



**Doping, the Inspection Game,
and Bayesian Monitoring.**

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Abstract

Doping tests create a signal of whether the athlete has acted fraudulently. If the signal is costly, but perfect, then the doping enforcer and the athlete play an “inspection game,” which has no equilibrium in pure strategies. This paper presents a modification of that game: The “Bayesian monitoring” model rests on the assumption that signals are available without cost, but vulnerable to two types of errors. Both the inspection game and the new model assume that the enforcer is interested in fostering compliant behavior and making correct decisions. While the inspection game has only one mixed strategy equilibrium, three perfect Bayesian equilibria exist under Bayesian monitoring (one in pure strategies, two in mixed). These outcomes can be described with respect to their punishment styles: tyrannic, draconian, and lenient. The equilibrium probability of compliant behavior is lowest under a tyrannic regime, and highest under a lenient regime. Total deterrence of doping behavior is impossible. An increase of punishment does not increase the probability of compliant behavior.

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1 Introduction

This paper analyzes the enforcement of compliant (“good”) behavior by benevolent but imperfect decision-making enforcers. Doping is defined as “bad” behavior as it causes a negative externality which exceeds the increased benefit internalized by the offender; it therefore implies a welfare loss by assumption. Thus, good behavior is efficient. The “benevolence” of the doping enforcer is characterized by two assumptions. First, he prefers higher social welfare and, hence, is interested in avoiding welfare losses and encouraging good behavior. Second, he prefers correct judgments over incorrect ones.¹ The first assumption would be justified when the compensation

¹Similar assumptions can be justified with regard to judges, see Posner (1994, 23f.) who argues that judicial decisions may, among other factors, be governed by intrinsic motivation; see also Macey (1994), Alexander (1994), and Anderson/Shugart/Tollison (1989).

of the enforcers is linked to the social product generated in the sports industry. The second assumption reflects their preference for justice.²

Another property that characterizes the enforcer in this paper is imperfect decision-making.³ I assume that doping officers are not perfectly informed about the actual behavior of the athlete, although they are not totally blind. Through examination of a case, and in particular through doping tests, they generate an informative signal which is correlated with what the athlete has actually done.⁴

In a world with perfect and costless signals on the one hand, and benevolent enforcers on the other hand, doping would hardly exist. The explanation of non-compliant behavior, thus, requires to relax at least one of these three assumptions. The incentive problem is not a topic covered by this paper, which rather focuses on the signal quality and on the cost of the signal. Moreover, it focuses on the game between the athlete and the doping enforcer instead of modeling games between athletes.⁵

If the signal produced by a doping test is perfect, but costly, the athlete and the enforcer would play an “inspection game.” This well explored game is briefly analyzed in Section 3.

In my model, however, I assume the signal to be costless.⁶ Introducing moderate monitoring costs would complicate the equilibrium analysis without altering the qualitative results. However, the signal is imperfect, as two types of errors may arise: false convictions or false acquittals.⁷ The error probabilities provide a measure for the enforcer’s monitoring (or: detection) skill.⁸ Just as it is the case in the “inspection game,” the Bayesian monitoring model assumes the absence of incentive problems; thus, erroneous decisions of the doping enforcer are only due to information problems.

In section 4, I set up the Bayesian monitoring model. It reflects the fact that the interaction between the doping enforcer and the athlete often has

²Feddersen/Pesendorfer (1998, 24) assume a negative judicial payoff component for wrong judgments.

³See Kirstein/Schmidtchen (1997).

⁴The ability to base the decision between punishment or acquittal on an informative signal can also be interpreted as judicial competence; see Hadfield (1994).

⁵Berentsen (2002) sets up a game between athletes that only has a mixed strategy equilibrium, and draws regulatory conclusions. In Haugen (2004), the game is set up as a prisoners’ dilemma; Eber (2008) incorporates fair play norms into this game.

⁶According to Feess/Schumacher (2006), costless auditing may reduce welfare, as it may crowd out the supervised firm’s own monitoring effort. With zero monitoring costs, my model is even closer to an ideal world than the one in Usman (2002).

⁷See Tullock (1980), (1994), Kirstein/Schmidtchen (1997), or Jost (1997) on basic models of binary judicial decision-making under two error types.

⁸See Heiner (1983, 1986).

a sequential nature, even if the action of the athlete cannot be perfectly observed by the enforcer. Both the inspection game as well as models in the tradition of Becker (1968) disregard this fact.⁹ This approach disregards the strategic interaction between potential offender and a rational enforcer who would update his beliefs in a Bayesian way. The literature on contract theory acknowledges the problem of rational updates of monitoring agents.¹⁰

In section 4.2, I will derive the three perfect Bayesian equilibria. In none of these equilibria is the athlete motivated to choose compliant (“good”) behavior with certainty. The intuition for this result is that a rational enforcer would not base his decision on an imperfect signal if he expects the athlete to have chosen good behavior. Bayesian updating should induce the enforcer to not punish the athlete, even if the signal says otherwise. The athlete’s best reply to a sure acquittal, however, would be to commit bad behavior (in other words: doping).

Just as in the inspection game, the enforcer is unable to induce good behavior of the athlete with probability one. This result holds even though it is assumed that the incentives of enforcers are perfectly aligned, and the doping test is costless.

2 Related literature

The enforcement model of Becker (1968) focuses in a price-theoretical manner on the decision-making of the potential offender, taking the parameters of the enforcement system as exogenous. If the size of the sanction is exogenously given, as well as the probability with which non compliant behavior is punished, then deterrence is just a matter of the expected sanction. If it exceeds the expected benefit from wrongdoing, then the risk-neutral potential offender is deterred.

Expected sanction is the product of enforcement probability and absolute sanction. Usually, it is costly to increase the probability, while it is possible to increase monetary sanctions without cost. Hence, it would be cost-minimizing for society to set the absolute fine as high as possible, and to reduce the enforcement probability such that the expected fine just exceeds the athlete’s expected benefit. This is a brief version of Becker’s “maximum fine” result. In the context of doping, this result would imply that the doping enforcer can save resources by reducing the probability of testing if the

⁹See Polinsky/Shavell (2000) for a survey of the traditional enforcement model.

¹⁰See, e.g., Baiman/May/Mukherji (1990), Cheng (1990), Mirrlees (1999), Faure-Grimaud/Laffont/Martimort (2003).

monetary sanction case of a positive result is maximal (the maximum would be the total lifetime income of the athlete).

This result has met several objections. Among them is the empirical observation that increased fines do not really increase deterrence (whereas the probability of detection seems to have a significant influence on potential offenders).¹¹ Moreover, the social cost of wrong convictions may be enormous if fines are maximal. Another objection refers to a hidden assumption of the Becker model: expected fines deter from wrongdoing only if the enforcer is committed to his sanction strategy. This is, however, not always the case in reality, and in particular not in the context of sports and doping.¹² Without a commitment to enforcement probability, enforcer and athlete play an inspection game, see Section 3.

A monitoring problem that is very similar to the problem highlighted in my Bayesian monitoring model has been discussed (in a rather informal manner) by Schwartz (1995) and Shavell (1995, 1996) with respect to appeals courts. Schwartz has pointed out that the analysis of appeals courts in Shavell (1995) is based on the assumption that the judges do not draw Bayesian inferences from the fact that an appeal has been brought. If judges, however, are modeled as strategic actors, then a separating equilibrium (according to which only legitimate appeals are brought), cannot prevail. The reply of Shavell (1996) to Schwartz mixes up normative and positive analysis: As the judge's Bayesian update would lead to undesirable results, it is not desirable that he use it.¹³ To sustain his view, Shavell points out that judges have to base their decision on written opinions, and Bayesian inference would be unacceptable as a written opinion. However, the verbal representation of the results of judicial reasoning does not necessarily have to reflect the line of reasoning. Decision theory predicts that rational judges use Bayesian inference when making their decisions; an experienced judge will certainly be able to find an acceptable wording for any opinion he has formed this way. Certainly, no judge will come to a decision only after he has written down the reasons for it.¹⁴

My model differs from the one in Reinganum/Wilde (1986), which is

¹¹See Ben-Shahar and Harel (1995).

¹²Eber (2002) has pointed out that anti-doping policy faces a credibility problem very similar to that identified for the conduct of the monetary policy.

¹³In his earlier article, Shavell has already stressed this point: "...such information should not be considered in judging..." see Shavell (1995, 363).

¹⁴Modern literature on auditing takes it as self-evident that regulators are not committed to a specific reaction to the signals they receive; see, e.g., Khalil (1997, 639) or Baliga/Corchon/Sjöström (1997).

a signaling game and not a monitoring game. In their model, the athlete knows his type and chooses a report which is sent to the enforcer after having observed the good signal realization in the lenient equilibrium. In a separating equilibrium, the enforcer can infer the athlete's type from the report. This signaling model (as well as the inspection game) addresses the enforcer's commitment problem. The traditional enforcement theory assumes that commitment to a specific enforcement probability is possible.¹⁵ If, however, an enforcer is unable to commit to a specific monitoring strategy, then perfect enforcement cannot be part of an equilibrium as the enforcer always has an incentive to not monitor when he expects the athlete to comply with certainty.¹⁶

Another model which is different from the one presented here is analyzed in Berentsen/Brügger/Lörtschler (2008). They start with an interaction between two athletes. The result of this interaction provides a signal to an enforcer who then decides whether to monitor or not. Their model acknowledges the existence of monitoring costs but assumes the monitor's signal to be perfect. Thus, their model follows the tradition of the "inspection game" and leaves out of focus the imperfectness of doping tests (operationalized by its probabilities of error). These parameters play a central role in my model.

Of the three Perfect Bayesian equilibria derived in section 4.2, one is an equilibrium in pure strategies. This equilibrium is identical to the one of the corresponding simultaneous game, as it was shown in Bagwell (1995).¹⁷ The existence of a pure strategy equilibrium is a crucial difference from the original inspection game. If the enforcer punishes regardless of the signal, then the athlete's best choice (anticipating the decision of the enforcer) is to commit bad behavior.

The other two equilibria involve mixed strategies. In one of these equi-

¹⁵See, among the many contributions to this literature, Becker (1968) or Reinganum/Wilde (1985); also see Graetz/Reinganum/Wilde (1985).

This assumption has been challenged by Baker/Miceli (2005) who find that apprehension rates are lower if the enforcer can commit to a punishment strategy and only uses fines. Commitment, however, may lead to higher apprehension rates if both fines and jail are used.

¹⁶Also see Khalil/Lawarrée (1995, 443). Melumad/Mookherjee (1989) demonstrate that it can pay for a benevolent government to commit to an audit strategy by delegation to an independent audit authority (instead of using a direct commitment device while retaining the audit authority).

¹⁷van Damme/Hurkens (1995) have shown that, if one allows for mixed strategies, then the equilibrium converges towards the pure strategy equilibrium of this simultaneous game if the error probabilities decrease. These two papers, however, address the commitment problem of the first mover, while the starting point of my paper is the inability of the monitor, who moves second, to commit to a monitoring strategy.

libria, the enforcer punishes with certainty if he receives the bad signal, and with positive probability if the signal indicates good behavior. Such a “draconic” strategy results in a low likelihood of good behavior on the athlete’s side. In the other mixed strategy equilibrium, the enforcer does not punish after receiving the good signal, but does punish with positive probability if the signal is bad. This “lenient” strategy induces the athlete to choose good behavior with a higher probability than under the draconic strategy.

Comparative statics analysis demonstrates that the athlete’s equilibrium probability of choosing good or bad behavior is independent of his own pay-off parameters. The size of the sanction and the athlete’s benefit from a transgression only influence the strategy choice of the monitor (the inspection game leads to the same comparative static results). These results are in clear contradiction to the predictions of the Becker model, which gives scope to empirical testing.

3 Inspection game between enforcer and athlete

In an inspection game between doping enforcer and an athlete, the former can perfectly observe the behavior of the latter if monitoring costs are borne. Enforcer and athlete decide simultaneously: the enforcer chooses whether or not to administer a doping test, without knowing the athlete’s choice. The athlete chooses whether to use forbidden substances (“bad”) or not (“good”), without knowing the enforcer’s choice when making his own decision.

Assume that the enforcer prefers to inspect if he expects the athlete to dope. If, however, the enforcer expects the athlete to choose good behavior, he would rather save the inspection costs. Assume furthermore that the athlete prefers good behavior when inspected, and bad behavior should the enforcer abstain from inspection. Under these assumptions, the simultaneous one-shot game has no Nash equilibrium in pure strategies. It only has a unique equilibrium in mixed strategies.¹⁸ Thus, perfect deterrence can never be part of a Nash equilibrium.

Formally, the inspection game is played between two risk-neutral and rational players, a athlete (S) and a doping enforcer (E), who play a simultaneous one-shot game. The athlete chooses good behavior ($d = g$) with

¹⁸See Wittman (1985) or Tsebelis (1989), (1990a), (1990 b). Holler (1990) has questioned the usage of mixed strategies as the solution concept and proposed Maximin strategies as more profitable for the players. Wittman (1993), however, has argued that Maximin strategies do not constitute a Nash equilibrium and, therefore, lack stability.

probability γ , or bad behavior ($d = b$) with $1 - \gamma$. Assume that bad behavior incurs an internalized benefit, denoted as $B > 0$, and a negative externality $X > B$, while the cost and benefit of good behavior are normalized to zero.

The enforcer decides whether to inspect the athlete (with probability δ) or not (with $1 - \delta$). If the enforcer chooses inspection, he has to bear cost C with $X > C > 0$. If the athlete has chosen bad behavior (“doping”), this will be detected with certainty, and a sanction $P > B$ will be imposed. Punishment burdens the athlete, but yields no benefit for the enforcer, but detection of bad behavior reverses the negative externality. The enforcer is assumed to be benevolent, i.e., he prefers to sanction correctly. Let $W > P$ represent the initial wealth of the athlete.¹⁹

Table 1: Inspection game and Becker model

	E	inspect, δ	not, $1 - \delta$
S			
good, γ		$-C$	$\mathbf{0}$
bad, $1 - \gamma$		$\mathbf{0}$	0
		$-C$	$-X$
		B-P	\mathbf{B}

Obviously, the inspection game depicted in table 1 has no Nash equilibrium in pure strategies. The mixed strategy Nash equilibrium is given by:

$$\gamma^* = \frac{X - C}{X} \text{ and } \delta^* = \frac{B}{P}.$$

Closer inspection of γ^* shows that it is decreasing in the externality X and the monitoring cost C , but unaffected by the value of P . Hence, an increase of the absolute sanction has no impact on the athlete’s behavior. The only effect of an increase of the sanction would be less frequent inspections (as δ^* decreases in P). Hence, there is no marginal deterrence effect of punishment. More general: any player’s equilibrium behavior is independent of his own payoff parameters. This is a standard result of inspection games.²⁰

¹⁹The initial wealth plays no role in the subsequent equilibrium analysis, but eliminates a potential problem of limited liability (“judgment proofness”).

²⁰See Wittman (1985), Bianco, Ordeshook, and Tsebelis (1990) or Andreozzi (2004).

In a model following Becker (1968), the enforcer would be committed to a specific value of δ (and of P , just as in the inspection game). Then, the decision-situation of S would not have a simultaneous, but rather a sequential nature: First, society decides upon δ and P , then the athlete chooses his behavior.²¹ The athlete would be deterred if, and only if, δ and P were fixed such that $B < \delta P$. Without commitment from the enforcer, however, players S and E would be in a simultaneous game. Then, this inequality would govern E's behavior, as derived above, rather than the choice of S.

4 Bayesian Monitoring

4.1 Assumptions

Now assume that the interaction between athlete and the doping enforcer has a sequential nature. After the athlete has made his choice whether or not to use illegal substances, nature produces an informative signal, denoted as i . The signal may assume one out of two realizations, $i = g$ or $i = b$. The probability of receiving a particular signal realization is contingent on the athlete's choice: $r = \Pr(i = g|d = g)$, $w = \Pr(i = g|d = b)$ with $1 \geq r > w \geq 0$.²² The parameters r and w provide a measure for the monitoring skill of the enforcer:

- With $r = 1$ and $w = 0$, he monitors perfectly,
- with $r = w$, he has zero monitoring skill;
- the intermediate case $0 < w < r < 1$ reflects imperfect, but positive monitoring skill.

The enforcer is unable to observe the athlete's actual choice. He only observes the imperfect signal and updates his beliefs using Bayes' rule. Denote his ex-post beliefs as $\mu = \Pr(d = g|i = g)$, $\nu = \Pr(d = g|i = b)$. The enforcer then has to decide between a sanction ($j = s$) or an acquittal ($j = a$). I define the enforcer's behavioral strategies $\alpha = \Pr(j = s|i = g)$, $\beta = \Pr(j = s|i = b)$.

²¹In the Becker model, $\delta < 1$ can be due to a lack of detection technology, but may also be caused by non-aligned incentives (which is excluded in the inspection game).

²²The parameter configuration $r \leq w$ would also reflect a correlation between the signal realization and the athlete's choice. This symmetric case can, therefore, be excluded from consideration. The extreme cases $r = w$ and $r = 1, w = 0$ will be dealt with later.

Just as in the inspection game, the enforcer is assumed to be benevolent, i.e., he has an interest in sanctioning correctly. However, the enforcer cannot directly observe the actual behavior of the athlete, but only the imperfect signal. This may induce the doping enforcer to occasionally punish wrongfully. Benevolence in this context is represented by his payoff parameters $G > 0$ and $L > 0$. G reflects the officer's interest in a correctly issued sanction, i.e., if the athlete has indeed chosen bad behavior. On the other hand, $-L$ is the officer's loss from issuing an incorrect sanction (if the athlete's behavior actually was good). These payoffs are revealed only after the decision of the enforcer has taken place.

I assume sanctions to be inefficient, i.e., $P > G$: the enforcer's benefit from a justified sanction is smaller than the burden imposed on the athlete. Without this assumption, punishment would generate a welfare gain $G - P$. This gain might even outweigh the harm from "bad" deeds - under such circumstances, it would be welfare enhancing to encourage (and sanction) wrongdoing. All other payoff components are just the same as in the inspection game solved above.

Figure 1: Bayesian monitoring game

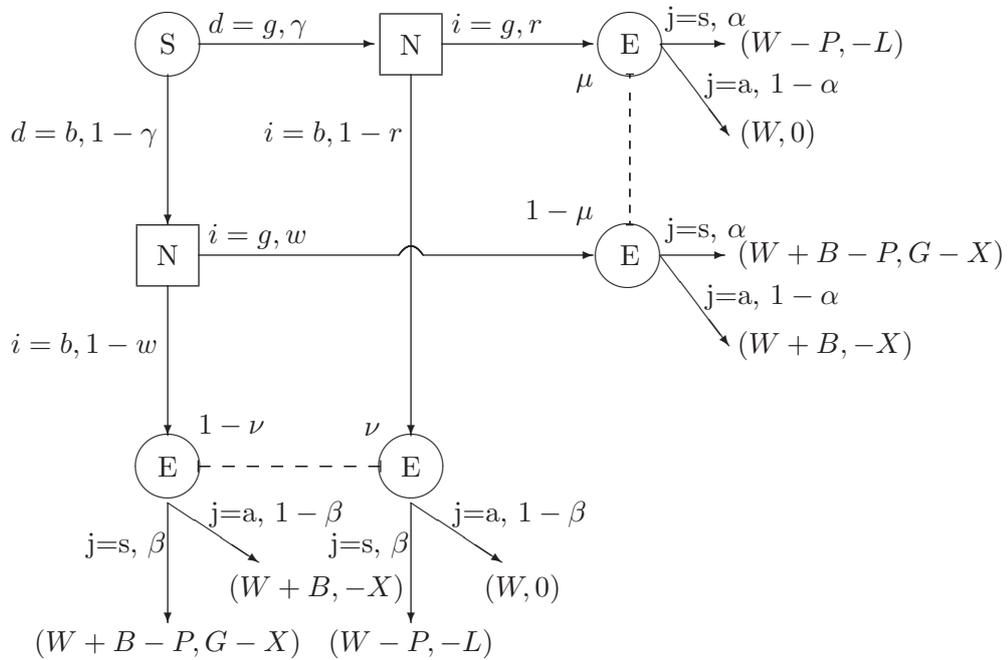


Figure 1 displays the interaction of the two players and the signals generated in the Bayesian Monitoring Game. Both players are assumed to be risk-neutral. The payoff parameters (W, P, B, X, G, L) and the signal quality parameters (r, w) are exogenously given and are common knowledge. Thus, the endogenous variables are $\alpha, \beta, \gamma, \mu$, and ν .

4.2 Equilibrium analysis

In this section, the Bayesian Nash equilibria $\{(\alpha^*, \beta^*); (\mu^*, \nu^*); \gamma^*\}$ will be derived. α^*, β^* and γ^* denote the enforcer's and the athlete's behavioral strategies in equilibrium, respectively, while μ^* and ν^* denote the enforcer's equilibrium beliefs.

4.2.1 The athlete's optimal choice

The athlete chooses his behavioral strategy γ so as to maximize his expected payoff, given the behavioral strategies (α, β) which he expects the enforcer to play. Thus, an equilibrium value γ^* maximizes

$$\{W - \gamma[r\alpha + (1-r)\beta]P + (1-\gamma)B - (1-\gamma)[w\alpha + (1-w)\beta]P\}. \quad (1)$$

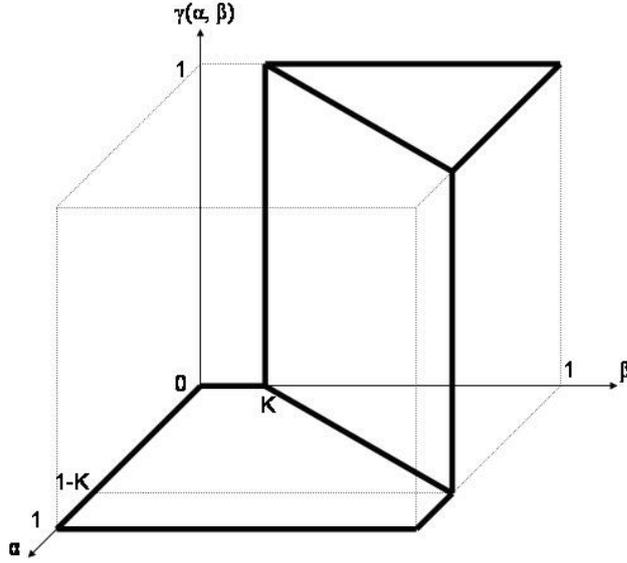
The first derivative with respect to γ is: $-P[r\alpha + (1-r)\beta] - B + P[w\alpha + (1-w)\beta]$. This expression can be rearranged to $P(r-w)(\beta - \alpha) - B$, which allows me to write the athlete's reaction function $\gamma^*(\alpha, \beta)$ as

$$\begin{aligned} \gamma^* = 1 &\Leftrightarrow \beta - \alpha > K \\ 0 < \gamma^* < 1 &\Leftrightarrow \beta - \alpha = K \\ \gamma^* = 0 &\Leftrightarrow \beta - \alpha < K \end{aligned}$$

with $K = \frac{B}{(r-w)P}$.²³ If $\gamma^* = 0$ or $\gamma^* = 1$, then the athlete chooses a pure strategy, while $0 < \gamma^* < 1$ represents the choice of a mixed strategy. Figure 2 illustrates the athlete's reaction function $\gamma^* = \gamma^*(\alpha, \beta)$. The bold pentagon in the lower left corner of the figure includes the (α, β) combinations to which the athlete's optimal reaction is $\gamma = 0$. The bold triangle in the upper right corner is situated above the (α, β) combinations which induce the athlete to choose $\gamma = 1$. Finally, the bold square in the center of the figure describes the (α, β) combinations that keep the athlete indifferent between all of his mixed strategies.

²³Division by $(r-w)$ is possible as $r = w$ is excluded.

Figure 2: Athlete's reaction function



4.2.2 Optimal choice of the doping enforcer

A rational doping enforcer chooses his action given his anticipation of the behavioral strategy chosen by the athlete. From a strategic point of view, the game is simultaneous, because no player can observe the respective other player's choice when making their own decision. However, the enforcer observes an imperfect signal triggered by the athlete's actual behavior. Therefore, he can make a decision contingent on the realization of this signal.

First, I examine the enforcer's optimal choice after having observed the signal realization $i = g$. In this case, Bayesian updating leads to the following ex-post belief:

$$\mu = \frac{r\gamma}{r\gamma + w(1 - \gamma)} \quad (2)$$

Taking these ex-post beliefs into account, the enforcer chooses his behavioral strategy α^* so as to maximize $[(1 - \mu)(\alpha G - X) - \mu\alpha L]$. The first derivative with respect to α is $G - \mu(G + L)$.²⁴ The relation between the

²⁴An interesting aspect of the analysis is that the negative externality X caused by the athlete's behavior is irrelevant for the enforcer's decision of whether to issue a sanction or

optimal α and the athlete's strategy choice γ becomes clear if equation (2) is used to substitute for μ . Then, the first derivative can be rewritten as

$$G - \frac{r\gamma}{r\gamma + w(1 - \gamma)}(G + L).$$

This is negative if, and only if,²⁵

$$\gamma > \frac{wG}{rL + wG}.$$

For simplicity, denote the right hand side of this inequality as γ_1 . The relation between the athlete's choice γ and the behavioral strategy of the enforcer after having observed $i = g$ is summarized in the following reaction function $\alpha^* = \alpha^*(\gamma)$:

$$\begin{aligned} \alpha^* = 1 &\Leftrightarrow \gamma < \gamma_1 \\ 0 < \alpha^* < 1 &\Leftrightarrow \gamma = \gamma_1 \\ \alpha^* = 0 &\Leftrightarrow \gamma > \gamma_1 \end{aligned}$$

given $w > 0$. If $w = 0$, the first derivative is negative and, thus, the enforcer chooses $\alpha^* = 0$. It would never observe the good signal incorrectly and, thus, never reacts with a sanction.

In the same vein, the optimal behavioral strategy $\beta^*(\gamma)$ can be derived for the case in which the enforcer observes the signal realization $i = b$. Bayesian updating induces the ex-post belief

$$\nu = \frac{(1 - r)\gamma}{(1 - r)\gamma + (1 - w)(1 - \gamma)}.$$

and the enforcer chooses β so as to maximize $[(1 - \nu)(\beta G - X) - \nu\beta L]$. The first derivative with respect to β is $G - \nu(G + L)$. Substitution of ν allows to state the reaction function $\beta^* = \beta^*(\gamma)$ as

$$\begin{aligned} \beta^* = 1 &\Leftrightarrow \gamma < \gamma_2 \\ 0 < \beta^* < 1 &\Leftrightarrow \gamma = \gamma_2 \\ \beta^* = 0 &\Leftrightarrow \gamma > \gamma_2 \end{aligned}$$

not. If the athlete has chosen bad behavior, X is sunk. From then on, only G and L and the enforcer's beliefs determine whether he chooses to punish or not.

²⁵The next inequality is equivalent to the "reliability condition" in Heiner (1983), (1986). See also Swets (1996).

with

$$\gamma_2 = \frac{(1-w)G}{(1-r)L + (1-w)G}.$$

Note that $r > w$, $G > 0$ and $L < \infty$ imply $\gamma_2 > \gamma_1$. Figure 3 displays the signal-contingent reaction functions of the enforcer for the case of $\gamma_1 > 0$ and $\gamma_2 < 1$. The reaction function $\alpha^*(\gamma)$ is represented by the dashed line. The straight line represents the reaction function $\beta^*(\gamma)$.

Figure 3: Reaction functions $\alpha^*(\gamma)$ and $\beta^*(\gamma)$ of the enforcer

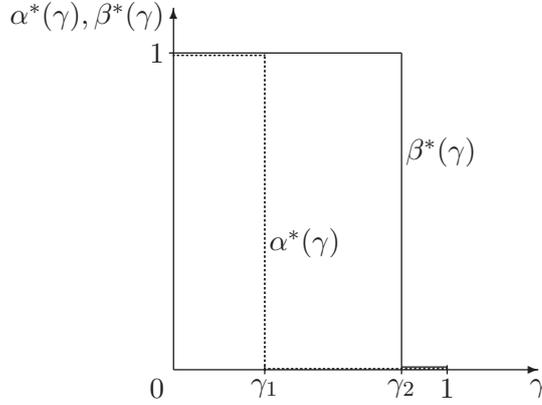


Figure 3 demonstrates that there is no value of γ for which $\alpha^*(\gamma) > \beta^*(\gamma)$ holds. Furthermore, for $\gamma = \gamma_1$ or $\gamma = \gamma_2$, exactly one of the enforcer's optimal behavioral strategies α^* or β^* is mixed while the other one is pure. These observations are important for the subsequent equilibrium analysis.

4.2.3 Bayesian Monitoring Equilibria

In the previous section, I have derived the reaction functions $\alpha^* = \alpha^*(\gamma)$ and $\beta^* = \beta^*(\gamma)$ of the enforcer, as well as the athlete's reaction function $\gamma^* = \gamma^*(\alpha, \beta)$. An equilibrium combination of behavioral strategies is given by $\alpha^*(\gamma^*), \beta^*(\gamma^*)$, and $\gamma^*(\alpha^*, \beta^*)$. The following proposition presents the (behavioral) strategy combinations which characterize the three perfect Bayesian equilibria, henceforth denoted as “Bayesian monitoring equilibria.”

Proposition 1: In the Bayesian Monitoring Game with $G, L > 0$ and $0 < w < r < 1$, three perfect Bayesian equilibria exist which are characterized by the following combinations of behavioral strategies:

i) **tyrannic:** $\gamma = 0$ and $\alpha = \beta = 1$,

ii) **draconian:** $\gamma = \gamma_1$ and $\alpha = 1 - K, \beta = 1$,

iii) **lenient:** $\gamma = \gamma_2$ and $\alpha = 0, \beta = K$,

$$\text{with } K = \frac{B}{(r-w)P}, \gamma_1 = \frac{wG}{rL+wG},$$

$$\text{and } \gamma_2 = \frac{(1-w)G}{(1-r)L+(1-w)G}.$$

Proof: i) According to the reaction function of S, his best reply to any $\beta - \alpha < K$ (in particular to $\alpha = \beta$) would be $\gamma = 0$ which is smaller than $\gamma_1 < \gamma_2$ (for $wG > 0$ and $r > w$). A's best reply to $\gamma < \gamma_1$ would be $\alpha = \beta = 1$.

ii) If E chooses α and β such that $\beta - \alpha = K$, then S is indifferent between his pure strategies. Should the enforcer choose $\gamma = \gamma_1$, his best reply would be $\beta = 1$ and, consequently, $\alpha = 1 - K$.

iii) If $\beta - \alpha = K$, and S chooses $\gamma = \gamma_2$, the best reply of E would be $\alpha = 0$ and, consequently, $\beta = K$.

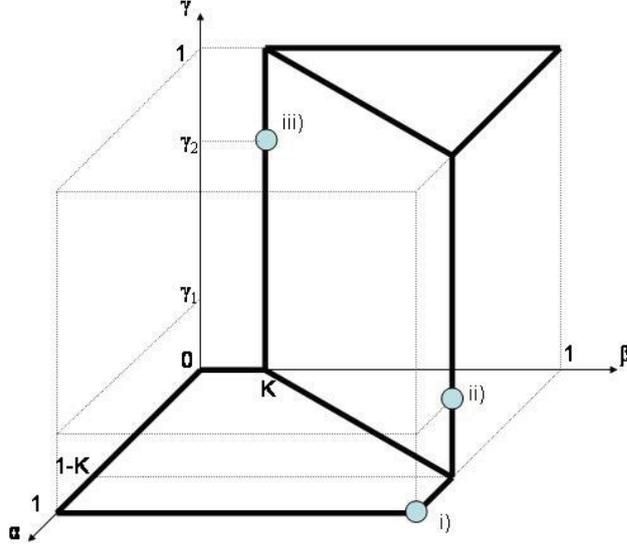
Other choices of γ do not lead to an equilibrium:

- If $0 < \gamma < \gamma_1$ then A's best reply is $\alpha = \beta = 1$, to which the best reply of S is $\gamma = 0$, hence this is not part of an equilibrium.
- If $\gamma_1 < \gamma < \gamma_2$ then A's best reply is $\alpha = 0$ and $\beta = 1$, to which the best reply of S is $\gamma = 1 > \gamma_2$, implying no equilibrium.
- If $\gamma_2 < \gamma \leq 1$ then A's best reply is $\alpha = \beta = 0$, to which the best reply of S is $\gamma = 0 < \gamma_2$, thus here is no equilibrium.

The tyrannic equilibrium is the only one in pure strategies. The draconian equilibrium is characterized by a low rate of compliant (“good”) behavior, as the athlete faces positive probability of being sanctioned after both signal realizations. In the lenient equilibrium, the threat of sanctions is rather moderate (only after the bad signal realizations, and then with a probability smaller than one), and the rate of good behavior is the highest. In Figure 4 the three equilibria are shown as bold dots.

The first result, labeled i), is rather straightforward: if the athlete chooses bad behavior, and the enforcer punishes regardless of the signal realization, both parties will be confirmed in their decisions and beliefs by the respective other party's behavior. In this equilibrium, the ex-post beliefs

Figure 4: Bayesian monitoring: equilibria



of the enforcer are $\mu = \nu = 0$. This is a surprising result, if compared to the inspection game which does not exhibit a pure strategy equilibrium.

The lenient equilibrium iii) establishes that it can be rational for a athlete to frequently choose good behavior under costless monitoring if the enforcer never punishes after having observed $i = g$ and only occasionally after $i = b$. In this equilibrium, A's ex-post beliefs are

$$\mu = \frac{r\gamma_2}{r\gamma_2 + w(1 - \gamma_2)} = \frac{r(1 - w)G}{r(1 - w)G + w(1 - r)L}$$

and

$$\begin{aligned} \nu &= \frac{(1 - r)\gamma_2}{(1 - r)\gamma_2 + (1 - w)(1 - \gamma_2)} \\ &= \frac{(1 - r)(1 - w)G}{(1 - r)(1 - w)G + (1 - w)(1 - r)L} = \frac{G}{L + G}. \end{aligned}$$

The equilibrium belief ν is independent of (r, w) , as the equilibrium choice of β neutralizes the signal's imperfection.

Result ii) has a rather strange property. The enforcer can induce the athlete to choose good behavior with probability γ_1 if he always punishes him after having observed the bad signal realization ($\beta^* = 1$), and he also punishes with a positive probability after having observed the good signal realization ($\alpha^* > 0$). However, the resulting probability of good behavior is lower than in the lenient equilibrium. In both mixed strategy equilibria, the monitor is required to choose $\beta > \alpha$. If the distance between these two signal-contingent punishment probabilities is calibrated adequately, then the athlete is indifferent between her pure strategies. This effect can either be attained by choosing $\alpha = 0$ and an adequately high $\beta < 1$, or by choosing $\beta = 1$ and an adequately low $\alpha > 0$. The monitor's ex-post beliefs in the second mixed strategy equilibrium are

$$\mu = \frac{r\gamma_1}{r\gamma_1 + w(1 - \gamma_1)} = \frac{rwG}{rwG + wrL} = \frac{G}{G + L}$$

and

$$\nu = \frac{(1 - r)\gamma_1}{(1 - r)\gamma_1 + (1 - w)(1 - \gamma_1)} = \frac{(1 - r)wG}{(1 - r)wG + (1 - w)rL}.$$

Now, it is the equilibrium choice of α that makes the ex-post belief μ independent of the signal quality. If a game exhibits multiple equilibria, using it as a positive tool would require criteria for equilibrium selection. However, it is beyond the scope of this paper to discuss this question in more detail.²⁶ The impossibility of implementing good behavior with certainty is directly implied by Proposition 1:

Corollary: In the Bayesian Monitoring Game with $0 < w < r < 1$, $G > 0$, and $L > 0$, the athlete's strategy $\gamma = 1$ is not part of a perfect Bayesian equilibrium.

This result formalizes what was mentioned verbally in the introduction: The enforcer could induce the athlete to choose $\gamma = 1$ by setting $\alpha = 0$ and $\beta = 1$. However, A's best reply to $\gamma = 1$ would be ($\alpha = \beta = 0$), but not ($\alpha = 0, \beta = 1$): If A expects the athlete to choose good behavior with certainty, then it would be irrational to rely on an imperfect signal when deciding whether or not to punish. To follow the signal would imply that the athlete might falsely be punished with a positive probability (namely w). The enforcer can, however, avoid the wrongful punishment with certainty

²⁶See Harsanyi/Selten (1988) on this topic.

if he simply disobeys the signal realization and refrains from punishment ($\alpha = \beta = 0$). However, this induces the athlete to choose bad behavior. Note that this result occurs even though the enforcer is assumed to be benevolent (i.e. prefers correctly issued sanctions) and the monitoring signal is costless. The reason for this result is the assumption of imperfectness of the monitoring signal.

4.3 Discussion

4.3.1 Comparative statics of the lenient equilibrium

The equilibrium analysis reveals that the enforcer's behavior does not depend on his own payoff parameters. The only parameters which influence his behavioral strategies in the equilibria are B, P, r and w . In the **lenient equilibrium**, the probability of punishment after having observed the bad signal realization ($i = b$) is $\beta^* = K$ with $K = B/[(r - w)P]$. Comparative statics shows that

$$\frac{\partial \beta^*}{\partial B}, \frac{\partial \beta^*}{\partial w} > 0 > \frac{\partial \beta^*}{\partial P}, \frac{\partial \beta^*}{\partial r}.$$

Thus, a better signal quality (i.e., a higher r or a lower w), a lower athlete's benefit B , or a higher sanction P would decrease the probability of punishment after the enforcer has observed the bad signal realization in the lenient equilibrium. The athlete chooses good behavior with probability γ_2 , which depends on r, w, L and G . The signs of the partial derivatives are

$$\frac{\partial \gamma_2}{\partial G}, \frac{\partial \gamma_2}{\partial r} > 0 > \frac{\partial \gamma_2}{\partial L}, \frac{\partial \gamma_2}{\partial w}.$$

The probability of good behavior can be increased by increasing the enforcer's reward for correct decisions, by lowering his disutility from wrong decisions, or by increasing the signal quality. In the context of doping, this result implies: A higher test quality would induce S to comply with higher probability, whereas the enforcer would mete out his punishment with a lower probability. These intuitive results, however, are limited to the lenient equilibrium.

4.3.2 Comparative statics of the draconian equilibrium

The comparative statics of the results of the **draconian equilibrium** are different. Here, the enforcer chooses a mixed punishment strategy $\alpha^* = 1 - K$ after observing the good signal realization (and punishes with $\beta = 1$ after

having observed the bad realization). The signs of the partial derivatives are:

$$\frac{\partial \alpha^*}{\partial B}, \frac{\partial \alpha^*}{\partial w} < 0 < \frac{\partial \alpha^*}{\partial P}, \frac{\partial \alpha^*}{\partial r}.$$

The athlete chooses good behavior with probability γ_1 . Again, this probability only depends on r, w, L and G . The signs of the partial derivatives are

$$\frac{\partial \gamma_1}{\partial G}, \frac{\partial \gamma_1}{\partial w} > 0 > \frac{\partial \gamma_1}{\partial L}, \frac{\partial \gamma_1}{\partial r}.$$

In the draconian equilibrium, a higher signal quality, i.e., $dr > 0 > dw$, would increase the probability of wrongful punishment α^* , which is rather counter-intuitive. Moreover, higher signal quality would decrease the probability of good behavior γ_1 in this equilibrium. This is certainly not what enforcers try to implement.

4.3.3 Perfect or Uninformative Signals

The next result is concerned with two extreme cases of signal quality which were excluded in the above analysis: an uninformative and a perfect signal.

Proposition 2: Consider the Bayesian Monitoring Game with $G > 0$ and $L > 0$.

- i) If $r = w$, then the equilibrium behavioral strategies are $\gamma = 0$ and $\alpha = \beta = 1$.
- ii) If $r = 1$ and $w = 0$, then the subgame perfect equilibria are characterized by $\alpha = 0$, $\beta > B/P$, and $\gamma = 1$.

Proof: i) With $r = w$, the first derivative of the athlete's yield function (1) would be $-B < 0$, hence the corner solution is $\gamma^* = 0$. A's best reply to this is $\alpha = \beta = 1$.

ii) With $r = 1$ and $w = 0$, the derivative is $-P\alpha - B + \beta P = (\beta - \alpha)P - B$. If A chooses a behavioral strategy combination with $\beta > \alpha + B/P$, then this is strictly positive, driving S to choose $\gamma = 1$. With $r = 1$ and $w = 0$, this is equal to $\gamma_2 = G/G$. A's best reply to S choosing γ_2 is to choose $\alpha = 0$ and some $\beta \in [0, 1]$. Thus, $\alpha = 0$, $\beta > B/P$, and $\gamma = 1$ is a subgame perfect equilibrium.

The first part of this proposition considers an enforcer without monitoring skill. The second part addresses an enforcer of perfect monitoring skill,

who evaluates the case without errors ($r = 1$ and $w = 0$). This reduces the game to one with perfect information, a sequential version of the inspection game (with zero monitoring cost). A's best reply to the good signal realization is acquittal, while after observing the bad realization the athlete will be punished with positive probability. Draconic punishment, combined with $\gamma = 0$ would also be a Nash equilibrium of this game, but perfect compliance as described in the proposition is the only subgame perfect equilibrium.

Hence, a Bayesian doping enforcer can only induce the athlete to comply with certainty if the doping test is perfect. In the real world, such doping tests are unavailable. If, however, the two error types occur with positive probability, even if it is very small), then this leads to the situation described in Proposition 1: Three equilibria exist, in two of which the probability of good behavior is smaller than one (and zero in the third equilibrium). Small deviations from perfect doping tests, thus, may dramatically drive down the compliance rates.

4.3.4 Impact of Benevolence Payoffs

The derived results depend on the assumptions $G > 0$ and $L > 0$: The enforcer derives utility from correct judgments, and disutility from wrong ones. Without such incentives ($G = L = 0$), monitoring would have no impact on the athlete's behavior, as this would imply $\gamma_1 = \gamma_2 = 0$. However, it is sufficient for the derived results that G and L are positive, even if the values of these parameters are negligible compared to B , X , and P .

5 Conclusions

The results of the Bayesian monitoring model presented here are different from enforcement models in the tradition of Becker (1968) or the inspection game:

- While in a simple Becker model the doping enforcer would either produce perfect deterrence or no deterrence at all, total deterrence is never part of a Bayesian monitoring equilibrium.
- Furthermore, the maximum fine result of Becker (1968) is rejected by this model. In all three perfect Bayesian equilibria derived above, the probability of compliant behavior is smaller than one. Thus, if the doping test is characterized by positive probabilities of error, then it is unavoidable that doping occurs with positive probability.

- In the model presented here, there is no simple trade-off between the sanction and the probabilities with which punishment is issued. The athlete's incentives (and his mixed equilibrium strategy) are not influenced by the level of the sanction P . The only effect of a higher fine would be a reduction of the enforcer's probability of punishment after having observed the good signal realization in the lenient equilibrium, or an increase of the probability of punishment after having observed the bad signal realization in the draconic equilibrium.
- Other than in the inspection game, an equilibrium in pure strategies exists (with non-compliance and "tyrannic" punishment). In the inspection game, the doping test is costly and perfect, while in the monitoring game it is costless and imperfect.
- The equilibrium probability of good behavior is independent of the external damage, the initial wealth, and the internalized benefit from bad behavior. The athlete's equilibrium strategy only depends on the parameters which reflect the preference of the enforcer for correct judgments as well as the quality of the doping test.
- While a perfect doping test would induce the athlete to choose good behavior with certainty (if the enforcer is benevolent), even very small error probabilities may dramatically drive down the compliance rates: Three Bayesian equilibria exist, in which the probability of good behavior can be dramatically lower than one, or even zero.

Thus, even if the signal is costlessly available, and the enforcer is benevolent, perfect enforcement cannot be reached if the doping test commits errors. The best outcome which can be attained is a high probability of good behavior (in the lenient equilibrium). The Bayesian monitoring game can, therefore, provide a positive explanation for the occurrence of doping even in a close-to-ideal world.

A policy implication from these results would be that sport associations should employ means to facilitate equilibrium selection of the lenient monitoring style.²⁷ In this equilibrium, an investment in higher signal quality may improve athletes' compliance. In the draconian equilibrium, however, such an investment would be even counterproductive.

²⁷Equilibrium selection is left out of the focus of this paper; see, e.g., Harsanyi and Selten (1988) on this topic.

References

- Anderson, G.M., Shugart, W.H., Tollison, R.D., 1989. On the Incentives of Judges to Enforce Legislative Wealth Transfers. *Journal of Law and Economics* 32, 215-228.
- Andreozzi, L., 2004. Rewarding Policemen Increases Crime. Another Surprising Result From the Inspection Game. *Public Choice* 121, 69-82.
- Bagwell, K. 1995., Commitment and Observability in Games. *Games and Economic Behavior* 8, 271-280.
- Baiman, S., May, J.H., Mukherji, A., 1990. Optimal Employment Contracts and the Returns to Monitoring in a Principal-Agent Context. *Contemporary Accounting Research* 6(2), 761-799.
- Baker, M.J., Miceli, T.J., 2005. Credible Criminal Enforcement. *European Journal of Law and Economics* 20, 5-15.
- Baliga, S., Corchon, L.C., Sjöström, T., 1997. The Theory of Implementation When the Planner Is a Player. *Journal of Economic Theory* 77, 15-33.
- Becker, G.S., 1968. Crime and Punishment: An Economic Approach. *Journal of Political Economy* 76, 169-217.
- Ben-Shahar, A. and Harel, A. 1995. Blaming the Victim: Optimal Incentives for Private Precautions against Crime. *Journal of Law, Economics, and Organization* 11(2), 434-455.
- Berentsen, A., 2002. The Economics of Doping. *European Journal of Political Economy* 18(2), 109-127
- Berentsen, A., Brügger, E., Lörtschler, S., 2008. On Cheating, Doping, and Whistleblowing. *European Journal of Political Economy* 24(2), 415-436.
- Bianco, W.T., Ordeshook, P.C., Tsebelis, G., 1990. Crime and Punishment: Are One-Shot, Two-Person Games Enough? *American Political Science Review* 84(2), 569-586.
- Eber, N., 2002. Credibility and Independence of the World Anti-Doping Agency. A Barro-Gordon-Type Approach to Antidoping Policy. *Journal of Sports Economics* 3(1), 90-96.
- Eber, N., 2008. The Performance-Enhancing Drug Game Reconsidered A Fair Play Approach. *Journal of Sports Economics* 3(1), 90-96.
- Faure-Grimaud, A., Laffont, J.-J., Martimort, D., 2003. Collusion, Delegation and Supervision with Soft Information. *Review of Economic Studies* 70, 253-279.
- Feddersen, T., Pesendorfer, W., 1998. Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting. *American Political Science Review* 92(1), 23-35
- Feess, E., Schumacher, C., 2006. Why Costless Auditing May Reduce Social Welfare. *Economics Letters* 90, 407-411.
- Graetz, M.J., Reinganum, J.F., Wilde, L.L., 1986. The Tax Compliance

- Game: Toward an Interactive Theory of Law Enforcement. *Journal of Law, Economics, and Organization* 2(1), 1-32.
- Hadfield, G.K., 1994. Judicial Competence and the Interpretation of Incomplete Contracts. *The Journal of Legal Studies* 23, 159-184.
- Harsanyi, J.C., Selten, R., 1988. *A General Theory of Equilibrium Selection in Games*. Cambridge, MA.
- Haugen, K. K., 2004. The Performance-Enhancing Drug Game. *Journal of Sport Economics* Vol. 5 (5) 67-86.
- Heiner, R.A., 1983. The Origin of Predictable Behavior. *American Economic Review* 73, 560-595
- Heiner, R. A., 1986. Imperfect Decisions and the Law: On the Evolution of Legal Precedent and Rules. *The Journal of Legal Studies* 15, 227-261
- Holler, M., 1990. The Unprofitability of Mixed-Strategy Equilibria in Two-Person-Games. A Second Folk Theorem. *Economics Letters* 32, 319-323.
- Jost, P.-J., 1997. Regulatory Enforcement in the Presence of a Court System. *International Review of Law and Economics* 17(4), 491-508
- Khalil, F., 1997. Auditing without Commitment. *RAND Journal of Economics* 28(4), 629-640.
- Khalil, F., Lawarrée, J., 1995. Collusive Auditors. *The American Economic Review* 85(2), 442-446.
- Kirstein, R., Schmidtchen, D., 1997. Judicial Detection Skill and Contractual Compliance. *International Review of Law and Economics* 17(4), 509-520
- Macey, J.R., 1994. Judicial Preferences, Public Choice, and the Rules of Procedure. *The Journal of Legal Studies* 23, 627-646.
- Maenning, W., 2002. On the Economics of Doping in International Sports. *Journal of Sport Economics* Vol. 3(1) 61-89.
- Melumad, Nahum D., Mookherjee, D., 1989. Delegation as Commitment: The Case of Income Tax Audits. *The RAND Journal of Economics* 20(2), 139-163.
- Mirrlees, J.A., 1999. The Theory of Moral Hazard and Unobservable Behavior: Part I. *The Review of Economic Studies* 66(1), 3-21.
- Polinsky, A.M., Shavell, S., 2000. The Economic Theory of Public Enforcement of Law. *Journal of Economic Literature* 38, 45-76.
- Posner, R.A., 1994. What do Judges and Justices Maximize? (The Same Thing Everybody Else Does). *Supreme Court Economic Review* 3, 1-41.
- Reinganum, J.F., Wilde, L.L., 1986. Equilibrium Verification and Reporting Policies in a Model of Tax Compliance. *International Economic Review* 27(3), 739-760.
- Reinganum, J.F., Wilde, L.L., 1985. Income Tax Compliance in a Principal-Agent Framework. *Journal of Public Economics* 26, 1-18.

- Schwartz, E.P., 1995. A Comment on "The Appeals Process as a Means of Error Correction", by Steven Shavell. *Legal Theory* 1, 361-363.
- Shavell, S., 1995. The Appeals Process as a Means of Error Correction. *The Journal of Legal Studies* 24(2), 379-426.
- Shavell, S., 1996. Reply to a Comment on "The Appeals Process as a Means of Error Correction". *Legal Theory* 2, 83-85.
- Swets, J.A., 1988. Measuring the Accuracy of Diagnostic Systems. *Science* 240, 1285-1293.
- Tsebelis, G., 1989. The Abuse of Probability in Political Analysis: The Robinson Crusoe Fallacy. *American Political Science Review* 83(1), 77-91.
- Tsebelis, G., 1990a. Are Sanctions Effective? A Game-Theoretic Analysis. *Journal of Conflict Resolution* 34(1), 3-28.
- Tsebelis, G., 1990b. Penalty Has no Impact on Crime: A Game-Theoretic Analysis. *Rationality and Society* 2(3), 255-286.
- Tullock, G., 1980. *Trials on Trial. The Pure Theory of Legal Procedure*; New York.
- Tullock, G., 1994. Court Errors. *European Journal of Law and Economics* 1, 9-21.
- van Damme, E., Hurkens, S., 1995. Games with Imperfectly Observable Commitment. *Games and Economic Behavior* 21, 282-308.
- Wittman, D., 1985. Counter-Intuitive Results in Game Theory. *European Journal of Political Economy* 1(1), 77-89.
- Wittman, D., 1993. Nash Equilibrium vs. Maximin. A Comparative Statics Analysis. *European Journal of Political Economy* 9, 559-565.
- Usman, M., 2002. Verifiability and Contract Enforcement: A Model with Judicial Moral Hazard. *Journal of Law, Economics, and Organization* 18(1), 67-94.