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Labor Supply and Growth Effects of Environmental Policy under Technological Risk

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Abstract

This paper analyzes the effects of technological risk on long–run growth when labor supply is elastic and production gives rise to a pollution externality. For the social planner as well as for the market economy we show that the randomness of production as well as the endogeneity of labor supply matter with respect to the equilibrium solution. The direction in which changes in the model parameters as well as changes of policy instruments influence labor supply and growth depends crucially on the volatility of output.

Keywords: stochastic growth, pollution, abatement, elastic labor supply

JEL classification: Q5, O4, D8, D9

1 Introduction

As the debate on climate change shows very clearly, the consequences of environmental degradation for economic development is subject to a large degree of uncertainty. This uncertainty arises from a variety of different sources such as ecological and technological risks, but also the stochastic dynamics of population. Yet, although uncertainty features prominently in the current debate on sustainability, its potential implications have rarely been analyzed in the literature on growth and the environment. In this paper we show that incorporating uncertainty might not only affect the optimal static and dynamic characteristics of optimal policy design, but might also alter the growth implications of, for example, environmental taxation.

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To keep the analysis tractable we concentrate on one potential source of uncertainty, technological risk, and show that in the presence of this risk the effects of environmental policy might even be reversed compared to the a deterministic setting. In contrast to the main body of literature in this field, we assume labor supply to be endogenously determined. We show that neglecting the reaction of the labor-leisure choice to changes in environmental policy might result in a wrong assessment of policy implications.[1]

In recent years an large body of literature has dealt with the mutual interdependency of economic growth and environmental degradation. Especially the development of endogenous growth theory has renewed the interest in this field, leading to an extensive analysis of the general conditions under which long–run growth can be feasible and optimal in the presence of environmental restrictions. These restrictions originate from a large number of sources such as rival and non–rival productivity effects as well as environmental amenities of renewable resources (e.g. Bovenberg and Smulders, 1995, 1996; Smulders, 1998; Grimaud, 1999; Elíasson and Turnovsky, 2004), exhaustible resources (e.g. Aghion and Howitt, 1998; Scholz and Ziemes, 1999; Schou, 2000) and stock or flow pollution (e.g. Gradus and Smulders, 1993; Smulders, 1996; Stokey, 1998).[2] The vast majority of this literature, however, does not consider the effects of uncertainty, but rather assumes that technological as well as ecological components are deterministic.

There are some exceptions to this rule however. Baranzini and Bourguignon (1995), for instance, consider a non–zero probability of extinction while Beltratti *et al.* (1998) and, more recently, Ayong Le Kama and Schubert (2004) include uncertainty about future preferences. Technically closest to our analysis is probably Soretz (2003, 2004, 2007) who discusses perception and policy issues of environmental pollution in an *AK*–type framework, but disregards trade–off effects between consumption and leisure, as well as how individual household's savings decision relates to a differentiated factor income risk.

In this paper we combine the traditional environmental economics literature on growth and the environment with the strand of literature dealing with labor supply in a stochastic setting. The analysis is motivated by the well–known result from the literature that the riskiness of capital returns and labor income is an important determinant of the intertemporal savings decision of risk averse agents. In his pioneering work Leland (1968) stressed the role of *precautionary savings*, which he defined as savings, a risk averse household additionally undertakes in order to self–insure against the riskiness of future income flows. Especially in the context of mod-

[1]For a general overview of the role of uncertainty in economics see Pindyck (2006).

[2]An extensive review of the related literature can be found in Pittel (2002).

ern growth theory, this draws a link between intertemporal choice, risk, and growth. Sandmo (1970) was the first to point out the importance of factor–specific risk related to the degree of risk aversion for the emergence of precautionary saving.

The majority of modern contributions dealing with continuous–time stochastic growth, where the economy follows a stochastic trend, suffers from the important shortcoming that they confine their analysis to a single income type (mostly *capital risk*), in order to maintain analytical tractability (cf Obstfeld, 1994; Smith, 1996; Soretz, 2003, 2004, 2007). Others either view the intertemporal flow of labor income as human wealth and treat it as a 'quasi accumulating' hedgeable asset (Corsetti, 1997) or assume labor incomes to be instantaneously deterministic Turnovsky (2000, 2003). Notably exceptions for the case of inelastic labor supply are Clemens and Soretz (2004) and Clemens (2004*b*, 2005). Only recently Clemens (2004*a*) and Turnovsky and Smith (2006) succeeded in deriving closed–form solutions for the equilibrium growth path of an economy with endogenous labor–leisure choice, where households simultaneously are subject to capital and income risk.

Besides the phenomenon of precautionary savings, the presence of risk substantially alters the policy implications derived within deterministic environments. This is due to the fact that taxes (or transfers respectively) also affect the riskiness of the policy target under consideration. The higher–order effects from the variance of the underlying risk might even reverse the direction of impact of policy instruments in a stochastic environment. The insurance effect of taxation was first discussed by Domar and Musgrave (1944) and Stiglitz (1969), or in a continuous–time growth context by Turnovsky (1993), Smith (1996) or Clemens and Soretz (1997).

Our model is a stochastic version of the Romer (1986) endogenous growth model with endogenous labor supply and a negative pollution externality. Pollution is generated from production activities and can be reduced by devoting part of output to abatement. Production and abatement are subject to a random disturbance that stems from an aggregate productivity shock. The economy follows a stochastic trends with the assumed uncertainty leading to second–order effects on expected labor supply and growth. The relatively simple model structure with constant private returns to scale and linearity in capital allows us to derive closed–form solutions.

The paper proceeds as follows: Section 2 introduces the model for which the socially optimal growth path is derived as a benchmark solution in Section 3. Section 4 focuses on the market economy. It derives an optimal policy mix and regards the general implications of regulatory activities on growth and labor supply. Section 5 then compares the results of Section 3 and 4 to the case of exogenous labor supply. Section 6 concludes.

2 The Model

We assume a closed economy in which a homogeneous good is produced from labor and capital. Individual production is stochastic, i.e. at each increment of time, the economy is subject to an aggregate productivity shock. The production and investment processes generate two types of externalities: First, we assume that production is subject to learning by doing. Production of a single producer is positively affected by aggregate production experience, and investment activities in privately–owned capital create a positive externality by raising the productivity of all firms. For simplicity it is assumed that this positive spillover effect is represented one–to–one by the aggregate level of capital input. This is the standard type of Romer (1986) model. A second externality arises from environmental pollution $\bar{P}(t)$. Production leads to a flow of pollution generating a negative effect on production which can be mitigated by abatement activities. The production technology is assumed to be of the stochastic Cobb–Douglas type

$$\mathrm{d}Y(t) = K(t)^{\alpha}\bar{K}^{1-\alpha}(1-l(t))^{1-\alpha}\bar{P}(t)^{-\eta}\left(\mathrm{d}t + \mathrm{d}z(t)\right) \ . \tag{1}$$

$\mathrm{d}z(t)$ is the serially uncorrelated increment to a standard Wiener process $z(t)$ with zero mean and an instantaneous variance of $\sigma^2\,\mathrm{d}t$. Due to the productivity shock, the returns to the two factors of production are stochastic. In terms of Sandmo (1970), the household is subject to a capital risk and an income risk.

To generate the instantaneous output flow $\mathrm{d}Y(t)$, producers employ physical capital, $K(t)$, and labor, $1-l(t)$, as a fraction of time endowment. The production displays constant returns to scale in $K(t)$ and $1-l(t)$ on the individual firm level. Aggregate capital accumulation, $\bar{K}(t)$, exerts a positive effect on productivity. In macroeconomic equilibrium $K(t)$ equals $\bar{K}(t)$, as we normalize the population to unity. Production is linear in capital on the aggregate level which ensures that the conditions for ongoing growth of per capita incomes are met. This, together with the assumption that the productivity shock is proportional to the mean rate of output, implies that the randomness of production does not disappear asymptotically as the output grows. The economy evolves according to a stochastic trend.

The negative pollution externality is represented by $P(t)$ with the effective flow of aggregate pollution being given by the ratio of mean output over aggregate abatement $\bar{A}$:

$$\bar{P}(t) = \frac{K^{\alpha}\bar{K}(t)^{1-\alpha}(1-l(t))^{1-\alpha}}{A(t)}. \tag{2}$$

Following Pittel (2002) we assume an elasticity of substitution of unity between abatement and raw pollution as a prerequisite for balanced growth to be consistent with non–increasing effective pollution in the long–run. Pollution is assumed to be a flow variable[3] that can be reduced by devoting a share of the output to abatement activities. As we assume perfect competition with a large number of producers, the effect of individual production on aggregate pollution is negligible such that, on the individual level, producers take pollution as exogenous to their production decision. Consequently, producers would—in the absence of environmental regulation—not conduct abatement as their perceived marginal return would be zero and effective pollution would asymptotically grow to infinity. Along the equilibrium growth path, aggregate pollution should be constant and equal to $\bar{P} = 1/a$, with a denoting the abatement ratio.

The economy is populated by a continuum $[0,1]$ of identical infinitely–lived individuals who maximize their intertemporal utility out of consumption and leisure

$$\mathrm{E}_0 \int_0^\infty \left[\ln C(t) + \frac{l(t)^{1-\delta}}{1-\delta} \right] e^{-\beta t}\, \mathrm{d}t, \qquad \text{if} \qquad \delta > 0, \delta \neq 0 \tag{3}$$

and $\ln c(t) + \ln l(t)$, if $\delta = 1$. $l(t)$ denotes leisure time with δ measuring the household's disliking of labor. $C(t)$ is individual consumption and β the rate at which agents discount future utility.

This intertemporal utility function comprises a number of important characteristics: First, (3) is log–linear in consumption which simultaneously implies that households are risk averse, with the Arrow/Pratt measure of relative risk aversion R_R being equal to unity. From the literature on precautionary savings under uncertainty (cf. Levhari and Srinivasan, 1969; Sandmo, 1970) it is well–known that—in this case—the intertemporal income and substitution effects from capital risk completely offset. In a model without a preference for leisure, the randomness of production would then generate certainty–equivalence results regarding the allocation of personal income on consumption and saving. The equilibrium expected growth rate of this economy would be identical to the growth rate in a deterministic economy, although the household suffers a welfare loss due to the presence of uncertainty. Since our model also takes account of risky labor incomes, the chosen specification allows us to focus entirely on the growth and policy effects of labor income risk. As will become obvious below, the riskiness of wage incomes affects the labor–leisure choice and influences optimal pollution as well as optimal pollution taxation.[4]

Second, by assuming the preferences of agents to be additively separable, the cross derivatives vanish and the effects of leisure on the marginal utility of consumption

[3]In the context of deterministic growth models Smulders (1996) have shown that the qualitative implications of a flow or stock formulation of pollution are equivalent as long as the focus is on balanced growth.

[4]The effect of the capital risk on pollution and the optimal policy mix were analyzed, for example, by Soretz (2003, 2004, 2007).

and *vice versa* are eliminated. Finally, (3) is consistent with a balanced growth path along which the time share devoted to leisure as well as the return to capital is constant while consumption grows at a constant rate King and Rebelo (cf 1999).

The aggregate capital stock follows the Itô diffusion process

$$\mathrm{d}K(t) = \mathrm{d}Y(t)\,(\mathrm{d}t + \mathrm{d}z(t)) - C(t)\,\mathrm{d}t - \bar{A}(t)\,[\mathrm{d}t + \mathrm{d}z_A(t)] \ , \tag{4}$$

where $\bar{A}(t)$ denotes aggregate abatement expenditure, which also follow a stochastic process $dz_A(t)$ to be endogenously determined in equilibrium.

We now proceed with the derivation of the Pareto–optimal growth path of the economy which serves as a benchmark solution. The subsequent sections then are devoted to the analysis of an economic and environmental policy aiming at mimicking the Pareto–optimal path. It will be shown that the Pareto–efficient allocation can be implemented by means of a subsidy on physical capital and a pollution tax giving rise to incentives to engage in abatement, combined with lump–sum payments. Two instruments will be sufficient to induce the efficient time path in a knife–edge scenario.

3 Social Planner

The benevolent social planner internalizes the two externalities present in the economy and also takes account of the fact that the diffusion process of abatement is governed by the exogenous productivity shock, such that $dz_A = dz$. In contrast to individual producers who only take account of the private returns to capital, the social planner considers the social return and chooses the intertemporal consumption path, working time, and abatement efforts such that the spillover effects are internalized, and capital is payed its social return. In contrast to the standard Romer (1986) model, where the private capital return unambiguously falls short of the social returns to investment, this is not necessarily the case in our setting, the results depending on whether or not the positive learning spillovers are outweighed by the negative pollution effects.

The maximization problem of the social planner reads[5]

$$\max_{C,l} \quad \mathrm{E}_0 \int_0^\infty \left[\ln C(t) + \frac{l^{1-\delta}}{1-\delta} \right] e^{-\beta t}\,\mathrm{d}t, \tag{5}$$

$$s.t. \quad \mathrm{d}K = \mathrm{d}Y(\mathrm{d}t + \mathrm{d}z) - C\,\mathrm{d}t - A\,(\mathrm{d}t + \mathrm{d}z], \qquad K(0) > 0, z(0) = 0 \tag{6}$$

[5] In what follows, we drop the time index of variables for expository convenience.

The stochastic Hamiltonian can be set up as follows (cf. Malliaris and Brock, 1982, ch. 2.10):

$$\mathscr{H}\left(C,K,A,l,\lambda,\frac{\partial\lambda}{\partial K}\right)=e^{-\beta t}\left[\ln C+\frac{l^{1-\delta}}{1-\delta}\right]+\lambda\left[K^{1-\eta}(1-l)^{1-\alpha}A^{\eta}-C-A\right)\right]+\frac{\sigma_K^2}{2}\frac{\partial\lambda}{\partial K}$$

with $\sigma_K^2=\left(K^{1-\eta}(1-l)^{(1-\alpha)(1-\eta)}-A\right)^2\sigma^2$. Maximization yields the following FOCs, where pollution is already substituted with the abatement ratio $a=1/\overline{P}$:

$$\frac{\partial\mathscr{H}}{\partial C}=e^{-\beta t}C^{-1}-\lambda=0 \tag{7}$$

$$\frac{\partial\mathscr{H}}{\partial l}=e^{-\beta t}l^{-\delta}-(1-\eta)(1-\alpha)K(1-l)^{-\alpha}a^{\eta}\left(\lambda+\frac{\partial\lambda}{\partial K}K(1-l)^{1-\alpha}\sigma^2\left(a^{\eta}-a\right)\right)=0 \tag{8}$$

$$\frac{\partial\mathscr{H}}{\partial A}=\left(\eta a^{\eta-1}-1\right)\left(\lambda+\frac{\partial\lambda}{\partial K}K(1-l)^{1-\alpha}\sigma^2\left(a^{\eta}-a\right)\right)=0 \tag{9}$$

$$\begin{aligned}\mathrm{d}\lambda&=-\frac{\partial\mathscr{H}}{\partial K}\mathrm{d}t+\frac{\partial\lambda}{\partial K}\sigma_K\,\mathrm{d}z\\&=-(1-\eta)(1-l)^{1-\alpha}a^{\eta}\left(\lambda+\frac{\partial\lambda}{\partial K}\sigma^2K(1-l)^{1-\alpha}\left(a^{\eta}-a\right)\right)\mathrm{d}t\\&\qquad+\frac{\partial\lambda}{\partial K}K(1-l)^{1-\alpha}\left(a^{\eta}-a\right)\mathrm{d}z\end{aligned} \tag{10}$$

together with the transversality condition

$$\lim_{t\to\infty}\mathrm{E}_t[\lambda(t)K(t)]=0.$$

Conditions (7) and (8) relate the marginal utility of consumption and leisure respectively to the shadow price λ, but (8) also accounts for the random nature of labor productivity. From condition (9) follows the optimal level of abatement activities. Equation (10) is a modified version of the optimality condition usually derived for the state variable K. It describes the stochastic evolution of the shadow price over time, which also follows a diffusion process.

The solution procedure for the stochastic system (7) to (10) is similar to the one well–known for deterministic models. We proceed with differentiating (7) with respect to time to obtain a second expression for the law of motion of the shadow price λ, which later on can be equated to (10). Application of Itô's lemma yields the following expression for $\mathrm{d}\lambda$:

$$\mathrm{d}\lambda=e^{-\beta t}C^{-1}\left(-\beta\,\mathrm{d}t-\frac{\mathrm{d}C}{C}+\frac{(\mathrm{d}C)^2}{C^2}\right). \tag{11}$$

With aggregate output being subject to a technological disturbance, consumption and saving become stochastic too. The associated diffusion process for consumption, $\mathrm{d}C$, can be obtained by applying Itô's lemma

$$\mathrm{d}C=C'(K)\,\mathrm{d}K+\frac{1}{2}C''(K)(\mathrm{d}K)^2. \tag{12}$$

Since a balanced growth path of the economy is characterized by a time–invariant growth rate, the consumption-wealth ratio, $\mu = C/K$, the abatement ratio a, as well as the time fractions allotted to labor and leisure, l and $1-l$, have to be constant over time, too. Otherwise the conditions for balanced growth would not be met. The solution conjecture of a time–invariant μ is consistent with the underlying isoelastic preferences and typical for this macroeconomic version of the CCAPM (Eaton, 1981; Turnovsky, 1993). Hence $C'(K) = \mu$, $C''(K) = 0$, $\mathrm{d}C = \mu \mathrm{d}K$, and $(\mathrm{d}C)^2 = \mu^2(\mathrm{d}K)^2$. Using the Itô multiplication rules[6] finally yields

$$\mathrm{d}\lambda = \lambda\left[\left(-\beta+\mu-(1-l)^{1-\alpha}(a^{\eta}-a)\left(1-\sigma^2(1-l)^{1-\alpha}(a^{\eta}-a)\right)\right)\mathrm{d}t-(1-l)^{1-\alpha}(a^{\eta}-a)\,\mathrm{d}z\right]. \tag{13}$$

Equating (10) to (13), dividing by λ, and sorting with respect to deterministic and stochastic components results in

$$\Big[-\beta+\mu+a(1-l)^{1-\alpha}\left(1-\eta a^{\eta-1}\right)+\sigma^2(1-l)^{2(1-\alpha)}(a^{\eta}-a) \\ \times\left(a^{\eta}-a+(1-\eta)a^{\eta}\frac{\partial\lambda}{\partial K}\frac{K}{\lambda}\right)\Big]\mathrm{d}t = -a^{\eta}(1-l)^{1-\alpha}\left(\frac{\partial\lambda}{\partial K}\frac{K}{\lambda}+1\right)\mathrm{d}z. \tag{14}$$

For μ to be non-stochastic over time, the random components on the RHS of (14) have to exactly offset, which is only the case if

$$\frac{\partial\lambda}{\partial K} = -\frac{\lambda}{K}\,. \tag{15}$$

Employing this condition and rearranging finally gives the following expression for the consumption-capital ratio, reflecting the consumption–saving tradeoff[7]

$$\mu_1^* = \beta + a(1-l)^{1-\alpha}\left(\eta a^{\eta-1}-1\right)\left(1-\sigma^2(1-l)^{1-\alpha}(a^{\eta}-a)\right). \tag{16}$$

Going back to the first-order condition related to labor–leisure choice, (8), utilizing (15) and rearranging, we derive a second condition for μ, this time reflecting the consumption–leisure tradeoff

$$\mu_2^* = l^{\delta}\frac{(1-\alpha)(1-\eta)a^{\eta}}{(1-l)^{\alpha}}\left(1-\sigma^2(1-l)^{1-\alpha}(a^{\eta}-a)\right). \tag{17}$$

In order to have a positive value for (17) and $\eta < 1$, the last term on the RHS has to be of positive sign. By (10) this represents the certainty equivalent to capital return

$$r_s^* = (1-\eta)a^{\eta}(1-l)^{1-\alpha}\left(1-\sigma^2(1-l)^{1-\alpha}(a^{\eta}-a)\right)\,. \tag{18}$$

The certainty equivalent to capital return is the real interest rate of a (hypothetical) safe asset, which falls below the rental rate to capital by the amount of the risk

[6] $\mathrm{d}t \times \mathrm{d}t = 0$, $\mathrm{d}z_i \times \mathrm{d}z_j = \rho_{ij}\sigma_i\sigma_j\,\mathrm{d}t$ for $i \neq j$, and $\mathrm{d}z_i \times \mathrm{d}z_j = \sigma^2\,\mathrm{d}t$ for $i = j$.

[7] In what follows asterisks denote the Pareto–efficient values of the macroeconomic variables.

premium $a^{\eta}(1-\eta)(1-l)^{2(1-\alpha)}\,(a^{\eta}-a)\,\sigma^2$, because risk averse households demand a higher expected return for bearing the risk of capital accumulation.[8]

Utilizing this in the first-order condition for A, (9), we find that this is only satisfied if $1=\eta a^{\eta-1}$, i.e. when abatement activities take place at the optimal level, if the marginal damage generated by pollution equals the marginal costs of abatement. Solving for the optimal level of the abatement ratio we get

$$a^* = \eta^{\frac{1}{1-\eta}} \,. \tag{19}$$

a^* is solely determined by the pollution elasticity of production. As economic intuition suggests, the more vulnerable output with respect to pollution, the higher the abatement ratio, i.e. the higher the share of production used for abatement purposes.

Using this information allows us to rewrite (16) and (17) which reduce to

$$\mu_1^* = \beta \,, \tag{20}$$

$$\mu_2^* = l^{\delta}\frac{(1-\alpha)(1-\eta)\eta^{\frac{\eta}{1-\eta}}}{(1-l)^{\alpha}}\left(1-\sigma^2(1-l)^{1-\alpha}(1-\eta)\eta^{\frac{\eta}{1-\eta}}\right) \,. \tag{21}$$

μ_1^* and μ_2^* are functions of the model primitives and the time allocation only. They have to be equal in order to be consistent with balanced growth, which also implies that the time share devoted to leisure has to be time–invariant, too. Since (21) is a nonlinear function in working time, the optimal time allocation is only implicitly determined by $\mu_1^*=\mu_2^*$ and cannot be derived explicitly. Equation (20) also reflects the well–known certainty equivalent result, which is typical for logarithmic preferences. Since the social planner internalizes the external effects, capital accumulation is rewarded the social return to capital, which amounts to a pure capital risk. The optimal consumption–capital ratio is solely determined by the rate of time preference, and the intertemporal income and substitution effects originating from the riskiness of the income source exactly offset. This also implies that the consumption-accumulation decision of the household is independent from the pollution generated through production.

By substituting (20) and (19) into (21), we get an expression, which implicitly describes the optimal allocation of time on labor and leisure in the Pareto–efficient economy:

$$l^{-\delta} = \frac{(1-\alpha)(1-\eta)\eta^{\frac{\eta}{1-\eta}}}{\beta(1-l)^{\alpha}}\left(1-\sigma^2(1-l)^{1-\alpha}(1-\eta)\eta^{\frac{\eta}{1-\eta}}\right) \,. \tag{22}$$

An equilibrium growth path is characterized by capital and consumption growing at a common stochastic rate, that is $\mathrm{d}K/K=\mathrm{d}C/C$. The equilibrium expected growth

[8] The safe asset is purely hypothetical, since we are dealing with an aggregate risk which—by assumption—cannot be diversified away. There is no safe outside option available.

rate can be derived by employing the aggregate resource constraint, (4), together with (20) and (21) and taking expectations. We obtain two expressions for the optimal expected growth rate, which—similarly to the consumption–capital ratios μ_1^* and μ_2^*—implicitly determine the equilibrium allocation of labor:

$$g_1^* = (1-\eta)\eta^{\frac{\eta}{1-\eta}}(1-l)^{1-\alpha} - \beta \tag{23}$$

$$g_2^* = (1-\eta)\eta^{\frac{\eta}{1-\eta}}(1-l)^{1-\alpha}\left(1 - \frac{(1-\alpha)\,l^{\delta}}{1-l}\left(1-\sigma^2(1-\eta)\eta^{\frac{\eta}{1-\eta}}(1-l)^{1-\alpha}\right)\right) \tag{24}$$

The economy is in equilibrium if the two growth rates are equal, i.e. $g_1^* = g_2^* = g^*$, or equivalently

$$\Delta g^* = g_1^* - g_2^* = -\beta + \frac{(1-\alpha)(1-\eta)\eta^{\frac{\eta}{1-\eta}}\,l^{\delta}}{(1-l)^{\alpha}}\left(1-\sigma^2(1-\eta)\eta^{\frac{\eta}{1-\eta}}(1-l)^{1-\alpha}\right) = 0 \tag{25}$$

Proposition 1 *A unique balanced growth path exists, if the certainty equivalent to capital return is positive and Δg^* satisfies the following conditions:*

(i) *Δg^* is a continuous and monotonic function in the domain $l \in (0,1)$.*

(ii) *The limits of Δ^* are of opposite sign, that is*

$$\operatorname{sgn}\lim_{l\to 0}\Delta g^* = -\operatorname{sgn}\lim_{l\to 1}\Delta g^* \,. \tag{26}$$

Proof: Differentiation of (25) with respect to l gives

$$\frac{\partial \Delta g^*}{\partial l} = (1-\alpha)l^{\delta}\left[\frac{r_s}{(1-\eta)(1-l)}\left(\frac{\delta}{l}+\frac{\alpha}{1-l}\right)+\alpha(1-\eta)\left(\frac{\eta^{\frac{\eta}{1-\eta}}}{(1-l)^{\alpha}\sigma}\right)^2\right]$$

For $r_s > 0$, Δg^* is monotonically decreasing in $l, l \in (0,1)$. The limits of Δg^* with respect to $l \to 0$ and $l \to 1$ are given by:

$$\lim_{l\to 0}\Delta g^* = -\beta \qquad \text{and} \qquad \lim_{l\to 1}\Delta g^* = \infty$$

□

Optimal leisure is implicitly determined by (20), (21) and (23):

$$l^* = 1 - \left(\eta^{\frac{\eta}{1-\eta}}(g^*+\beta)\right)^{\frac{1}{1-\alpha}} \,. \tag{27}$$

Figure 1 illustrates the result of Proposition 1 and shows that there is an interior solution for the optimal time allocation.

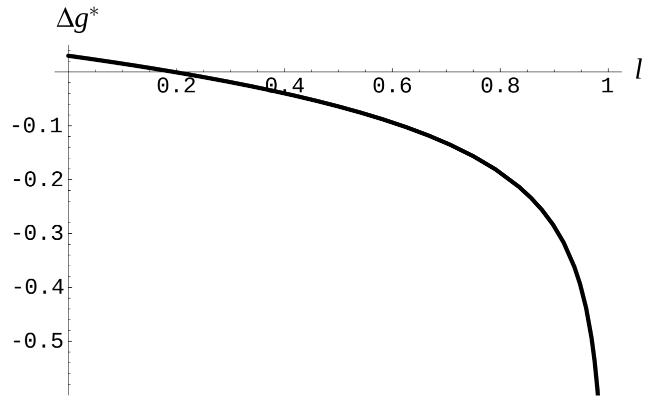


Figure 1: Unique equilibrium labor supply in the Pareto–efficient economy

Comparative static analysis of the Pareto–efficient allocation While the optimal consumption–capital–ratio (20) is only determined by the rate of time preference, optimal growth and leisure depend on the other model parameters. By employing the implicit function theorem to (22) we can show how optimal leisure—and labor input respectively—responds to changes in the model parameters. We focus on those parameters which seem most interesting to us: η, which reflects the vulnerability of production with respect to pollution; σ^2 which measures the impact of changes in the riskiness of production; and finally δ, representing the elasticity of marginal utility with respect to leisure.

From (22), we get the following comparative static results for a variation in the three model primitives:

$$\frac{\mathrm{d}l^*}{\mathrm{d}\sigma^2} = \frac{l^\delta}{\beta A}\frac{1-\alpha}{(1-l)^{2\alpha-1}}\left((1-\eta)\eta^{\frac{\eta}{1-\eta}}\right)^2 > 0\,, \tag{28}$$

$$\frac{\mathrm{d}l^*}{\mathrm{d}\delta} = -\frac{\ln l}{A} > 0\,, \tag{29}$$

$$\frac{\mathrm{d}l^*}{\mathrm{d}\eta} = -\frac{l^\delta}{\beta A}\frac{1-\alpha}{(1-l)^\alpha}\left(1-2\sigma^2(1-l)^{1-\alpha}(1-\eta)\eta^{\frac{\eta}{1-\eta}}\right)B \gtreqless 0 \tag{30}$$

with

$$A = \frac{\delta}{l} + \frac{\alpha}{1-l} + \frac{\sigma^2(1-\eta)\eta^{\frac{\eta}{1-\eta}}\frac{1-\alpha}{(1-l)^\alpha}}{1-\sigma^2(1-\eta)\eta^{\frac{\eta}{1-\eta}}\frac{1-\alpha}{(1-l)^\alpha}} > 0 \qquad \text{and} \qquad B = \eta^{\frac{\eta}{1-\eta}}\frac{\ln\eta}{1-\eta} < 0\,.$$

(28) and (29) show that the planner responds qualitatively in the same way to an increase in the variance of the technology shock as to an increase in the utility parameter δ.

In the presence of logarithmic preferences with respect to consumption, an increase in σ^2 does not have an effect on the household's optimal propensity to consume. Nevertheless, second–order effects from the productivity shock on the labor–leisure trade–off can be observed. By facing a higher technological risk, the planner substitutes leisure for labor time in order to compensate for a higher variance associated with labor input. The increase in σ^2 affects the shadow price between consumption and leisure. This becomes obvious, if we go back to equation (22), implicitly determining the optimal time allocation and derived by equating (7) to (8). The RHS of (22) becomes smaller. In order to still satisfy the first–order condition (8), the LHS of (22), measuring marginal utility of leisure, U_l, has to become smaller too, which is only the case for the underlying concave function and δ given, if the amount of leisure consumed increases.

As can be seen from (23) the associated decrease in working time causes negative growth effects. It reduces the net output–capital ratio (output minus abatement effort) while leaving μ unchanged at the cost of savings and growth

$$\frac{\mathrm{d}g^*}{\mathrm{d}\sigma^2} = -(g^* + \beta)\frac{1-\alpha}{1-l}\,\frac{\mathrm{d}l}{\mathrm{d}\sigma^2} < 0 \,.$$

An increase in δ reflects an increase in the marginal utility of leisure which also induces the planner to substitute leisure for labor. As (23) shows, this, too, reduces savings and growth

$$\frac{\mathrm{d}g^*}{\mathrm{d}\delta} = -(g^* + \beta)\frac{1-\alpha}{1-l}\,\frac{\mathrm{d}l}{\mathrm{d}\delta} < 0 \,.$$

The response to a change in the pollution elasticity of output, η, crucially depends on whether the first–order effects (stemming from the mean) or the second–order (variance) effects prevail

$$\frac{\mathrm{d}g^*}{\mathrm{d}\eta} = -(g^* + \beta)\frac{1-\alpha}{1-l}\frac{\mathrm{d}l}{\mathrm{d}\eta} + (1-l)^{1-\alpha}B \gtreqless 0 \,.$$

If we consider the benchmark of a riskless economy ($\sigma = 0$), an increase in η unambiguously increases leisure and decreases growth. In this case, a higher η leads to a reduction in the marginal productivities of the input factors capital and labor, which—in a market economy—would amount to a decline in the associated factor prices, thus making working efforts less attractive. The growth effect is further aggravated by an associated increase in optimal abatement (19). In a stochastic setting, we additionally have a second–order effect from the risk premium, which becomes obvious, if we substitute (18) into (22), which then reduces to $l^{-\delta} = (1-\alpha)r_s/(\beta(1-l))$. The social planner takes account of the fact that the households are risk averse and dislike deviations from a smooth income flow. Changes in η also affect the expected risk premium, by this translating into labor supply and growth effects, which counteract (and even may outweigh) the negative effects from the mean return. As a result, a higher pollution reagibility of output might raise labor supply and—given that this increase is sufficiently large—even raise expected growth.

4 The Market Economy

We now proceed with discussing the market economy. Households ultimately own firms. Since we initially assumed all individuals to be identical, we will confine our analysis to the representative consumer, who chooses his intertemporal consumption flow, working and leisure time, as well as his abatement efforts in order to maximize his intertemporal welfare (3) subject to his budget constraint, while treating public policy as exogenous.

We assume that the household is subject to environmental taxation. As a firm owner he pays a pollution tax but disregards his individual contribution to the overall level of pollution as a by–product of production activities. The firm owner is subject to a linear pollution tax at the rate τ^p. Since the firm–specific flow of pollution, $P(t)$, generated through production also is subject to the geometric Brownian motion, we postulate the following diffusion for individual tax payments:

$$\mathrm{d}T^p(t) = \tau^p P(t)\left(\mathrm{d}t + \mathrm{d}z(t)\right) . \tag{31}$$

We assume identical rates for the taxes levied on the deterministic and the random components of pollution.

The household furthermore receives a subsidy on capital accumulation. The subsidy rate τ^k is constant and proportional to the level of investment undertaken, such that subsidy payments follow the diffusion:

$$\mathrm{d}T^k(t) = \tau^k K(t)\left(\mathrm{d}t + \mathrm{d}z(t)\right) . \tag{32}$$

Net government revenues (may they be positive or negative) are redistributed to households in a lump–sum fashion. The diffusion for lump–sum payment $T(t)$ in case of a balanced government budget is then given by

$$\mathrm{d}T(t) = \mathrm{d}T^p(t) - \mathrm{d}T^k(t) = T(t)\left(\mathrm{d}t + \mathrm{d}z(t)\right) \tag{33}$$

and, depending on the expenditure and revenue flows, can be stochastic too. We require the government budget constraint to be met in each period of time, so there is no government debt or surplus.

The representative agent maximizes individual welfare (3) subject to his budget constraint $\mathrm{d}K = \mathrm{d}Y - C\,\mathrm{d}t - A(\mathrm{d}t + \mathrm{d}z) - \mathrm{d}T^p + \mathrm{d}T^k + \mathrm{d}T$, or equivalently

$$\begin{aligned} \mathrm{d}K = &\left(K^\alpha\left(\bar{K}(1-l)\right)^{1-\alpha}\bar{P}^{-\eta} - C - A + T - \tau^p\left(\frac{K^\alpha\left(\bar{K}(1-l)\right)^{1-\alpha}}{A}\right) + \tau^k K\right)\mathrm{d}t \\ &+ \left(K^\alpha\left(\bar{K}(1-l)\right)^{1-\alpha}\bar{P}^{-\eta} - A + T - \tau^p\left(\frac{K^\alpha\left(\bar{K}(1-l)\right)^{1-\alpha}}{A}\right) + \tau^k K\right)\mathrm{d}z , \end{aligned} \tag{34}$$

while taking prices, tax rates and lump-sum payments as exogenously given.

The stochastic Hamiltonian can be set up as follows:

$$\mathscr{H}\left(C,K,A,l,\lambda,\frac{\partial\lambda}{\partial K}\right)=e^{-\beta t}\left(\ln C+\frac{l^{1-\delta}}{1-\delta}\right)$$
$$+\lambda\left(K^{\alpha}\bar{K}^{1-\alpha}(1-l)^{1-\alpha}\bar{P}^{-\eta}-C-A+T-\tau^{p}\left(\frac{K^{\alpha}\left(\bar{K}(1-l)\right)^{1-\alpha}}{A}\right)+\tau^{k}K\right)+\frac{\sigma_K^2}{2}\frac{\partial\lambda}{\partial K}$$

where aggregate pollution is exogenous, and

$$\sigma_K^2=\left(K^{\alpha}\left(\bar{K}(1-l)\right)^{1-\alpha}a^{\eta}-A+T-\tau^{p}\left(\frac{K^{\alpha}\left(\bar{K}(1-l)\right)^{1-\alpha}}{A}\right)+\tau^{k}K\right)^{2}\sigma^{2}. \quad (35)$$

Maximization leads to the following FOCs, where we already haven taken account of $P=1/a$ and the government budget constraint (33):

$$\frac{\partial\mathscr{H}}{\partial C}=e^{-\beta t}C^{-1}-\lambda=0 \quad (36)$$

$$\frac{\partial\mathscr{H}}{\partial l}=e^{-\beta t}l^{-\delta}-\lambda K\frac{1-\alpha}{1-l}\left(a^{\eta}(1-l)^{1-\alpha}-\frac{\tau^{p}}{aK}\right)\left(1+\sigma^{2}\frac{\partial\lambda}{\partial K}\frac{K}{\lambda}(1-l)^{1-\alpha}(a^{\eta}-a)\right)=0 \quad (37)$$

$$\frac{\partial\mathscr{H}}{\partial A}=\left(\tau^{p}\frac{1}{aA}-1\right)\left(1+\sigma^{2}\frac{\partial\lambda}{\partial K}\frac{K}{\lambda}(1-l)^{1-\alpha}(a^{\eta}-a)\right)=0 \quad (38)$$

$$\begin{aligned}\mathrm{d}\lambda&=-\frac{\partial\mathscr{H}}{\partial K}\mathrm{d}t+\frac{\partial\lambda}{\partial K}\sigma_K\,\mathrm{d}z\\&=-\lambda\Bigg[\left(\alpha a^{\eta}(1-l)^{1-\alpha}+\tau^{k}-\alpha\frac{\tau^{p}}{aK}\right)\left(1+\sigma^{2}\frac{\partial\lambda}{\partial K}\frac{K}{\lambda}\left((1-l)^{1-\alpha}(a^{\eta}-a)\right)\right)\mathrm{d}t\\&\quad+\frac{\partial\lambda}{\partial K}\frac{K}{\lambda}\left((1-l)^{1-\alpha}(a^{\eta}-a)\right)\mathrm{d}z\Bigg].\end{aligned} \quad (39)$$

The first–order conditions (36) and (37) with respect to consumption and leisure are identical in structure compared to the associated conditions of the planner problem. The pollution tax is tied to individually generated pollution. The representative agent knows that he can avoid/reduce tax payments by voluntarily undertaking abatement efforts, which is reflected in condition (38).

The solution procedure is similar to the one already outlined above. From (39) follows the certainty equivalent to capital return, r_s:

$$r_s=\left(\alpha a^{\eta}(1-l)^{1-\alpha}+\tau^{k}-\alpha\frac{\tau^{p}}{aK}\right)(1-\sigma^{2}(1-l)^{1-\alpha}(a^{\eta}-a)). \quad (40)$$

The equilibrium riskless rate of a (hypothetical) safe asset nicely demonstrates the multiple ways of how fiscal policies affect accumulation in a risky environment. The two policy instruments have a twofold impact on the riskless rate. Both affect the mean return as well as the risk premium on capital holdings, the latter reflecting the second–order effects stemming from the variance of the technological disturbance.[9] First, the subsidy payed on capital accumulation generally makes savings more attractive. Therefore, the mean real interest rate of a (hypothetical) safe asset has to increase too, due to the general equilibrium nature of our approach. Second, by raising the expected capital return, a subsidy also increases the volatility of future capital income flows, which risk averse agents dislike and makes them demand a larger risk premium on capital holdings. Although the induced intertemporal income and substitution effects of changes in the real return to capital offset for the case of logarithmic preferences in consumption (see (41) and (42) below), they still are present in the equilibrium value of r_s. The effects reverse, if it comes to the pollution tax. Being tied to current production and putting a burden on capital accumulation, the pollution tax reduces the deterministic part of the interest rate while raising the stochastic component. A reduction in the volatility of future capital incomes leads to a lower risk premium on capital accumulation.

In order to have a positive value of the certainty equivalent to capital return, we need the first–order effects to prevail, such that the last term on the RHS of (40) is of positive sign.[10]

If we next differentiate (36) with respect to time and equate the resulting diffusion process $\mathrm{d}\lambda$ for the shadow price to condition (39), we obtain the desired relationship for the consumption–capital ratio μ, which we expect to be constant along the balanced growth path. By additionally taking into account that the government has to run a balanced budget, we arrive at:

$$\mu_1 = \beta + \left((1-l)^{1-\alpha}((1-\alpha)a^{\eta} - a) + \tau^k + \alpha\frac{\tau^p}{aK}\right)[1 - \sigma^2(1-l)^{1-\alpha}(a^{\eta} - a)]. \tag{41}$$

As before, another expression for μ can be obtained from the first–order condition for leisure:

$$\mu_2 = \frac{(1-\alpha)\,l^{\delta}}{1-l}\left(a^{\eta}(1-l)^{1-\alpha} - \frac{\tau^p}{aK}\right)\left[1 - \sigma^2(1-l)^{1-\alpha}(a^{\eta} - a)\right]. \tag{42}$$

A comparison between the propensity to consume out of wealth chosen by the social planner (20) with the one of decentralized market economy (41), shows very clearly the impact of factor income risk on intertemporal consumption choice, because both measure the consumption–saving tradeoff. Whereas the social planner rewards capital its social return—which equals the value of output—and therefore indirectly neglects labor income risk, labor and capital inputs of the market economy are payed

[9] Recall that $r = r_s +$ risk premium.

[10] A positive sign of r_s is important for existence and uniqueness of the steady state. See also Clemens (2004*a*) for an extensive discussion of the feasibility of balanced growth paths in continuous–time stochastic growth models with elastic labor supply.

according to their marginal product in production. As Leland (1968) pointed out, decreasing absolute risk aversion is necessary and sufficient for households to save out of precautionary motives in the presence of *pure income risk*. This condition is met for any positive value of the coefficient of relative risk aversion $R_R > 0$, which in our model equals unity by the assumption of log–utility in consumption. Furthermore, as demonstrated by Levhari and Srinivasan (1969) as well as by Sandmo (1970), households have to be sufficiently risk averse in the presence of a *pure capital risk* in order to undertake buffer stock savings, which for the underlying isoelastic preferences corresponds to all values of $R_R > 1$. There is no savings effect from the riskiness of capital incomes for $R_R = 1$. Consequently, if we observe any impact from risk on intertemporal consumption choice, this can entirely be attributed to the presence of labor income risk.

The underlying tax–transfer system indirectly redistributes income between labor and capital, since it subsidizes accumulation while simultaneously taxing pollution which is created as a by–product of private capital and labor inputs in production. For any given policy–mix not mimicking the Pareto–efficient allocation, the effects of labor income risk on consumption and saving prevail and we find μ to be smaller if compared to a riskless environment (i.e. $\sigma = 0$). This indicates the presence of precautionary savings the household undertakes in order to self–insure against the fluctuations of future income flows. Compared to the Pareto–efficient allocation and other things equal, μ is more likely of being too large, because the households only take into account the lower market returns. They neglect the Marshallian knowledge externality in their intertemporal decision and consequently save too little.

From the first–order condition for A, (38), we are able to derive an expression for the optimal relation between l and a:

$$0 = \left(\frac{\tau^p}{A} - a\right)\left(1 - \sigma^2(1-l)^{1-\alpha}(a^\eta - a)\right) . \tag{43}$$

which only is satisfied for a positive value of r_s, if $\tau^p = aA$. From (43) it can be seen that a constant abatement ratio over time is only consistent with household optimization if the tax rate increases over time. Due to the accumulation of capital, the marginal value of a unit of pollution rises over time. Consequently, to keep pollution from increasing over time, its costs in terms of the tax have to increase as well (see also Pittel, 2002). At the same time the tax on pollution serves as an implicit subsidy on abatement which increases in a growing economy. Along any balanced path the growth rate of the tax has to equal to joint growth rate of abatement and capital. Substituting $\tau^p = aA$ into (31) shows that tax revenues exactly suffice to pay for abatement expenditures.

Considering (41) shows that a constant propensity to consume, which is a prerequisite for balanced growth, requires the subsidy rate on capital to be constant over time. However, although the rates of the two policy instruments develop differently over time, their growth rates are of course identical as in the subsidy case the subsidy basis grows at the same rate as the pollution tax. Nevertheless a balanced budget of the regulating authority without any lump–sum transfers, i.e. $T = 0$, can only hold for an optimal policy in a knife edge case, as will be shown below.

The expected growth rate of the decentralized economy can be obtained from the aggregate resource constraint (34) under consideration of a balanced government budget and the two expressions for the propensity to consume, (41) and (42):

$$g_1 = (1-l)^{1-\alpha}(a^{\eta} - a) - \mu_1 \tag{44}$$

$$g_2 = (1-l)^{1-\alpha}(a^{\eta} - a) - \mu_2 \,. \tag{45}$$

The impact of risk, which negatively affects consumption is of opposite sign in the expected growth rate of the economy (44), thereby indicating the presence of precautionary saving, which is empirically supported, e.g. by Zeldes (1989), Caballero (1990), and Hubbard *et al.* (1994).

The economy is in equilibrium if the two growth rates are equal, i.e. $g_1 = g_2 = g$, which implicitly determines the equilibrium level of labor supply:

$$\Delta g = g_1 - g_2 = -\beta + \left[1 - \sigma^2(1-l)^{1-\alpha}\left((\tau_A^p)^{\eta} - \tau_A^p\right)\right] \times \left[(1-l)^{1-\alpha}(1-\alpha)\left((\tau_A^p)^{\eta} - \tau_A^p\right)\left(\frac{l^{\delta}}{1-l} - 1\right) - \tau^k\right] = 0 \tag{46}$$

where we considered $\tau^p = aA$ from (43). The associated conditions for existence and uniqueness of a macroeconomic equilibrium along the balanced growth path closely resemble those stated for the Pareto–efficient solution in Proposition 1 and therefore are relegated to the Appendix.

The optimal policy The growth path of the social planner can be replicated by the appropriate choice of the pollution tax and the capital subsidy. Similar to the standard deterministic economy, a policy is chosen optimally if we set the two instruments equal to the marginal externalities of pollution and capital:

$$\tau^{p*} = \eta a^{\eta} A \qquad \text{and} \qquad \tau^{k*} = (1-\eta)(1-\alpha)a^{\eta}(1-l)^{1-\alpha} \tag{47}$$

Substituting (47) into (43) yields the familiar condition

$$a\left(\eta a^{\eta-1} - 1\right)\left(1 - \sigma^2(1-l)^{1-\alpha}\left(a^{\eta} - a\right)\right) = 0 \,, \tag{48}$$

which is identical to (9), the condition for an optimal provision of A in the social planner case. Consequently we get for the above described policy:

$$a = a^* = \eta^{\frac{1}{1-\eta}} . \tag{49}$$

If the policy instruments are chosen according to (47) the decentralized economy replicates the efficient allocation with conditions for propensities to consume and expected growth rates equal to (20), (21), (23), and (24).

In contrast to a riskless environment, taxation and subsidization now both target equally at mean economic activities as well as at random fluctuations around the mean. This potentially opens up the alternative to examine more complex–structured tax–transfer–systems, for instance, by treating stochastic and deterministic activities differently at differentiated rates, but is beyond the scope of the present paper (see e.g. Clemens and Soretz, 1997, 2004; Soretz, 2007).

Although the technology shock has zero mean, the variance of the capital stock increases over time. Neglecting the stochastic structure of production and abatement in the policy–mix, would leave polluting economic activities partly untaxed and these effects would accumulate over time. As the social planner takes account of the technology risk in his allocation decisions, a solely deterministic fiscal policy runs short of its target and never suffices to internalize the external effects.

Regarding (47), it can be seen that the pollution externality affects the optimal level of the capital subsidy. In contrast to an economy without pollution, the learning–by–doing spillover in our economy has a twofold effect on production: The positive direct effect on the social return and an indirect negative effect from the repercussions of capital accumulation on pollution. The optimal subsidy rate in (47) corrects for the net of the two effects.

Substituting (47) to (49) into (33) shows that lump–sum transfers/taxes are non–zero except for the knife–edge case in which the negative pollution externality is exactly offset by the positive net capital externality, i.e. iff $\alpha = (1-\eta)^{-1}$. In all other cases

$$T \gtreqless 0 \qquad \Longleftrightarrow \qquad \eta \gtreqless (1-\eta)(1-\alpha) . \tag{50}$$

Comparative static analysis of the decentralized allocation In contrast to the social planner case, the equilibrium propensity to consume differs from the rate of time preference in the decentralized setting. As households do not internalize the production externalities and capital is only awarded its private return, the labor income risk is not neutralized and influences the consumption decision of households. This also implies that the consumption–accumulation decision of the household now depends not only on the pollution elasticity of production and the other model parameters, but also on the tax and subsidy rates.[11]

To compute the comparative statics of l and g, we equate μ_1 to μ_2 from (41) and (42), and additionally take regard of $\tau^p = aA$, such that

$$\beta = \left[1-\sigma^2(1-l)^{1-\alpha}\left((\tau_A^p)^\eta - \tau_A^p\right)\right]\left[(1-l)^{1-\alpha}(1-\alpha)\left((\tau_A^p)^\eta - \tau_A^p\right)\left(\frac{l^\delta}{1-l}-1\right)-\tau^k\right] , \quad (51)$$

where we define $\tau_A^p \equiv \frac{\tau^p}{A}$ for notational simplicity, which measures. As $\beta > 0$ and $r_s > 0$ for feasibility reasons, such that also $\frac{l^\delta}{1-l} > 1$, the second term on the RHS is positive in equilibrium. Employing the implicit function theorem we can derive the following comparative static results for the policy instruments:

Proposition 2 *The equilibrium labor supply decreases with a rise in the subsidy rate payed on capital accumulation, if the technological risk does not become too large. The effect of a change in the pollution tax, as measured by τ_A^p, generally is of ambiguous sign*

$$\frac{\mathrm{d}(1-l)}{\mathrm{d}\tau^k} < 0 \qquad \textit{for} \qquad 1 > 2\sigma^2(1-l)^{1-\alpha}\left((\tau_A^p)^\eta - \tau_A^p\right) , \quad (52)$$

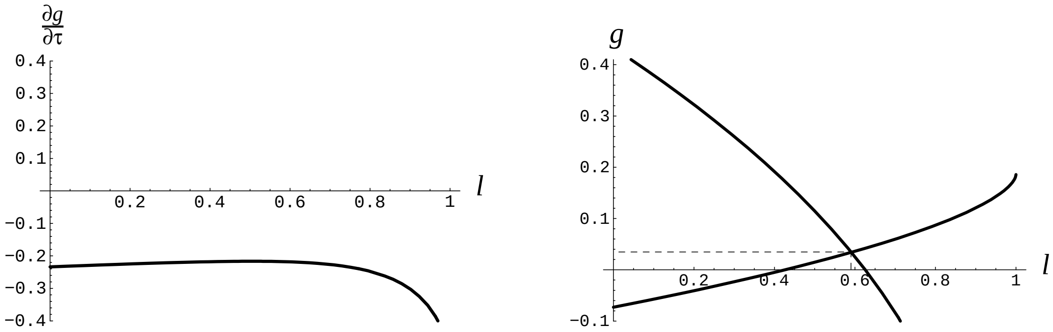
$$\frac{\mathrm{d}(1-l)}{\mathrm{d}\tau_A^p} \lesseqqgtr 0 \qquad \textit{for} \qquad \tau_A^{p*} \lesseqqgtr \tau_A^p . \quad (53)$$

Proof: see Appendix.

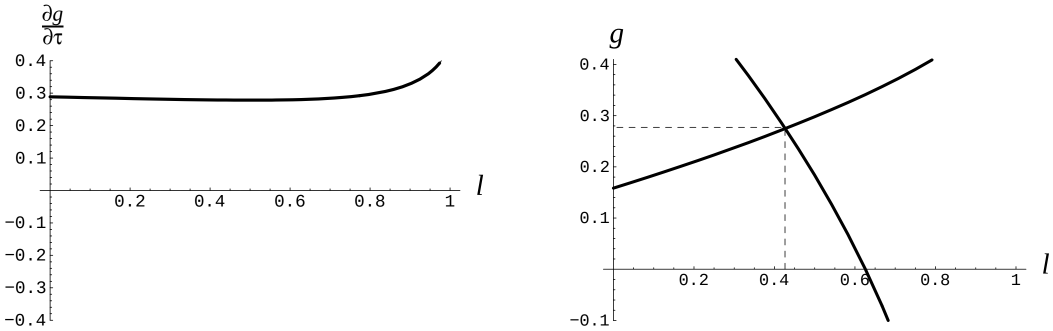
An increase in capital subsidization leads an increase in capital formation and a substitution of capital for labor, such that households can enjoy more leisure. To this extent our results coincide with the implications of a deterministic setting. However, besides this static effect, we have a negative dynamic effect of labor–leisure choice on expected growth. A reduction in labor supply not only reduces total wage income but also the associated wage income risk, thereby leading to a decline in precautionary savings. Hence we have counter–acting effects from capital subsidization on growth, such that the negative effect prevails, if we additionally take account of the government budget constraint. For a sufficiently large technology risk, the sign of (57) even may reverse without violating the imposed feasibility conditions on r_s.

Regarding the by–effects on expected growth of a change in τ_A^p, it is not possible to derive clear–cut results, due to the multitude of interacting adjustments of labor supply and the marginal productivity of abatement. Nevertheless, it is possible to identify the response of the equilibrium expected growth rate to a change in the policy instruments numerically, where the results can be shown to hold for a large range of parameter values.

[11] As the comparative static results of a variation in η, σ^2, and δ only get more complex without changing the qualitative results already derived above for the planner economy, we concentrate on the results for the policy instruments only.



(a) tax scenario 1 $(\tau_A^P = 0.5)$



(b) tax scenario 2 $(\tau_A^P = 0.05)$

Figure 2: *Comparative static results of a variation in the pollution tax*

The response of labor supply to an increase in τ_A^p crucially depends on $\tau_A^{p*} \gtreqless \tau_A^p$, i.e., whether or not the chosen policy overshoots or falls below its optimal value. Given that e.g. $1 > 2\sigma^2(1-l)^{1-\alpha}\left((\tau_A^p)^\eta - \tau_A^p\right)$, labor supply decreases due to a rise in the tax rate while growth increases if the pollution tax is below its optimal level. Raising τ_A^p induces an increase in marginal abatement that raises production by more than the marginal costs of abatement in terms of output. The opposite holds if taxation is above its optimal level. Figure 2 depicts two such scenarios in which the pollution tax is either higher (Figure 2(a)) or lower (Figure 2(b)) than the optimal pollution tax $\tau_A^p \approx 0.77$.[12] In both subfigures, the right–hand plot shows equilibrium leisure at the respective tax rate, while the left–hand plot displays the comparative static results for the growth rate with respect to a change in τ_A^p.

[12]The parameters in Figure 2 were set to $\alpha = 0.2$, $\beta = 0.1$, $\delta = 1$, $\eta = 0.1$, $\sigma = 0.1$. For simplicity, the capital subsidy was set to zero in both subfigures.

5 Exogenous labor supply

We want to conclude our analysis with some final remarks on the case of inelastic labor supply. When households have no preference for leisure, the entire time–endowment is devoted to working, i.e. $l = 0$. Consequently, the solution for the exogenous case can be obtained by setting $(1-l)$ equal to unity and disregarding the FOCs for leisure in the preceding sections.

Social planner It can immediately be seen from (9) and (20) that the optimal abatement ratio (i.e. optimal pollution, $P^{*e} = 1/a^{*e}$) as well as the optimal propensity to consume are unaffected by the endogeneity of labor supply:[13]

$$a^{*e} = \eta^{\frac{1}{1-\eta}}, \qquad \mu^{*e} = \beta \qquad g^{*e} = (1-\eta)\eta^{\frac{\eta}{1-\eta}} - \beta\,. \tag{54}$$

Although the optimal share of output devoted to abatement activities remains unchanged, optimal expected growth rises as production increases due to the higher input of labor:

$$g^{*e} - g^{*} = (1-\eta)\eta^{\frac{\eta}{1-\eta}}(1-(1-l)^{1-\alpha}) > 0\,. \tag{55}$$

The functional form and sign of this growth differential is independent from the technology risk. Nevertheless, the randomness of production has an indirect effect on (55) by affecting the optimal choice of l.

Let us now consider the growth effects of an increase in η, the pollution elasticity of production, under the two scenarios. If labor supply is exogenous, an increase in η unambiguously increases growth by raising the abatement ratio. In the endogenous labor setting however, the increase of η additionally changes the optimal labor choice. As we have seen before, the change in labor supply depends on whether or not the first–order effect is dominated by the second–order effect stemming from the variance of the productivity shock. If the first–order effect dominates, optimal labor supply increases due to an increase in η which raises growth and lowers the growth differential in (55). If the second–order effect dominates, growth is lowered by a decrease in labor supply. So while an increase in η unambiguously increases growth when labor supply is exogenous, it might lower growth for endogenous labor–leisure choice case. Whether growth rises or falls, crucially depends on the magnitude of the production risk σ^2.

Market economy As can be seen from (38) the equilibrium abatement ratio in the market economy is still solely determined by the chosen tax policy, $a = \tau_A^p = \frac{\tau^p}{A}$. Hence, setting $\tau_A^p = \tau_A^{pe}$ results in identical pollution levels for the exogenous and

[13]The superscript e refers to variable values in the exogenous labor scenario.

the endogenous labor setting. This also implies that the optimal tax policies are identical in both cases ($\tau_A^{p*e} = \tau_A^{p*} = \eta^{\frac{\eta}{1-\eta}}$). The non–normalized tax rates, $\tau^p = \eta^{\frac{\eta}{1-\eta}} A$, will however differ between the two scenarios, as A follows a different time path. Recalling that optimal growth is higher in the exogenous labor case, τ^{p*e} increases faster than τ^{p*}.

With respect to the optimal capital subsidy, its level is higher when labor is exogenously supplied ($\tau^{k*e} - \tau^{k*} = (1-\eta)(1-\alpha)\eta^{\frac{\eta}{1-\eta}}(1-(1-l)^{1-\alpha})$. This result originates from the fact that the marginal capital externality is higher for exogenous labor supply, where aggregate labor input is comparably larger.

With respect to the difference between growth rates in the endogenous and exogenous labor case, the difference again crucially depends on the technological risk:

$$g^e - g = [(\tau^p)^\eta - \tau^p](1-(1-l)^{1-\alpha}) \left[\alpha - \sigma^2((1-\alpha)(1-\eta)((\tau^p)^\eta - \tau^p)(1+(1-l)^{1-\alpha}) + \tau^k)\right] . \quad (56)$$

If production is deterministic, growth again is unambiguously higher when labor supply is inelastic, i.e. the growth differential is positive. Yet, if production is stochastic, the riskiness of labor income lowers the growth differential and may even reverse its sign.

As for the growth differential itself, the change in $g^e - g$ due to a change in the capital subsidy is unambiguous in a deterministic setting. In this case τ^k only has an indirect effect on growth via its impact on labor supply, leading to an increase in growth. If production is stochastic, however, an increase in the capital subsidy additionally increases the volatility of labor income which affects the growth differential negatively. Whether $g^e - g$ rises or falls due to an increase in τ^k again depends crucially on the magnitude of the production risk σ^2.

The effect of an increase in the pollution tax on the growth gap is not even unambiguous in the deterministic scenario. In this case, higher taxation increases the abatement ratio but lowers working time, the net effect depending crucially on the underlying parameter values. Matters get even more complicated in the stochastic setting, where not only the effect of taxation on labor can either be positive or negative, but where also the impact of the riskiness of production is of ambiguous sign.

6 Conclusions

This paper has analyzed the effects of technological risk on long-run growth when labor supply is elastic and production gives rise to two types of externalities. On the one hand, production generates a flow of pollution that the individual producer takes as exogenous and that can be reduced by abatement activities. On the other hand, the input of capital induces a positive knowledge spillover. For the described economy we have then examined the optimal as well as the decentralized balanced growth path.

With respect to the optimal solution, we have shown that expected growth is not affected by the aggregate technology shock. As preferences are logarithmic and the social planner perfectly internalizes the externalities, intertemporal substitution and income effects exactly offset. Along the balanced path the optimal abatement ratio is constant and solely determined by the pollution elasticity of output. Whether an increase in the pollution elasticity affects labor supply and growth positively or negatively depends largely on the volatility of technology. If production is deterministic, optimal labor supply and growth decrease due to an increase in the pollution reagibility of production. If production is stochastic, however, second–order effect arise from the risk premium. Given that the variance of the technology shock is sufficiently high, these effects might prevail and give rise to an increase in labor supply and growth.

For the market economy, we have first focused on optimal policies that replicate the social planner solution. It has been shown that the randomness of output requires optimal policy schemes to comprise a stochastic component to take account of the rising volatility of output. Deterministic and stochastic activities can be taxed/subsidized equally, but might also be treated differently by the policy maker. In the present paper, we have concentrated on a uniform treatment of stochastic and deterministic income parts and have left differentiated policies to future research. It has been shown that a combination of pollution taxation and capital subsidization can give rise to the optimal solution if the deterministic and stochastic part of the tax/subsidy rates are set equal to the respective marginal externalities.

Furthermore, we have examined more general properties of non–optimal taxation and subsidy policies. We showed that these properties can, but do not have to be in line with the well–known results from deterministic models of pollution and growth. The inclusion of the technology risk might lead to a reversal of the responses of labor supply and growth to changes in the model primitives and policy variables. A comparison of exogenous and endogenous labor supply has furthermore shown that it is the endogeneity of the labor supply that allows for these potential sign reversals.

In the paper at hand we have assumed pollution to be a flow, thereby allowing for a straightforward integration into the existing literature on stochastic growth. Yet, modeling pollution or resources as stock variables might give rise to interesting additional insights. Especially in the context of climate change, the consideration of risky stocks seems to be a promising field for future research.

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A Existence and Uniqueness of the Growth Equilibrium in the Market Economy

Proposition 3 *A unique balanced growth path exists for the decentralized economy if the certainty equivalent of the capital return is positive and* $1 > 2\sigma^2(1-l)^{1-\alpha}\left((\tau_A^p)^\eta - \tau_A^p\right)$*. Under these conditions*

(i) Δg *is a continuous and monotonically increasing function in the domain* $l \in (0,1)$*.*

(ii) The limits of Δ *are of opposite sign*

$$\operatorname{sgn}\lim_{l\to 0}\Delta g = -\operatorname{sgn}\lim_{l\to 1}\Delta g\,.$$

Proof: Differentiation of (46) with respect to l gives

$$\frac{\partial \Delta g^*}{\partial l} = (1-\alpha)(1-l)^{-\alpha}\left[(\tau_A^p)^\eta - \tau_A^p\right]\left(\left[1-2\sigma^2(1-l)^{1-\alpha}\left[(\tau_A^p)^\eta - \tau_A^p\right]\right](1-\alpha)\right.$$
$$\left. + \left[1-\sigma^2(1-l)^{1-\alpha}\left[(\tau_A^p)^\eta - \tau_A^p\right]\right] l^\delta\left(\frac{\delta}{l}+\frac{\alpha}{1-l}\right) + \sigma^2((1-\alpha)l^\delta(1-l)^{-\alpha}\left[(\tau_A^p)^\eta - \tau_A^p\right] - \tau^k)\right)$$

which is positive for $r_s > 0$ and $1 > 2\sigma^2(1-l)^{1-\alpha}\left((\tau_A^p)^\eta - \tau_A^p\right)$.The limits of Δg^* with respect to $l \to 0$ and $l \to 1$ are given by:

$$\lim_{l\to 0} \Delta g = -\beta - \left[1-\sigma^2\left((\tau_A^p)^\eta - \tau_A^p\right)\right](1-\alpha)\left((\tau_A^p)^\eta - \tau_A^p\right) < 0 \qquad \text{and} \qquad \lim_{l\to 1} \Delta g = \infty \qquad \square$$

Proof of Proposition 2

$$\frac{\mathrm{d}l}{\mathrm{d}\tau^k} = \frac{D}{C} > 0 \tag{57}$$
$$\frac{\mathrm{d}l}{\mathrm{d}\tau^p} = \frac{E}{C} \gtreqless 0 \qquad \text{for} \qquad \tau_A^{p*} \gtreqless \tau_A^p \tag{58}$$

with

$$C = (1-\alpha)(1-l)^{-\alpha}\left[(\tau_A^p)^\eta - \tau_A^p\right]\left(\left[1-2\sigma^2(1-l)^{1-\alpha}\left[(\tau_A^p)^\eta - \tau_A^p\right]\right](1-\alpha)\right.$$
$$+ \left[1-\sigma^2(1-l)^{1-\alpha}\left[(\tau_A^p)^\eta - \tau_A^p\right]\right] l^\delta\left(\frac{\delta}{l}+\frac{\alpha}{1-l}\right)$$
$$\left. + \sigma^2((1-\alpha)l^\delta(1-l)^{-\alpha}\left[(\tau_A^p)^\eta - \tau_A^p\right] - \tau^k)\right) > 0$$

$$D = \left[1-\sigma^2(1-l)^{1-\alpha}\left((\tau_A^p)^\eta - \tau_A^p\right)\right] > 0$$

$$E = -\left[\eta(\tau_A^p)^{\eta-1} - 1\right](1-l)^{1-\alpha} \times$$
$$\times \left[(1-\alpha)\left[1-2\sigma^2(1-l)^{1-\alpha}\left[(\tau_A^p)^\eta - \tau_A^p\right]\right]\left(\frac{l^\delta}{1-l} - 1\right) + \sigma^2\tau^k\right] \gtreqless 0 \quad \text{for} \quad \tau_A^{p*} \gtreqless \tau_A^p$$

From (51) it follows that $(1-\alpha)l^\delta(1-l)^{-\alpha}\left[(\tau_A^p)^\eta - \tau_A^p\right] > \tau^k$. The signs of C and E again depend on $1-2\sigma^2(1-l)^{1-\alpha}\left((\tau_A^p)^\eta - \tau_A^p\right)$. A sufficient condition for C and E to be positive (as postulated in (57) and (58)) is $1 > 2\sigma^2(1-l)^{1-\alpha}\left((\tau_A^p)^\eta - \tau_A^p\right)$. In case this condition does not hold, i.e., if the stochastic part dominates the deterministic part, the sign of C becomes ambiguous while the sign of E switches.