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Revenue Sharing, Competitive Balance and the Contest Success Function

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Abstract: This paper investigates revenue sharing in an asymmetric two team contest model of a sports league with Nash behavior of team owners. The innovation of the analysis is that it focuses on the role of the contest success function (CSF). In case of an inelastic talent supply, revenue sharing turns out to worsen competitive balance regardless of the shape of the CSF. For the case of an elastic talent supply, in contrast, the effect of revenue sharing on competitive balance depends on the specification of the CSF. We fully characterize the class of CSFs for which revenue sharing leaves unaltered competitive balance and identify CSFs ensuring that revenue sharing renders the contest closer.

JEL classification: D72, C72

Keywords: Revenue Sharing, Competitive Balance, Contest Success Function

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1 Introduction

Under a revenue sharing agreement in sports competitions, the revenue obtained by the individual competitor is partially redistributed to other competitors. Many major sports leagues in the U.S. use a variety of such redistribution schemes. For example, in the U.S. National Football League (NFL) 40% of designated gate revenue is assigned to the visiting team. A similar agreement is in operation in the U.S. Major League Baseball (MLB). Moreover, revenue sharing is not restricted to gate revenue. Many European sports leagues like, e.g., the German Soccer League (DFL) centrally sell television rights to broadcasting stations and distribute the resulting revenue among the teams in the league. Compared to the decentral sale of television rights, a central solution usually possesses redistributive elements and is therefore similar to a revenue sharing agreement.

The basic motivation behind all kinds of revenue sharing is to level the playing field among the teams. The hope is that redistribution of resources supports the so-called weak drawing teams which have a rather small potential of raising revenue. The additional revenue should enable the weak teams to invest more in the quality of their players and to increase their probability of success so that the gap to stronger teams is reduced. Put differently, revenue sharing usually aims at improving competitive balance within a sports league. This may be beneficial for the league as a whole since it is widely believed that closer competitions in sports leagues are more exciting to potential viewers and more interesting for sponsors. Hence, an increase in competitive balance ceteris paribus increases attendance and the willingness of sponsors to give the league financial support.

The hypothesis that a more balanced competition is beneficial for a sports league is largely undisputed. However, in the previous literature there has been a long and controversial discussion on the impact of revenue sharing on competitive balance. The early literature derived the so-called invariance principle (e.g. El-Hodiri and Quirk, 1971, Quirk and El-Hodiri, 1974, Fort and Quirk, 1995, Vrooman, 1995). This principle states that introducing or intensifying revenue sharing leaves unaltered competitive balance. The argument is that with a fixed talent supply in a sports league, revenue sharing reduces the unit cost of talent while leaving unchanged the relation of the teams' investment in talent. However, Szymanski (2004) shows that it is more than the fixed talent supply assumption that drives the invariance result.[1] Previous authors additionally assume what Szymanski

[1] See also the survey of Szymanski (2003) and the discussion in Szymanski (2006) and Eckard (2006).

(2004) calls Walrasian fixed supply behavior. Under such a behavioral assumption each team owner takes into account that an increase in its own talent investment reduces talent investment of the other teams due to the fixed talent supply. This is equivalent to the conjectural variation of joint profit maximization and thereby deviates from Nash behavior typically assumed in non-cooperative game models. Szymanski and Késenne (2004) show that with Nash behavior the invariance principle breaks down and revenue sharing worsens competitive balance, regardless of whether talent supply is fixed or elastic.

This paper contributes to the discussion of the effects of revenue sharing on competitive balance in sports leagues. We follow Szymanski and Késenne (2004) and consider revenue sharing in an asymmetric two team contest model with Nash behavior of team owners. The main innovation of our analysis is that it focuses on the role of the so-called contest success function (CSF). This function gives a team's probability of success (e.g. probability of winning the championship) as a function of the talent levels of all teams. Szymanski and Késenne (2004) derive their main results with the help of a CSF that equals a team's talent level divided by the sum of talent levels of both teams. This is the well known Tullock (1980) ratio-form CSF. In contrast, we develop a model with a general logit CSF and discuss whether and, if so, how the insights of Szymanski and Késenne (2004) change when alternative specifications of the CSF are considered.

The answer to these questions turns out to depend mainly on the assumption regarding the elasticity of talent supply. If talent supply is perfectly inelastic, the result of Szymanski and Késenne (2004) holds regardless of the shape of the CSF. Introducing or intensifying revenue sharing will reduce competitive balance. The intuition is as follows. For constant unit cost of talent, revenue sharing has two effects. Due to the *revenue effect*, revenue sharing reduces the marginal return to talent investment so that both teams ceteris paribus invest less in talent. The *competition effect* states that, in contrast to the strong team, the weak team ceteris paribus has an incentive to increase talent investment since its opponent (the strong team) becomes weaker. The sum of both effects is unambiguously negative for the strong team and may be positive for the weak team. However, if talent supply is fixed, the unit cost of talent is not constant and we additionally obtain a *talent cost effect* of revenue sharing: The revenue and competition effects reduce aggregate talent demand of the teams so that the unit cost of talent has to fall in order to restore the equilibrium in the talent market. This, in turn, provides both teams with the incentive to increase talent investment again. The talent cost effect is stronger for the large team than for the small team since with lower (common) unit cost of talent the large team can exploit its advantage to a larger extent. It will turn out that the talent cost effect renders the sum of all three effects positive for the strong team and negative for the weak team, regardless of the shape of the CSF. As consequence, we obtain as a general result under an inelastic talent supply that revenue sharing worsens competitive balance in the sports league.

In contrast, the result of Szymanski and Késenne (2004) cannot be generalized to all specifications of the CSF if talent supply is perfectly elastic. In this case, the unit cost of talent is fixed and the talent cost effect disappears. The remaining revenue and competition effects of revenue sharing unambiguously lower the talent level of the large team, but the change in the talent level of the small team is ambiguous. The effect of revenue sharing on competitive balance therefore depends on the shape of the CSF. Perhaps most interestingly, we fully characterize the class of CSFs for which the invariance principle is restored, i.e. for which revenue sharing does not alter competitive balance. Note that we obtain this invariance result in the case of an elastic talent supply, not in the opposite case of a fixed talent supply as previous studies referred to above.[2] Moreover, with an elastic talent supply it is even possible to determine specifications of the CSF that ensure the intended positive effect of revenue sharing on competitive balance. Both a zero and a positive impact of revenue sharing on competitive balance can be generated with the help of CSFs that are rather similar to the Hirshleifer (1989, 1991) difference-form CSF. Beside the Tullock (1980) ratio-form CSF, the difference-form specification is the most used CSF in the contest literature (see e.g. Baik, 1998). An axiomatic foundation of this functional form is provided by Skarperdas (1996).

The results of our theoretical analysis have important implications for revenue sharing agreements in real world sports leagues. The case of an inelastic talent supply is usually seen relevant for the major sports leagues in the U.S. since there is only one league for each kind of sports and the whole talent is employed in this league. In Europe, in contrast, several leagues compete for scarce talent so that every individual league faces an elastic talent supply. With this interpretation, our results suggest that revenue sharing is a bad thing for the U.S. major leagues. Independent of the CSF, revenue sharing is counterproductive as it renders the league more unbalanced. In Europe, however, it may well be the case that revenue sharing improves competitive balance, depending on the

[2]The reason for this difference is that we do not use the assumption of Walrasian fixed supply behavior, but instead follow Szymanski and Késenne (2004) and assume Nash behavior throughout.

shape of the CSF. To the extent that league managers are able to manipulate the CSF, we may even argue that league managers can choose that shape of the CSF which ensures a positive effect of revenue sharing on competitive balance.

The remainder of the paper is organized as follows. In Section 2 we introduce the contest model of a sports league and characterize the Nash equilibrium. Sections 3 and 4 then investigate the effects of revenue sharing under the assumption of elastic and inelastic talent supply, respectively. Section 5 concludes the analysis.

2 Asymmetric Contest with Revenue Sharing

We use a variant of the model of Szymanski and Késenne (2004). The sports league is modeled as a contest. There are two risk-neutral teams competing for winning the championship. Team $i \in \{1, 2\}$ expends effort $t_i \geq 0$ that is usually interpreted as investment in or demand for talent. For given investment of team $j \neq i$, an increase in investment of team i increases the probability w_i that team i will win the championship. To specify this winning probability, we introduce a general logit function[3]

$$w_i = \frac{H(t_i)}{H(t_i) + H(t_j)}, \quad i \neq j, \tag{1}$$

with $H(t) \geq 0$ and $H'(t) > 0$ for all $t > 0$. Equation (1) is the CSF in our model as it determines team i's probability of success as a function of both teams' talent levels. Since the shape of the CSF is solely determined by the shape of the function H, we will subsequently denote H as CSF. In sporting contests, a possible interpretation of H is that it measures team i's performance. It represents the production technology of the contest showing how the input 'talent' is transformed into output. According to (1), team i's winning probability then equals the ratio of its performance to total performance of both teams. The condition $H'(t) > 0$ implies that higher investment in talent leads to higher performance and, ceteris paribus, to a higher probability of success. We follow Szymanski and Késenne (2004) and assume that H is the same for both teams since it seems reasonable that in industries like sports leagues all competitors adopt best practice.

In order to derive their main results, Szymanski and Késenne (2004) consider rather

[3] The most general function that can be used is $w_i = W(t_i, t_j)$. But this function would not yield clear cut results which may be the reason why a large part of the contest literature assumes some kind of logit specification. An axiomatic foundation of (1) is given, for example, in Skarperdas (1996).

general revenue functions of the two teams and focus on the specific CSF $H(t) = t$. As we are interested in the role of the CSF, we proceed the other way round. We consider linear revenue functions of the teams and investigate how the shape of the CSF influences the effects of revenue sharing. More specifically, team i's revenue is a linear function $v_i w_i$ of its winning probability. The marginal revenue $v_i > 0$ is constant for both teams.[4] If both teams have the same marginal revenue, the league will always be balanced and there is no scope for discussing whether revenue sharing improves or worsens competitive balance. The focus will therefore be on an asymmetric contest characterized by differences in marginal revenue. Without loss of generality, we assume that team 1 is the large team with a higher revenue potential than the small team 2, i.e. $v_1 > v_2$. Asymmetries in the revenue potential may be due to differences in, for example, the number of fans, the capacity of the stadium or the effectiveness of managers to promote the teams.

Revenue sharing is modeled by the redistribution parameter $\alpha \in]v_1/(v_1 + v_2), 1]$. This parameter gives the share of a team's revenue that this team is allowed to keep. The share $1 - \alpha$ of a team's revenue is redistributed to the other team. There will be no revenue sharing in the extreme case of $\alpha = 1$. The effects of revenue sharing on the contest outcome can be investigated by decreasing α to some value below 1. The assumption $\alpha > v_1/(v_1 + v_2)$ will ensure that the net marginal revenue (the marginal revenue after sharing) is positive for both teams. This is a necessary condition for both teams to demand a positive amount of talent in equilibrium. Team i's profit can be written as

$$\Pi^i(t_i, t_j) = \alpha v_i w_i + (1 - \alpha) v_j w_j - c t_i, \qquad i \neq j, \tag{2}$$

where $c > 0$ is the cost per unit of talent. Profit of team i in equation (2) equals the part of this team's revenue that is not redistributed to team j plus the revenue that team i

[4] Linear revenue functions are also used by Szymanski and Késenne (2004) to illustrate their results and by other authors (see the survey of Szymanski, 2003, for example). An argument often made in the sports economics literature is that the revenue function should be inverted U-shaped in order to reflect the fact that a rather unbalanced contest reduces revenue (see again Szymanski, 2003). This is certainly true, but it is an open question whether a team manager really takes into account this property of the revenue function (and, thus, reduces talent investment when his team becomes too dominant) or whether he instead works with a monotonically increasing revenue function (that can be approximated by a linear function). Moreover, linear revenue functions may be motivated by the assumption that the league is not too unbalanced so that the inverted U-shaped part of the 'true' revenue function is irrelevant for the analysis. Anyway, the assumption of linear revenue in our framework simply helps to focus on the role of the CSF without mixing it up with the impact of the revenue function.

gets from team j less the cost of talent. Using (1) in (2) and rearranging yields

$$\Pi^i(t_i, t_j) = (1-\alpha)v_j + z_i \frac{H(t_i)}{H(t_i) + H(t_j)} - ct_i, \qquad i \neq j, \tag{3}$$

with $z_i := \alpha v_i - (1-\alpha)v_j > 0$ due to $\alpha > v_1/(v_1 + v_2)$. Equation (3) shows that team i's revenue can be decomposed into two components. The first is the fixed part $(1-\alpha)v_j$. Independent of team i's own talent, this team will always get a part of team $j's$ revenue as long as revenue sharing takes place ($\alpha < 1$). The second revenue component is the variable part $z_i w_i$. A marginal increase in team i's winning probability increases revenue of team i by z_i. The net marginal revenue z_i reflects both the increase in team i's own revenue (caused by the rise in team i's winning probability) as well as the decrease in the revenue team i obtains from team j (caused by a drop in team j's winning probability). Our assumption $v_1 > v_2$ implies $z_1 > z_2$ so that the larger team has higher marginal revenue not only before revenue sharing but also thereafter. Note that the impact of introducing or intensifying revenue sharing on net marginal revenue is negative for both teams, i.e. $\mathrm{d}z_i/\mathrm{d}\alpha = v_1 + v_2 > 0$. The reason is that lowering α reduces αv_i which reflects the revenue gain from increasing team i's winning probability and increases $(1-\alpha)v_j$ which reflects team i's revenue loss from lowering team j's winning probability (by increasing team i's winning probability). Both effects have a negative impact on team i's net revenue.

As already mentioned in the introduction, we assume Nash behavior of the two teams throughout. Team i maximizes its profit (3) with respect to investment t_i. In doing so, it takes as given investment t_j of the other team. We are looking for an interior (pure-strategy) Nash equilibrium (t_1^*, t_2^*). This equilibrium is determined by the first and second order conditions of profit maximization

$$\Pi^i_{t_i}(t_i^*, t_j^*) = z_i \frac{H'(t_i^*)H(t_j^*)}{[H(t_i^*) + H(t_j^*)]^2} - c = 0, \qquad i \neq j, \tag{4}$$

$$\Pi^i_{t_i t_i}(t_i^*, t_j^*) = z_i H(t_j^*) \frac{[H(t_i^*) + H(t_j^*)]H''(t_i^*) - 2[H'(t_i^*)]^2}{[H(t_i^*) + H(t_j^*)]^3} < 0, \qquad i \neq j. \tag{5}$$

With the help of conditions (4) and (5) we can impose a restriction on the shape of the CSF. It is assumed that H satisfies

$$H(t)H''(t) - [H'(t)]^2 < 0 \qquad \text{for all} \quad t \geq 0. \tag{6}$$

For $H''(t) \leq 0$ this condition is always satisfied. For $H'' > 0$, condition (6) is a necessary condition for the existence of a Nash equilibrium. To see this, suppose the opposite.

Then $H'(t)/H(t)$ is non-decreasing in t. Equation (4) and $z_1 > z_2$ imply $H'(t_2^*)/H(t_2^*) > H'(t_1^*)/H(t_1^*)$ and, thus, $t_2^* \geq t_1^*$ and $H(t_2^*) \geq H(t_1^*)$. This yields $[H(t_1^*) + H(t_2^*)]H''(t_1^*) - 2[H'(t_1^*)]^2 \geq 2\{H(t_1^*)H''(t_1^*) - [H'(t_1^*)]^2\} \geq 0$. It follows $\Pi^1_{t_1 t_1}(t_1^*, t_2^*) \geq 0$, i.e. the second order condition of team 1 is violated and there does not exist an equilibrium. Hence, if an equilibrium exists, condition (6) has to be satisfied. $H'(t)/H(t)$ is then decreasing in t and we obtain $t_1^* > t_2^*$, $H(t_1^*) > H(t_2^*)$ and $w_1^* > w_2^*$. This means that team 1 demands more talent, has higher performance and a higher winning probability than team 2.

Our main interest is how revenue sharing affects competitive balance in the equilibrium of the league. We follow the large part of the sports economics literature and view competitive balance as the uncertainty of the contest outcome. Hence, competitive balance is the highest when both teams have the same chance of winning the championship. The larger the difference in the probabilities of success, the more unbalanced is the league. As Szymanski and Késenne (2004) we therefore use[5]

$$b^* := \frac{w_1^*}{w_2^*} = \frac{H(t_1^*)}{H(t_2^*)} \tag{7}$$

as a measure of competitive balance. Hence, competitive balance is the highest if $b^* = 1$. The larger the difference $b^* - 1$ in absolute terms, the smaller is the degree of competitive balance in the league. As $t_1^* > t_2^*$ and $H(t_1^*) > H(t_2^*)$, we always have $b^* > 1$. The question is then whether introducing or intensifying revenue sharing (lowering α) improves competitive balance (lowers b^*). Formally, this would be the case if $\mathrm{d}b^*/\mathrm{d}\alpha > 0$.

From (7) we can derive a first basic answer to this question. Let $\eta^i_{Ht} := t_i^* H'(t_i^*)/H(t_i^*)$ be the elasticity of team i's equilibrium performance with respect to its equilibrium talent level and $\eta^i_{t\alpha} := (\mathrm{d}t_i^*/\mathrm{d}\alpha)\cdot(\alpha/t_i^*)$ be the elasticity of team i's equilibrium talent with respect to the sharing parameter α. Differentiating b^* defined by (7) then yields

$$\frac{\mathrm{d}b^*}{\mathrm{d}\alpha} \gtreqless 0 \qquad \Leftrightarrow \qquad \eta^1_{Ht}\eta^1_{t\alpha} \gtreqless \eta^2_{Ht}\eta^2_{t\alpha}. \tag{8}$$

$\eta^i_{Ht}\eta^i_{t\alpha}$ is the elasticity of team i's equilibrium performance with respect to the sharing parameter α. Hence, (8) states that competitive balance is improved by revenue sharing if and only if a one percent reduction in α reduces performance of team 1 by a larger percentage than performance of team 2. The reason is that the winning probability of team 1 then falls while that of team 2 goes up with a decline in b^* as an end result.

[5] Note that measuring competitive balance by b^* is equivalent to measuring competitive balance directly by the difference in winning probabilities $\tilde{b}^* := w_1^* - w_2^*$, as done in Runkel (2006). This can be seen if we write the difference as $\tilde{b}^* = (b^* - 1)/(1 + b^*)$. Hence, $\tilde{b}^*$ increases if and only if b^* increases.

Whether this condition is satisfied or not depends mainly on the shape of the CSF. The functional form of H directly determines the elasticity η^i_{Ht} and indirectly the elasticity $\eta^i_{t\alpha}$ via the equilibrium conditions given by equation (4). This highlights the important role the CSF plays for the effects of revenue sharing on competitive balance in sports leagues. In the following sections we will investigate this role by considering several specifications of the CSF and their consequences for the expression in equation (8). In doing so, we follow the previous literature referred to in the introduction and distinguish the cases of perfectly elastic and perfectly inelastic supply of talent. The former case implies that the unit cost of talent c is constant. In the latter case, in contrast, we have to account for the effect revenue sharing exerts on c via changes in the teams' demand for talent. This will require to make explicit the market clearing condition for talent.

3 Elastic Talent Supply

Let us start with the case of a perfectly elastic talent supply. In this case, c is given and equation (4) determines the teams' equilibrium talent levels as functions of the sharing parameter α (which is contained in the net marginal revenue z_1 and z_2). Totally differentiating, using $\mathrm{d}z_i/\mathrm{d}\alpha = v_1 + v_2 > 0$ and rearranging gives the matrix equation

$$\begin{pmatrix} z_1 H_2 \dfrac{(H_1+H_2)H_1'' - 2(H_1')^2}{H_1+H_2} & z_1 H_1' H_2' \dfrac{H_1 - H_2}{H_1+H_2} \\ z_2 H_1' H_2' \dfrac{H_2 - H_1}{H_1+H_2} & z_2 H_1 \dfrac{(H_1+H_2)H_2'' - 2(H_2')^2}{H_1+H_2} \end{pmatrix} \begin{pmatrix} \mathrm{d}t_1 \\ \mathrm{d}t_2 \end{pmatrix}$$

$$= \begin{pmatrix} -(v_1+v_2)H_2 H_1' \\ -(v_1+v_2)H_1 H_2' \end{pmatrix} \mathrm{d}\alpha, \quad (9)$$

where $H_i := H(t_i^*)$, $H_i' := H'(t_i^*)$ and $H_i'' := H''(t_i^*)$. The determinant of the matrix on the LHS of equation (9) reads

$$\Delta = \frac{z_1 z_2 H_1 H_2}{(H_1+H_2)^2} \prod_{i \neq j} [(H_i + H_j)H_i'' - 2(H_i')^2] + \frac{z_1 z_2 (H_1' H_2')^2 (H_1 - H_2)^2}{(H_1+H_2)^2} > 0. \quad (10)$$

The sign of (10) follows from the second order condition (5). Applying Cramer's rule to the matrix equation (9) and using equation (4) to eliminate z_1 and z_2 yields the impact of revenue sharing on the teams' equilibrium investment levels

$$\frac{\mathrm{d}t_1^*}{\mathrm{d}\alpha} = \frac{c(v_1+v_2)H_1 H_1'(H_1+H_2)}{\Delta} \left\{ \frac{(H_1 - H_2)(H_2')^2}{H_2 H_1'} - \frac{H_2[(H_1+H_2)H_2'' - 2(H_2')^2]}{H_1 H_2'} \right\}, (11)$$

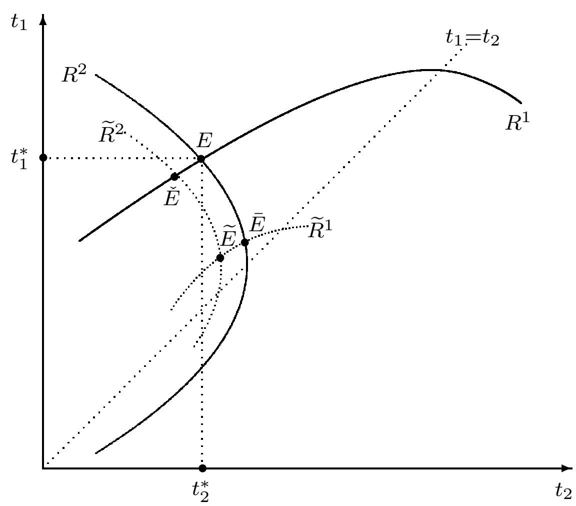
$$\frac{dt_2^*}{d\alpha} = \frac{c(v_1+v_2)H_2H_2'(H_1+H_2)}{\Delta}\left\{\frac{(H_2-H_1)(H_1')^1}{H_1H_2'} - \frac{H_1[(H_1+H_2)H_1''-2(H_1')^2]}{H_2H_1'}\right\}. \quad (12)$$

Taking into account the second order condition (5), $\Delta > 0$, $H_1 > H_2$ and $dH(t_i^*)/d\alpha = H_i' \cdot (dt_i^*/d\alpha)$ immediately proves

Lemma 1. *Suppose talent supply is perfectly elastic. Introducing or intensifying revenue sharing then reduces equilibrium investment and performance of team 1, that is* $dt_1^*/d\alpha > 0$ *and* $dH(t_1^*)/d\alpha > 0$*. The impact of revenue sharing on equilibrium talent investment and performance of team 2 is indeterminate, i.e.* $dt_2^*/d\alpha \gtreqless 0$ *and* $dH(t_2^*)/d\alpha \gtreqless 0$.

This lemma can best be illustrated with the help of Figure 1 which displays the reaction functions of the two teams. Team 1's reaction function R^1 increases up to the point where

Figure 1: Revenue Sharing with Elastic Talent Supply



$t_2 = t_1$ and decreases thereafter. A similar argument holds for team 2's reaction function R^2 if we exchange the two axes.[6] Since the equilibrium of the league satisfies $t_1^* > t_2^*$, it is in a point like E where R^1 has positive and R^2 negative slope.

Now suppose revenue sharing is introduced or intensified. As explained already above, this reduces team 2's net marginal revenue z_2. As consequence, the reaction function of

[6]Formally, the slopes of the reaction functions can be computed from the first order condition (4) as $(dt_1/dt_2)|_{R^1} = -\Pi^1_{t_1t_2}/\Pi^1_{t_1t_1}$ and $(dt_1/dt_2)|_{R^2} = -\Pi^2_{t_2t_2}/\Pi^2_{t_2t_1}$ with $\Pi^1_{t_1t_2}(t_1,t_2) = z_1H_1'H_2'(H_1 - H_2)/(H_1+H_2)^3$ and $\Pi^2_{t_2t_1}(t_2,t_1) = z_2H_1'H_2'(H_2-H_1)/(H_1+H_2)^3$.

team 2 is shifted to the left from R^2 to $\widetilde{R}^2$. Ceteris paribus the contest then reaches the point $\check{E}$ where the talent level of team 2 is reduced compared to the initial equilibrium E. This is the *revenue effect* for team 2. Moreover, the shift of team 2's reaction function also lowers team 1's talent investment. This is the *competition effect* for team 1: Team 1's opponent becomes weaker, so team 1 reduces investment since it is already the stronger team with the higher investment level. Analogously, revenue sharing reduces team 1's net marginal revenue z_1. Team 1's reaction function is therefore shifted downwards from R^1 to $\widetilde{R}^1$ so that the contest ceteris paribus moves from E to $\bar{E}$. This shift encompasses a *revenue effect* for team 1 and a *competition effect* for team 2. The revenue effect for team 1 is unambiguously negative implying that, compared to the initial equilibrium E, talent investment of team 1 is reduced. But the competition effect of team 2 is positive: Team 2 is the weaker team and its opponent becomes relatively weaker by the revenue effect. Hence, team 2 ceteris paribus receives the incentive to increase talent investment. The sum of revenue and competition effect is unambiguously negative for the strong team 1. In contrast, for the weak team 2 the competition effect may outweigh the revenue effect so that team 2's talent investment may increase upon introducing or intensifying revenue sharing. Such a situation is displayed in Figure 1 where the shift of team 2's reaction function is moderate and the new equilibrium of the contest is attained in point $\widetilde{E}$.

What are the implications for competitive balance? Differentiating equation (7), using equations (11) and (12) and rearranging yields

$$\frac{\mathrm{d}b^*}{\mathrm{d}\alpha} = \frac{2c(v_1+v_2)(H_1'H_2')^2}{\Delta}\,[G(t_1^*) - G(t_2^*)], \tag{13}$$

with

$$G(t) := H(t)\,\frac{H(t)H''(t) - [H'(t)]^2}{[H'(t)]^3}. \tag{14}$$

Hence, the impact of revenue sharing on equilibrium competitive balance is determined by the properties of the function $G(t)$ defined in (14) and, thus, by the shape of the CSF. As a benchmark, let us first consider the Tullock (1980) ratio-form CSF used by Szymanski and Késenne (2004). In our framework, this CSF is represented by the linear function $H(t) = t$ which implies $H'(t) = 1$, $H''(t) = 0$ and $G(t) = -t$. It follows $G(t_1^*) - G(t_2^*) = t_2^* - t_1^* < 0$ and $\mathrm{d}b^*/\mathrm{d}\alpha < 0$. Consistent with Szymanski and Késenne (2004), we can therefore conclude that for a linear CSF the introduction or intensification of revenue sharing worsens competitive balance. However, our analysis is not restricted to the linear CSF. A more general functional form of H is[7]

$$H(t) = k_1(k_2 t + k_3)^{1/(1-k_4)}, \qquad k_1, k_2 > 0,\ k_3 \in \mathbb{R},\ k_4 < 1. \tag{15}$$

The linear function is obtained as special case if $k_1 = k_2 = 1$ and $k_3 = k_4 = 0$. The function (15) also reflects the often used power function $H(t) = t^r$ if we set $k_1 = k_2 = 1$, $k_3 = 0$ and $k_4 = (r-1)/r$. Moreover, Runkel (2006) considers a gallery of further functional forms of H, for example,

$$H(t) = k \ln(1+t), \qquad k > 0, \tag{16}$$

$$H(t) = \exp\left\{ \int_0^t \exp\left\{-x^k\right\} dx \right\}, \qquad k > 1, \tag{17}$$

$$H(t) = \exp\left\{ \frac{\exp\{k_1(t+k_3)\} + k_2}{k_1} \right\}, \qquad k_1 < 0, k_2, k_3 \in \mathbb{R}. \tag{18}$$

For all these CSFs we obtain

Proposition 1. *Suppose talent supply is perfectly elastic. Under all CSFs* (15) – (18) *introducing or intensifying revenue sharing worsens competitive balance, i.e.* $\mathrm{d}b_1^*/d\alpha < 0$

Proof: The CSF (15) yields $H'(t) = k_1 k_2 (k_2 t + k_3)^{k_4/(1-k_4)}/(1-k_4)$, $H''(t) = k_1 k_2^2 k_4 (k_2 t + k_3)^{(2k_4-1)/(1-k_4)}/(1-k_4)^2$, $G(t) = -(1-k_4)^2(k_2 t + k_3)/k_2$ and $G'(t) = -(1-k_4)^2 < 0$. With the CSF defined by (16) we obtain $H'(t) = k/(1+t)$, $H''(t) = -k/(1+t)^2$, $G(t) = -(1+t)\ln(1+t)[1+\ln(1+t)]$ and $G'(t) = -[1+\ln(1+t)]^2 - \ln(1+t) < 0$. The CSF (17) implies $H'(t) = H(t)\exp\left\{-t^k\right\}$, $H''(t) = H(t)\left[\exp\left\{-2t^k\right\} - kt^{k-1}\exp\left\{-t^k\right\}\right]$, $G(t) = -kt^{k-1}\exp\left\{2t^k\right\}$ and $G'(t) = kt^{k-2}\exp\left\{2t^k\right\}\left[1-k-2kt^k\right] < 0$. Under the functional form (18), we obtain $H'(t) = H(t)\exp\{k_1(t+k_3)\}$, $H''(t) = H(t)\exp\{k_1(t+k_3)\}$ $[k_1 + \exp\{k_1(t+k_3)\}]$, $G(t) = k_1\exp\{-2k_1(t+k_3)\}$ and $G'(t) = -2k_1^2\exp\{-2k_1(t+k_3)\}$ < 0. Hence, all CSFs (15) - (18) imply $G'(t) < 0$, $G(t_1^*) < G(t_2^*)$ and $\mathrm{d}b^*/\mathrm{d}\alpha < 0$. ■

Proposition 1 shows that the result of Szymanski and Késenne (2004) can be generalized to further functional forms of the CSF. Revenue sharing is counterproductive in the sense

[7] All specifications of the CSF considered below satisfy condition (6) which is necessary for the existence of an equilibrium. Whether an equilibrium really exists, however, depends inter alia on the parameters of the CSFs (the ks). Unfortunately, it is not possible to impose general restrictions on the set of parameter constellations that ensure existence. But for each CSF it is possible to identify examples where an equilibrium exists. Details on these examples can be obtained upon request. Put differently, to ensure existence of an equilibrium in the sports league the set of parameter constellations given in each specification of the CSF has to be further restricted, but the restricted set is never empty.

of a declining competitive balance in the league not only for the Tullock CSF, but also for the CSFs in (15) – (18). The intuition can be traced back to the general insight derived by equation (8). Under all functional forms (15) – (18) of the CSF, the elasticity of equilibrium performance with respect to the sharing parameter is smaller in absolute terms for the large team 1 than for the small team 2. Hence, introducing or intensifying revenue sharing through reducing α by one percent lowers equilibrium performance of team 2 by more than that of team 1 and so renders the league more unbalanced.

Proposition 1 only presents a selection of CSFs for which the result of Szymanski and Késenne (2004) remains true. There are other functional forms that imply a negative impact of revenue sharing on competitive balance. However, we cannot prove this to be a general result. There may be specifications of the CSF for which the expression in (13) is non-negative. Given the discussion in the previous literature, it is of special interest whether there are CSFs that restore the invariance principle. The following proposition gives a full characterization of the class of CSFs ensuring such a result.

Proposition 2. *Suppose talent supply is perfectly elastic. Introducing or intensifying revenue sharing leaves unaltered competitive balance* ($\mathrm{d}b_1^*/\mathrm{d}\alpha = 0$) *if and only if*

$$H(t) = k_2 \exp\left\{\sqrt{2k_1 t + 2k_3}/k_1\right\}, \qquad k_1, k_2 > 0, k_3 \geq 0. \tag{19}$$

Proof: From (13) we see that $\mathrm{d}b^*/\mathrm{d}\alpha = 0$ if and only if $G(t_1^*) = G(t_2^*)$ or, equivalently, $G(t) = -k_1$ where k_1 is a given constant. Note that $k_1 > 0$ due to (6) and (14). The condition $G(t) = -k_1$ can be written as

$$H''(t)[H(t)]^2 - H(t)[H'(t)]^2 + k_1[H'(t)]^3 = 0. \tag{20}$$

This is a non-linear second order differential equation that has two solutions. The first is

$$H(t) = k_2 \exp\left\{-\sqrt{2k_1 t + 2k_3}/k_1\right\}. \tag{21}$$

$H(t) > 0$ requires $k_2 > 0$. But we then obtain $H'(t) = -H(x)/\sqrt{2k_1 t + 2k_3} < 0$. Hence, this cannot be a meaningful CSF as it violates our assumption $H'(t) > 0$. The second solution is (19). The condition $k_2 > 0$ ensures $H(t) > 0$ and $H'(t) > 0$. Moreover, $k_3 \geq 0$ together with our above assumption $k_1 > 0$ implies $2k_1 t + 2k_3 \geq 0$ for all $t \geq 0$ so that H is defined on the set of real numbers. It follows that the specification in equation (19) fully characterizes the class of CSFs implying $\mathrm{d}b_1^*/\mathrm{d}\alpha = 0$. ■

Proposition 2 shows that using another CSF than Szymanski and Késenne (2004) may restore the invariance principle derived by the early sports economics literature. If the CSF is given by (19), the elasticity of performance with respect to the sharing parameter α is the same for both teams. Hence, reducing α by one percent reduces equilibrium performance of both teams by the same percentage and so leaves unaltered the relation of the teams' probabilities of success and competitive balance within the league. To illustrate the meaning of this result, consider the special case of $k_1 = 2$, $k_2 = 1$ and $k_3 = 0$. The CSF in equation (19) then simplifies to $H(t) = \exp\left\{\sqrt{t}\right\}$. This CSF is not the same, but very similar to the Hirshleifer (1989, 1991) difference-form CSF $H(t) = \exp\{kt\}$ with $k > 0$. Beside the Tullock ratio-form CSF and its generalization $H(t) = t^r$, the difference-form specification is the most used CSF in the contest literature (see e.g. Baik, 1998).[8]

It is important to note that Proposition 2 makes the case for the invariance principle without changing the behavioral assumption made by Szymanski and Késenne (2004). The result is obtained from the first and second order conditions (4) and (5) which, in turn, are derived under the assumption of Nash behavior, i.e. each team is supposed to maximize profit with respect to its talent investment taking as given talent investment of the other team. Hence, the invariance principle derived in Proposition 2 does not refer to the Walrasian fixed supply assumption where each team takes into account that an increase in its own talent demand reduces the amount of talent that can be employed in the other team. The result in Proposition 2 is solely driven by changing the functional form of the CSF from a linear specification to some particular exponential specification. The assumption of Nash behavior in our model also explains why we derive the invariance principle in the case of a perfectly elastic talent supply, whereas the previous literature needs a perfectly inelastic talent supply to obtain the invariance principle.

The possibility of the invariance principle represented by Proposition 2 immediately raises the question whether we can also identify CSFs under which revenue sharing even improves competitive balance in the league. That this indeed may be a possible outcome is best illustrated by considering the specification $H(t) = \exp\left\{t^k\right\}$ with $k \in]0, 1[$. We then obtain $H'(t) = kt^{k-1}\exp\left\{t^k\right\}$, $H''(t) = kt^{k-2}(k-1+kt^k)\exp\left\{t^k\right\}$, $G(t) = (k-1)t^{1-2k}/k^2$ and $G'(t) = (k-1)(1-2k)t^{-2k}/k^2$. This immediately implies

[8] Note that the difference-form CSF of Hirshleifer (1989, 1991) implies $H'(t) = kH(t)$, $H''(t) = k^2H(t)$ and, thus, $H(t)H''(t) - [H'(t)]^2 = 0$. Hence, the necessary condition (6) for the existence of an equilibrium is violated and $H(t) = \exp\{kt\}$ is not a suitable CSF in our framework.

Proposition 3. *Suppose talent supply is perfectly elastic and the CSF is* $H(t) = \exp\left\{t^k\right\}$ *with* $k \in]0,1[$. *Introducing or intensifying revenue sharing then worsens (leaves unchanged, improves) competitive balance if and only if* $k \in]0, 0.5[$ ($k = 0.5$, $k \in]0.5, 1[$).

The first part of Proposition 3 gives a further CSF for which the result of Szymanski and Késenne (2004) is valid. For the exponential function $H(t) = \exp\left\{t^k\right\}$ with $k \in]0, 0.5[$ revenue sharing renders the league more unbalanced. The second part of Proposition 3 confirms Proposition 2 as the function $H(t) = \exp\left\{t^k\right\}$ with $k = 0.5$ is a special case of (19). Most interestingly, however, is the third part of Proposition 3. If $H(t) = \exp\left\{t^k\right\}$ with $k \in]0.5, 1[$, then the elasticity of equilibrium performance with respect to the sharing parameter α is higher in absolute terms for the large team 1 than for the small team 2. Hence, revenue sharing reduces performance of team 1 by more than performance of team 2 so that the competition in the league becomes closer. The importance of this result becomes obvious when we take into account that for increasing k we come closer and closer to the Hirshleifer difference-form CSF which would be obtained for $k = 1$.

To sum up, this section has shown that the result of Szymanski and Késenne (2004) can be extended to particular classes of CSFs. But we also derived the class of CSF which ensures the invariance result (under Nash behavior). Moreover, the CSF may be such that revenue sharing really improves competitive balance, as actually intended. In general, this ambiguity of the impact of revenue sharing on competitive balance under a perfectly elastic talent supply can be seen with the help of equations (13) and (14). From (13) follows that $\mathrm{d}b^*/\mathrm{d}\alpha \gtreqless 0$ if and only if $G(t_1^*) \gtreqless G(t_2^*)$. According to (14) and $t_1^* > t_2^*$, this is the case if and only if $G'(t) \gtreqless 0$. The above Propositions 1 – 3 show that each sign in the latter inequality is satisfied by a non-empty set of CSFs.

4 Inelastic Talent Supply

Let us now turn to the case of a perfectly inelastic talent supply. It is supposed that the talent supply is fixed at a value $\bar{t} > 0$. The talent market equilibrium condition is

$$t_1 + t_2 = \bar{t}. \tag{22}$$

This condition determines the equilibrium unit cost of talent c which is now no longer fixed. More specifically, the first order conditions in (4) yield the teams' talent demands as functions of the unit cost of talent. Inserting this talent demand into (22) determines the equilibrium c. Hence, in order to figure out the effect of revenue sharing on competitive balance under a perfectly inelastic talent supply we now have to totally differentiate not only the first order conditions in (4), but also the equilibrium condition (22). It is important to note that this procedure is not equivalent to the Walrasian fixed supply assumption. The Walrasian assumption states that each team takes into account the effect of its talent investment on the talent investment of the other team via the fixed supply restriction. In contrast, the analysis in this section sticks to the assumption of Nash behavior. Formally, our analysis is based on the first order conditions (4) that are derived under the assumption that each team takes as given talent demand of the other team. The difference to the analysis of an elastic supply in the previous section is that we (but not the teams!) take into account the talent market equilibrium condition (22).

Totally differentiating equations (4) and (22) and rearranging yields

$$\begin{pmatrix} z_1 H_2 \dfrac{(H_1+H_2)H_1''-2(H_1')^2}{H_1+H_2} & z_1 H_1' H_2' \dfrac{H_1-H_2}{H_1+H_2} & -(H_1+H_2)^2 \\ z_2 H_1' H_2' \dfrac{H_2-H_1}{H_1+H_2} & z_2 H_1 \dfrac{(H_1+H_2)H_2''-2(H_2')^2}{H_1+H_2} & -(H_1+H_2)^2 \\ 1 & 1 & 0 \end{pmatrix} \times$$

$$\times \begin{pmatrix} \mathrm{d}t_1 \\ \mathrm{d}t_2 \\ \mathrm{d}c \end{pmatrix} = \begin{pmatrix} -(v_1+v_2)H_2H_1' \\ -(v_1+v_2)H_1H_2' \\ 0 \end{pmatrix} \mathrm{d}\alpha. \quad (23)$$

The determinant of the matrix on the LHS now reads

$$\widetilde{\Delta} = (H_1+H_2)\left\{ \sum_{i\neq j} z_i H_j[(H_i+H_j)H_i''-2(H_i')^2] - H_1'H_2'(H_1-H_2)(z_1-z_2) \right\} < 0. \quad (24)$$

The sign of $\widetilde{\Delta}$ follows from $z_1 > z_2$, $H_1 > H_2$ and the second order condition (5). Applying Cramer's rule and rearranging yields

$$\frac{\mathrm{d}t_1}{\mathrm{d}\alpha} = -\frac{\mathrm{d}t_2}{\mathrm{d}\alpha} = -\frac{(v_1+v_2)(H_1+H_2)^2(H_2H_1'-H_1H_2')}{\widetilde{\Delta}}. \quad (25)$$

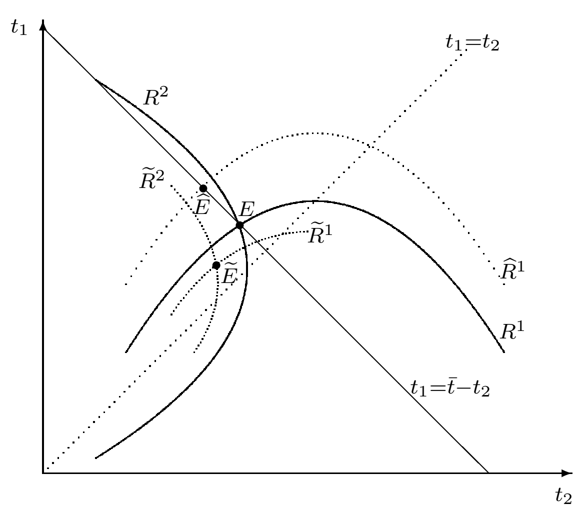
Taking into account $\widetilde{\Delta} < 0$ from (24), $H_2'/H_2 > H_1'/H_1$ from (4) and $\mathrm{d}H(t_i^*)/\mathrm{d}\alpha = H_i' \cdot (\mathrm{d}t_i^*/\mathrm{d}\alpha)$ for $i \in \{1,2\}$ immediately proves

Lemma 2. *Suppose talent supply is perfectly inelastic. Introducing or intensifying revenue sharing then increases equilibrium investment and performance of team 1, but decreases equilibrium investment and performance of team 2, that is* $\mathrm{d}t_1^*/d\alpha = -\mathrm{d}t_2^*/d\alpha < 0$, $\mathrm{d}H(t_1^*)/\mathrm{d}\alpha < 0$ *and* $\mathrm{d}H(t_2^*)/\mathrm{d}\alpha > 0$.

Lemma 2 shows that with an inelastic talent supply the effects of revenue sharing on the teams' investment in talent may be diametrically different from the effects in case of an elastic talent supply determined by Lemma 1. With an inelastic supply team 1 increases its talent investment, while with an elastic supply revenue sharing induces team 1 to reduce investment. For team 2 this argument may be reversed. With an inelastic talent supply the effect of revenue sharing on team 2's investment is unambiguously negative, whereas an elastic talent supply may induce team 2 to increase its talent investment when revenue sharing is introduced or intensified.

The reason for this change in results may best be illustrated with the help of Figure 2. This figure is similar to Figure 1, but it additionally displays the talent market equilibrium

Figure 2: Revenue Sharing with Inelastic Talent Supply



condition $t_1 = \bar{t} - t_2$ as a falling line with slope equal to -1. The initial equilibrium E must be located on this new line as the talent market is always in equilibrium. For a given unit cost of talent c, revenue sharing shifts the reaction function of team 1 downwards from R^1 to $\widetilde{R}^1$ and that of team 2 to the left from R^2 to $\widetilde{R}^2$. These movements cause the revenue and competition effects already described in Figure 1. The two teams are then in a situation like $\widetilde{E}$. However, this point lies below the talent market equilibrium line and thereby implies that talent demand falls short of talent supply. As a consequence, we obtain an additional *talent cost effect*. Due to the excess talent supply, the unit cost of talent goes down and both reaction functions are shifted back. For the stronger team (team 1) this upward shift more than compensates the initial downward shift so that the new reaction function is $\widehat{R}^1$ which lies above the initial reaction function R^1. The league therefore attains a new equilibrium in a point like $\widehat{E}$ where team 1 invests more and team 2 less than in the initial equilibrium.

The driving force behind this result are the asymmetric consequences of the talent cost effect for the two teams, i.e. the reduction in unit cost gives the large team a stronger incentive to increase talent investment than the small team. This observation is consistent with the findings in Runkel (2006) in a model without revenue sharing. There a uniform increase in unit effort cost (caused by an increase in a cost parameter which is the same for both players and under the control of the contest designer) reduces the effort level of the strong player by more than that of the weak player. Our model reflects the reversed effect. A uniform reduction in unit talent cost (caused by revenue sharing and the resulting oversupply of talent) raises talent investment of the strong team by more than talent investment of the weak team. The intuition is the same in both models. The higher (lower) the unit cost of effort (talent), the less (more) able is the strong player (team) to exploit its comparative advantage resulting from the higher revenue potential.

The implications for competitive balance between the two teams in the league are straightforward. Since t_1^* and $H(t_1^*)$ increase while t_2^* and $H(t_2^*)$ decrease, b^* defined in equation (7) must go up. This insight immediately leads to

Proposition 4. *Suppose talent supply is perfectly inelastic. Introducing or intensifying revenue sharing then worsens competitive balance, regardless of the shape of the CSF.*

According to Lemma 2, revenue sharing affects the talent investment of the two teams asymmetrically if the talent supply is perfectly inelastic. The stronger team increases talent investment, while the weaker team invests less. The consequence is a fall in competitive balance as shown by Proposition 4. It is important to note that this result is true independent of the shape of the CSF. This is in contrast to the case of a perfectly elastic talent supply. The reason is as follows. In both cases, lowering α triggers revenue and competition effects that reduce talent investment of the strong team while the effect on the weak team's investment is ambiguous. In case of an inelastic talent supply, we additional have a talent cost effect. This renders the total effect on the stronger team's investment positive and the effect on the weaker team's investment negative so that competitive balance deteriorates. But the adjustment through the unit talent cost is not present in case of an elastic talent supply so that we remain in a situation where the effect of revenue sharing on competitive balance is ambiguous and depends on the shape of the CSF.

To sum up, Proposition 4 shows that the result which Szymanski and Késenne (2004) derive under the assumption of a fixed talent supply and a linear CSF can be extended to a general CSF. But note the difference to the result obtained from the Walrasian fixed supply model. This model also assumes a fixed talent supply but obtains the invariance result. The reason for the difference to our result is that the Walrasian model does not use the assumption of Nash behavior. Hence, in contrast to our Lemma 2, the fall in unit cost of talent (caused by the oversupply of talent) increases talent investment of both teams to the same extent so that competitive balance remains unchanged. In our model, as in Szymanski and Késenne (2004), the fall in unit cost raises the stronger team's investment by more than that of the weaker team and so worsens competitive balance.

5 Conclusion

This paper reexamines the impact of revenue sharing on competitive balance in sports leagues. We use a two team asymmetric contest model with Nash-behavior of team owners. In contrast to the previous sports economics literature, our analysis is not restricted to the Tullock ratio-form CSF, but also considers other specifications of this function. If talent supply is fixed, revenue sharing renders the league more unbalanced regardless of the functional form of the CSF. In case of an inelastic talent supply, however, the unit cost of talent is fixed and revenue sharing may have any effect on competitive balance. We fully characterize the class of CSFs under which the effect vanishes and also identify CSFs under which revenue sharing has the intended effect of reducing the gap in winning probabilities between strong and weak teams.

The ambiguous results in case of an elastic supply immediately raise the question how the CSF looks like in real world sports leagues. The contest and sports economics literature frequently uses the Tullock ratio-form CSF in its general power form $H(t) = t^r$. One reason may be that this specification renders the analysis tractable and usually allows for reduced form solutions of the model. However, this does not imply that the power form CSF is the most relevant functional form in real world contests. Analytical tractability does not say anything about empirical relevance. Another often used argument in favor of the power form CSF is its axiomatic foundation provided by Skarperdas (1996).[9] Axiomatic foundations are helpful in identifying the implicit assumptions underlying specific functional forms of the CSF. But they, too, cannot make statements about empirical relevance. Moreover, Skarperdas (1996) not only provides an axiomatic foundation of the power CSF, but also of the Hirshleifer (1989, 1991) difference-form CSF.[10] Skarperdas (1996) shows that the power CSF needs an homogeneity axiom stating that multiplying the talent levels of all teams with the same factor does not change the teams' winning probabilities. The difference-form CSF, in contrast, is based on an axiom which requires that adding the same quantity to the talent levels of all teams does not alter the teams' winning probabilities.

Whether this latter property or the homogeneity axiom is more relevant is in the end an empirical question that cannot be answered by a theoretical analysis. Hirshleifer (1989, 1991) refers to 'stylized facts' from military warfare in order to defend his specification of the CSF. Even though this points to the relevance of CSFs used in our above analysis, it has to be admitted that this, too, is not a good indicator for the 'empirically right' CSF since Hirshleifer (1989, 1991) does not present a sound econometric analysis and since military conflicts may be different from other types of contests, e.g., sports contests. In sum, an empirical analysis of CSFs for different kinds of contests is lacking in the literature. Such an analysis is both an important and promising task of further research.

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[9]Clark and Riis (1998) extend the axiomatic foundation to so-called unfair contests. Axiomatic foundations for contests where each player uses more than one instrument are given by Rai and Sarin (2008) and Arbatskaya and Mialon (2008). Münster (2008) provides an axiomatic foundation of CSFs in group contests. See also the related studies of Amegashie (2006) and Malueg and Yates (2006).

[10]For group contests, an axiomatic foundation of the Hirshleifer CSF can be found in Münster (2008).

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