the buyer's will use their private information strategically, instead of sharing them truthfully with their suppliers. However, experimental research shows that this behavior is not always observable [Inderfurth et al [12]]. Especially the strategic and long-term effects of investments in setup cost reduction might influence the supplier's behavior and therefore this theory's predictions.

References

[1] J.F. Affisco, M. Paknejad, and F. Nasri. Quality improvement and setup reduction in the joint economic lot size model. *Eur. J. Oper. Res.*, 142(3):497–508, 2002.

[2] A. Banerjee. A joint economic-lot-size model for purchaser and vendor. *Decision Sci.*, 17(3):292–311, 1986.

[3] J.B. Brimberg and W.J. Hurley. A note on the assumption of constant order size in the basic EOQ model. *Production and Operations Management*, 15(1):171–172, 2006.

[4] C.J. Corbett. Stochastic inventory systems in a supply chain with asymmetric information: Cycle stocks, safety stocks, and consignment stock. *Oper. Res.*, 49(4):487–500, 2001.

[5] C.J. Corbett and X. de Groote. A supplier's optimal quantity discount policy under asymmetric information. *Management Sci.*, 46(3):444–450, 2000.

[6] C.J. Corbett and C.S. Tang. Designing supply contracts: Contract type and information asymmetry. In R. Ganeshan S. Tayur and M. Magazine, editors, *Quantitative Models for Supply Chain Management*. Kluwer Academic Publishers,2003, 2003.

[7] C.J. Corbett, D. Zhou, and C.S. Tang. Designing supply contracts: Contract type and information asymmetry. *Management Sci.*, 50(4):550–559, 2004.

[8] G. Fandel and J. Reese. Just-in-time logistics of a supplier in the car manufacturing industry. *Int. J. Product. Econ.*, 24:55–64, 1991.

[9] S.K. Goyal. An integrated inventory model for a single supplier-single customer problem. *Int. J. Product. Res.*, 15(1):107–111, 1977.

[10] A.Y. Ha. Supplier-buyer contracting: Asymmetric cost information and cutoff level policy for buyer participation. *Naval Res. Logistics*, 48(1):41–64, 2000.

[11] C.H. Hahn, D.J. Bragg, and D. Shin. Impact of the setup variable on capacity and inventory decisions. *Academy of Management Review*, 13(1):91–103, 1988.

[12] K. Inderfurth, A. Sadrieh, and G. Voigt. The impact of information sharing on supply chain performance in case of asymmetric information. *FEMM Working Paper Series, No. 01/2008*, pages 1–38, 2008.

[13] G. Keller and H. Noori. Justifying new technology acquisition through its impact on the cost of running an inventory policy. *IIE Transactions*, 20(20):284–291, 1988.

[14] S.L. Kim, C. Hayya, and J.D. Hong. Setup reduction in the economic production model. *Decision Sci.*, 23(2):500–508, 1992.

[15] J.P. Leschke. An empirical study of the setup-reduction process. *Production and Operations Management*, 5(2):121–131, 1996.

[16] J.P. Leschke and E.N. Weiss. The multi-item setup-reduction investment-allocation problem with continuous investment-cost functions. *Management Sci.*, 43(6):890–894, 1997.

[17] X. Liu and S. Cetinkaya. A note on “quality improvement and setup reduction in the joint economic lot size model“. *Eur. J. Oper. Res.*, 182(1):194–204, 2007.

[18] J.P. Monahan. A quantity discount pricing model to increase vendor profits. *Management Sci.*, 30(6):720–726, 1984.

[19] R. Myer. Suppliers - manage your costumers. *Harvard Bus. Rev.*, November - December:160–168, 1989.

[20] F. Nasri, J.F. Affisco, and M.J. Paknejad. Setup cost reduction in an inventory model with finite-range stochastic lead times. *Int. J. of Product. Res.*, 28(1):199–212, 1990.

[21] M.J. Paknejad and J.F. Affisco. The effect of investment in new technology on optimal batch quantity. *Proceedings of the Northeast Decision Science Institute*, pages 118–120, 1987.

[22] M.J. Paknejad, F. Nasri, and J.F. Affisco. An analysis of setup cost reduction in a two stage systeme with power investment function. *Mathematical Methods of Operations Research*, 43(3):389–401, 1996.

[23] E.L. Porteus. Investing in Reduced setups in the EOQ Model. *Management Sci.*, 31(8):998–1010, 1985.

[24] D. Sappington. Limited liability contracts between principal and agent. *J. Econ. Theory*, 29:1–21, 1983.

[25] R.J. Schonberger and M.J. Schniederjans. Reinventing inventory control. *Interfaces*, 14(3):76–83, 1984.

[26] A.M. Spence. Multi-product quantity-dependent prices and profitability constraints. *The Review of Economic Studies*, 47(5):821–841, 1980.

[27] E. Sucky. A bargaining model with asymmetric information for a single supplier-single buyer problem. *Eur. J. Oper. Res.*, 171(2):516–535, 2006.

[28] P. van Beek and C. van Putten. OR Contributions to Flexibility Improvement in Production/Inventory Systems. *Eur. J. Oper. Res.*, 31(1):52–60, 1987.

6 Appendix

6.1 Karush-Kuhn-Tucker Conditions

$$\frac{\partial L}{\partial Q_i} = -p_i \frac{f_i}{Q_i^2} + \mu_i \frac{h_i}{2} + \sum_{j=1, j\neq i}^{n} (\lambda_{ij} \frac{h_i}{2} - \lambda_{ji} \frac{h_j}{2}) \leq 0 \tag{19}$$

$$\frac{\partial L}{\partial T_i} = p_i - \mu_i + \sum_{j=1, j\neq i}^{n} (\lambda_{ji} - \lambda_{ij}) \leq 0 \tag{20}$$

$$\frac{\partial L}{\partial f_i} = \frac{pi}{Q_i} + p_i \cdot r \cdot \frac{\partial k(f_i)}{\partial f_i} + \gamma_i - \alpha_i \leq 0 \tag{21}$$

$$\frac{\partial L}{\partial Q_i} \cdot Q_i = 0 \tag{22}$$

$$\frac{\partial L}{\partial Q_i} = 0 \tag{23}$$

$$\frac{\partial L}{\partial T_i} T_i = 0 \tag{24}$$

$$\frac{\partial L}{\partial f_i} \cdot f_i = 0 \tag{25}$$

$$\frac{\partial L}{\partial \mu_i} = \frac{h_i}{2} Q_i - C_{AS} - T_i \leq 0 \tag{26}$$

$$\frac{\partial L}{\partial \mu_i} \mu_i = 0 \tag{27}$$

$$\frac{\partial L}{\partial \lambda_{ij}} = \frac{h_i}{2} Q_i - T_i - \frac{h_i}{2} Q_j + T_j \leq 0 \tag{28}$$

$$\frac{\partial L}{\partial \lambda_{ij}} \lambda_{ij} = 0 \tag{29}$$

$$\frac{\partial L}{\partial \alpha_i} = f_{min} - f_i \leq 0 \tag{30}$$

$$\frac{\partial L}{\partial \alpha_i} \alpha_i = 0 \tag{31}$$

$$\frac{\partial L}{\partial \gamma_i} = f_i - f_{max} \leq 0 \tag{32}$$

$$\frac{\partial L}{\partial \gamma_i} \gamma_i = 0 \tag{33}$$

$$\alpha_i, \gamma_i, \mu_i \geq 0, \lambda_{ij} \geq 0 \tag{34}$$

6.2 The optimal menu of contracts

Solving (20) for μ_i and substituting in (19) while considering $Q_i \geq 0$ results in

$$p_i(\frac{h_i}{2} - \frac{f_i}{Q_i^2}) + \frac{1}{2}\sum_{j=1,j\neq i}^{n} \lambda_{ji}(h_i - h_j) \quad = \quad 0 \tag{35}$$

From (20) and Sappington's [24] (see section 2) results $\mu_i = 0 \quad \forall i = 1, ..., n-1,\ \mu_n = 1$ and $\lambda_{ij} = 0$, for $j < i$ and $j > i+1$ it follows that

$$\begin{aligned}
\text{for} \quad i \quad &= \quad n \\
p_n \quad &+ \quad \lambda_{n-1,n} = 1 \\
\text{for} \quad i \quad &= \quad n-1 \\
p_{n-1} \quad &+ \quad \lambda_{n-2,n-1} - \lambda_{n-1,n} = 0 \Rightarrow \lambda_{n-2,n-1} = 1 - p_n - p_{n-1} = \sum_{t=1}^{n-2} p_t \\
\text{for} \quad i \quad &= \quad n-2 \\
p_{n-2} \quad &+ \quad \lambda_{n-3,n-2} - \lambda_{n-2,n-1} = 0 \Rightarrow \lambda_{n-3,n-2} = 1 - p_n - p_{n-1} - p_{n-2} = \sum_{t=1}^{n-3} p_t \\
\text{for} \quad i \quad &= \quad 2 \\
p_2 \quad &+ \quad \lambda_{12} - \lambda_{23} \Rightarrow \lambda_{12} = 1 - p_n - ... - p_2 = p_1
\end{aligned}$$

Substituting this result into (35) and solving for Q gives:

$$Q_i^* \quad = \quad \sqrt{\frac{2 \cdot f_i^*}{h_i + \phi_i}} \tag{36}$$

$$\text{where} \quad \phi_i \quad = \quad \frac{\sum_{t=0}^{i-1} p_t}{p_i}(h_i - h_{i-1}) \quad \forall \quad i = 1, ..., n \quad \text{and} \quad p_0, h_0 = 0 \tag{37}$$

$$\forall \phi_i \quad < \quad \phi_{i+1}, \quad i = 1, ..., n-1 \tag{38}$$

where (38) follows directly from assumption (1).

As $\mu_n = 1$ (see [24]) it follows that

$$T_n^* = \frac{h_n}{2} Q_n^* - C_{AS} \tag{39}$$

Furthermore, $\lambda_{ij} = 0$, for $j < i$ and $j > i+1$ and $\lambda_{ij} > 1$ for $i = j-1$ holds (see [24]) and it follows from (28) :

$$T_i^* = \frac{h_i}{2}(Q_i^* - Q_{i+1}^*) + T_{i+1}^* \quad \forall i = 1, ..., n-1 \tag{40}$$

From (21) and $\alpha_i = 0, \gamma_i = 0$ (i.e. as long as the optimal investment level is an interior solution) it follows that:

$$\frac{1}{Q_i} = -r \cdot \frac{dk(f_i)}{df_i} \tag{41}$$

For $\alpha_i > 0$ ($\gamma_i > 0$) the optimal setup costs are f_{min} (f_{max}).

Substituting (36) into (41) we see that the optimal setup cost level f_i^* is obtained by solving the following equation (as long as the optimal investment level is an interior solution):

$$\sqrt{\frac{h_i + \phi_i}{2 \cdot f_i^*}} \quad = \quad -r \cdot \left. \frac{dk(f_i)}{df_i} \right|_{f_i = f_i^*} \tag{42}$$

.

6.3 Power Cost Function for $n = 2$

The expressions (16) and (17) follow directly from Porteus [23]. The expressions (13)-(15) and (18) are simply obtained by exerting (36)-(41) for $n = 2$. Expression (11) is obtained by solving (6), i.e.

$$\sqrt{\frac{h_2+\phi_2}{2\cdot f_r}} = -r\cdot a\cdot(-b)\cdot f_r^{-b-1} \tag{43}$$

$$\Rightarrow f_r = \left(\sqrt{\frac{2}{h_2+\phi_2}}\cdot r\cdot a\cdot b\right)^{\left(\frac{1}{b+0.5}\right)} \tag{44}$$

6.4 Overinvestment in case of a convex investment function: numerical example

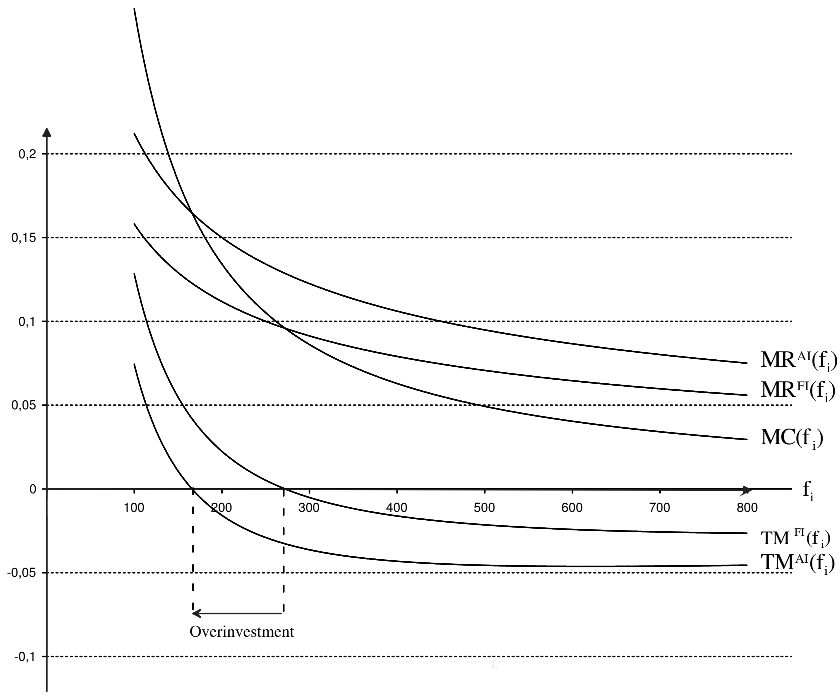

Figure 11: Marginal analysis for the numerical example and $i = 2$

6.5 Mathematical formulation for the change in coordination deficit ΔCD, the overinvestment effect OE and the setup cost effect SE

$$
\begin{aligned}
\Delta CD &= (K_2(SC)(Q_2^*(SC), f_2^*(SC)) - K_2(SC)(Q_2^*, f_2^*)) \\
&- K_2(SC)(Q_2^{f_{max}}(SC), f_{max}) - K_2(SC)(Q_2^{f_{max}}, f_{max})
\end{aligned}
$$

$$OE = r \cdot (k(f_2^*) - k(f_2^*(SC)))$$

$$
\begin{aligned}
SE &= \left(\left(\frac{f_{max}}{Q_2^{f_{max}}(SC)} + \frac{h_2}{2} \cdot Q_2^{f_{max}}(SC) \right) - \left(\frac{f_{max}}{Q_2^{f_{max}}} + \frac{h_2}{2} \cdot Q_2^{f_{max}} \right) \right) \\
&- \left(\left(\frac{f_2^*(SC)}{Q_2^*(SC)} + \frac{h_2}{2} \cdot Q_2^*(SC) \right) - \left(\frac{f_2^*}{Q_2^*} + \frac{h_2}{2} \cdot Q_2^* \right) \right)
\end{aligned}
$$

6.6 Comparative static analysis

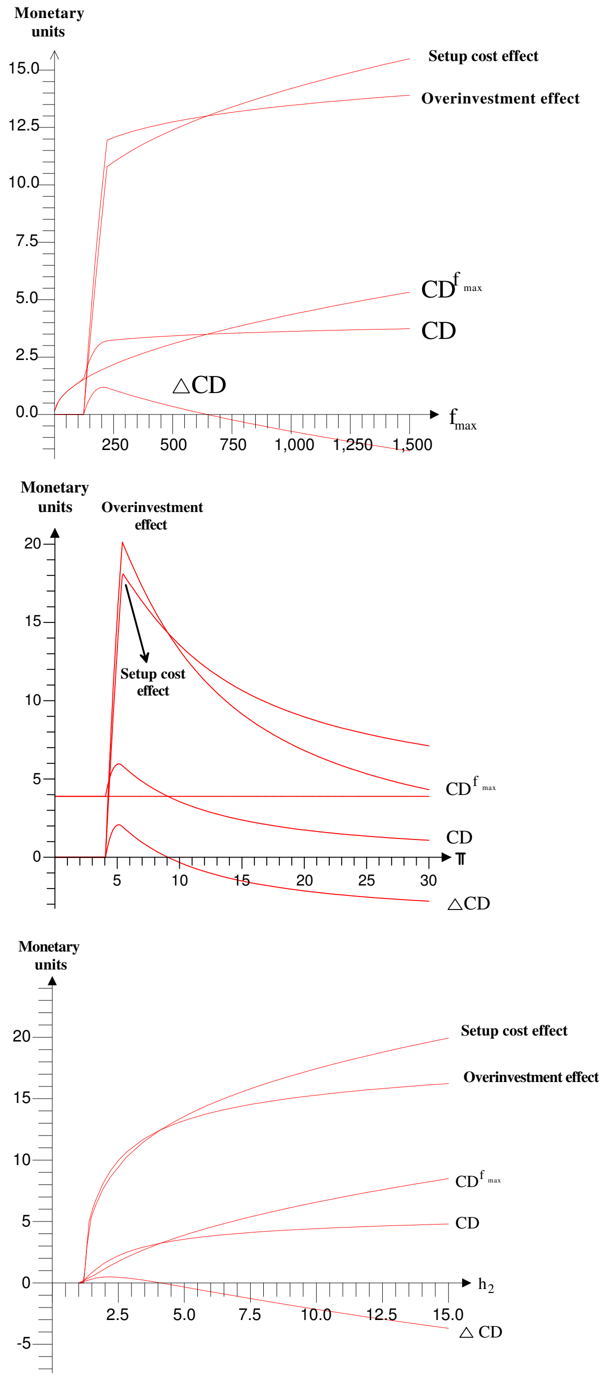

Figure 12: Coordination deficit in dependence of f_{max}, π and h_2

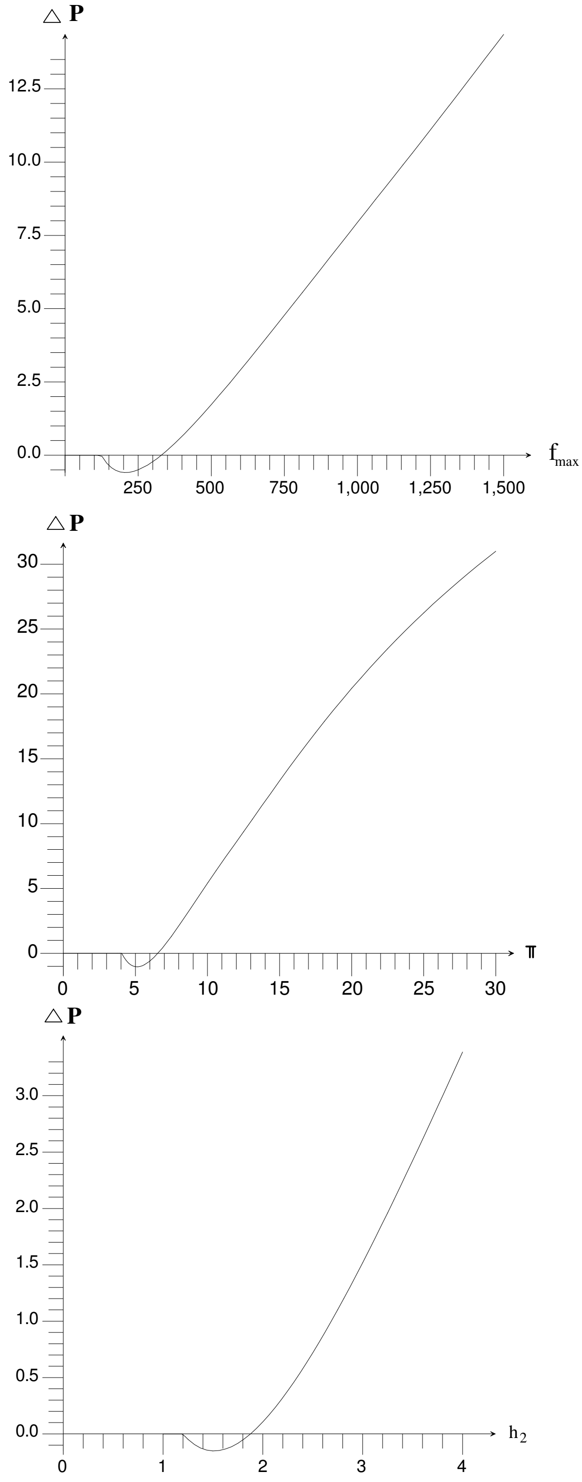

Figure 13: Performance changes in dependence of f_{max}, π and h_2

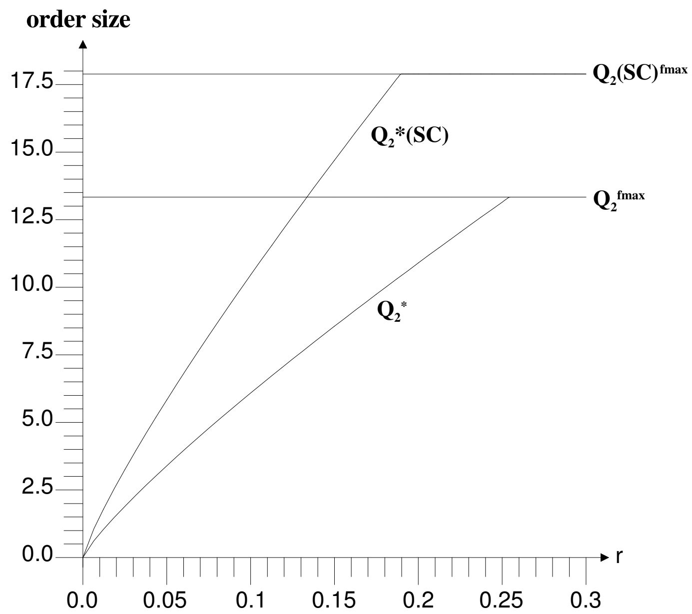

Figure 14: Comparative static analyses - order sizes in dependence of interest rate r