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Serving the Many or Serving the Most Needy?

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Abstract

For free, subsidized or cost-covering? The decision on how much to charge for a good or service is fundamental in social business planning. The higher the fee paid by the recipient, the more people in need can be served by the additional revenues. But charging a fee means simultaneously to exclude the very poor from consumption. This paper argues that the entrepreneur's trade off between both effects is governed by her level of poverty aversion, i.e., her preference intensity for the service of needy people with different incomes. Additionally, we account for the possibility of excess demand for the provided good and assume that applicants are rationed by non-price allocation mechanisms. We thereby contribute to the extensive literature on the pricing and rationing behaviour of nonprofit firms. Within our theoretical model, we find ambiguous reactions of the entrepreneur to a cut in donations. Given a sufficiently low level of status-quo donations, entrepreneurs with relatively high poverty aversion tend to increase the project volume, while those with relatively low poverty aversion do the opposite.

Keywords: allocation mechanism, donation, nonprofit, poverty aversion, social entrepreneur, user fee

JEL Classification: L31, H41, D45

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1. Introduction

Social entrepreneurship takes place where basic human needs are left unsatisfied by the market mechanism. Austin et al. (2006, p. 2) suggest that "this is often due to the inability of those needing the services to pay for them." The entrepreneur satisfies the necessities by offering subsidized goods or services as complements to the market supply. Examples are manifold: Soup kitchens distribute balanced food, homeless shelters offer nighttime residence, charity shops sell donated second-hand goods, and micro-insurance schemes provide basic health securities. However, the availability of third-party funds to finance those businesses might be insufficient to meet the entire demand. In coping with the arising problems of congestion and rationing, the social entrepreneur is confronted with two general decisions: Which and how many needy people will be served? The provision of eligible customers must then be ensured through an adequate mix of rationing instruments. According to Steinberg and Weisbrod (1998), such instruments are diverse forms of user fees, the formulation of eligibility criteria, queues, waiting lists, quality dilution, product bundling etc.

In this paper we propose a positive model of the social entrepreneur's pricing decision in the light of other exogenously given third-party funds[1]. We examine two effects of charging uniform user fees on the composition and quantity of recipients. On the one hand, charging a fee excludes the lowest-income individuals, who are typically considered the most needy. On the other hand, the entrepreneur's budget is enlarged and enables her to serve more needy individuals. Given that excess demand is not completely dissolved by the user fee, we assume the entrepreneur to have a non-price rationing instrument at hand to achieve a utility maximizing allocation.

A similar approach is taken by Steinberg and Weisbrod (2005). They characterize pricing and rationing decisions of nonprofit organizations which seek to maximize the weighted sum of the consumers' surpluses. In their model they allow for price discrimination and analyze equilibrium prices in comparison to marginal costs and reservation prices. A similarly defined utility function can be found in Le Grand (1975).

[1] Aside from user fees, nonprofit organizations typically generate income from additional sources, which can be clustered into donations and unrelated business income (Steinberg and Weisbrod, 1998). There is a thrust of literature dealing with aspects of each of the sources and the interactions between them. Exemplarily, contributions to the field of public or private donations highlight the role of lead donors (Andreoni, 1998, 2006), fundraising strategies (List and Lucking-Reiley, 2002), united charities (Fisher, 1977; Bilodeau, 1992), and the interaction between government grants and fundraising success (Rose-Ackerman, 1987; Andreoni and Payne, 2003). Work on unrelated business income points to disutility from engaging in commercial activities (Schiff and Weisbrod, 1991; Weisbrod, 1998) and agency problems within the organization (Du Bois et al., 2004).

However, the proposed objectives do not reflect the satisfaction of basic human needs, which social entrepreneurs typically consider strongly. Economic theory suggests that individuals satisfy those needs first, provided their budget is sufficiently large. Different reservation prices, as a part of consumer surplus, thus, generally point to different incomes and not to differently intense preferences. It is straight forward to conclude that a given user-fee level results in a higher surplus for wealthier recipients. Although the nonprofit organization, as analyzed in Steinberg and Weisbrod (2005), might weight wealthier consumers less than poorer, it is unclear why it should consider consumer surplus at all, since this is an inadequate proxy for consumer utility in social contexts. In the extreme case, the provision of individuals without any liquidity to bid for the good or service does not help to fulfill the firm's goal even if the good is allocated to them costlessly. Consequently, they are served last, if at all.

We overcome the pinpointed problem by assuming the social entrepreneur to attribute values to each individual, which reflect her attitude towards poverty and, thus, are negatively correlated to the recipients' income. The entrepreneur's objective is to maximize the aggregated value of served individuals. This assumption goes in line with Nichols et al. (1971, p. 316), who claim that "[…] the poorer a person is, the more willing the public is to provide him […]". Therefore, we refer to the entrepreneurial attitude as *poverty aversion*.

In addition to Steinberg and Weisbrod (2005), there are various other attempts to characterize the objectives of social entrepreneurs and nonprofit organizations, ranging from the maximization of service, budget, and usage to the maximization of the number of users (Steinberg, 1986; Brooks, 2005; Ansari et al., 1996). All these approaches describe a social entrepreneur who extends the project size by charging recipients a fee until all applying individuals are served. However, they do not explain why many organizations charge no user fee but simultaneously face congestion.

As a second modification of the Steinberg and Weisbrod (2005) model, we exclude price discrimination from consideration and instead analyze uniform user fees for a number of reasons. From a pragmatic perspective, one can find many examples of social enterprises typically offering their goods or services at uniform prices, e.g. soup kitchens, charity shops or homeless shelters. From a theoretical perspective, a detection of reservation prices might be prohibitively costly. In those cases, price discrimination is no option and other rationing instruments to implicitly allocate the good to poorer applicants, such as queues or waiting lists, constitute alternatives.[2] From a technical perspective, a uniform user fee simplifies the model. However, we indicate in the subsequent analysis that all derived results can be obtained with a consideration of price discrimination.

The organization and results of the paper are given as follows. In section 2 we introduce a model of the entrepreneur's decision calculus which accounts for the level of poverty aversion, the structure of the market for the good and the applicable rationing mechanisms. In section 3 we provide optimality conditions and formally prove the existence of corner solutions and interior utility maxima implying positive user fees. Section 4 analyzes a variation in third-party funds. We find three entrepreneurial reactions. First, there is a particular level of poverty aversion at which user-fee revenues are reduced to exactly the same amount by which third-party funds are increased. Hence, the project volume remains unchanged. In contrast, entrepreneurs with a higher poverty aversion react with a reduction of the project volume and entrepreneurs with a lower aversion widen the scope of their service. We conclude in section 5 with a discussion of these results.

2. The Model

Consider a group of individuals unable to satisfy a specific basic human need due to their insufficient incomes. A social entrepreneur discovers the deficit and plans to allocate a need-oriented and subsidized good on a nonprofit basis. The entrepreneur is characterized as a poverty-averse person, valuing the provision to an individual higher, the poorer the person is. This assumption goes in line with the extensive literature on the allocation of public goods which often assumes equity considerations or the desire to serve the poor as the driving force behind this activity.[3] A similar characterization is given by Nichols et al. (1971, p. 316), who claim that "[…] the poorer a person is, the more willing the public is to provide him […]". The social entrepreneur maximizes her utility by determining the user fee for the good. More specifically, in the optimum the total of the income-dependent values she attributes to each served individual is maximized. This is formalized in the following model.

The constant marginal costs of producing the good are $c \in R_+^*$. They must be covered by the entrepreneur's income, which might include third-party funds, e.g. government grants,

[2] Nichols et al. (1971), Lindsay and Feigenbaum (1984), and in a later version Cullis and Jones (1986) provide theoretical analyses of the effects of rationing by waiting.
[3] See for example Alderman (1987), Glazer and Niskanen (1997), Kulshreshtha (2007), Le Grand (1975), and Sah (1987).

private donations or mission unrelated business incomes. We simply subsume those funds under *donations* $D \in R_+$ and assume that their total level is exogenously given. In case this level is insufficient to serve all individuals, there is a need to ration applicants. We model two rationing instruments: a uniform user fee as the entrepreneur's decision variable and a non-price allocation mechanism which is automatically applied if further rationing arises. The uniform user fee f, with $f \in [0, f_{max}] \subset R_+$ and $f_{max} > c$, mitigates excess demand by excluding individuals with lower reservation prices and enlarging the entrepreneur's budget. The non-price rationing instrument helps the entrepreneur to identify and directly serve only the poorest individuals with the ability to pay the fee.

We do not consider price discrimination for a number of reasons. There are many examples of social businesses typically offering their good at a uniform price. One might hypothesize that those enterprises principally sell low-involvement products to a large number of individuals, such as food providing services or charity shops. Since here a detection of each applicant's income, or rather reservation price, is prohibitively costly, price discrimination is infeasible. Even in cases where several income classes can be defined and different user fees are charged, a further segmentation of heterogeneous subgroups may be desirable but not possible. For example, the allocation of food in a university cafeteria is accompanied by a differentiation of prices between students, members of the university and external visitors. Examination of eligibility is done by student identity cards and service cards. Although students differ in their wealth and poorer students should be subsidized more, a further segmentation according to income would be too costly. In those cases, other rationing instruments, which implicitly allocate the good to the poorest applicants, like queues, are implemented.[4] A perceptible simplification of the model constitutes another reason for analyzing uniform user fees. Subsequently, we argue that all derived results can be likewise shown with a consideration of price discrimination.

The demand for the good is given by $\overline{n}(f)$, with $\overline{n}:[0,f_{max}] \to R_+$, $\overline{n}(f_{max}) = 0$, $\overline{n}(c) \geq 1$,[5] $\overline{n}(0) < \infty$, $\overline{n}_f := \overline{n}'(f) < 0$ and $\overline{n}_{ff} := \overline{n}''(f) > 0$. It is important to note that reservation prices are uniquely determined by the individual's ability to pay. Microeconomic theory suggests that a low reservation price is the result of a low income or a weak preference for the good. In contrast, a prerequisite for high reservation prices is a sufficiently large income.

[4] Nichols et al. (1971), Lindsay and Feigenbaum (1984) and in a later version Cullis and Jones (1986) provide theoretical analyses of the effects of rationing by waiting.

[5] The assumption $\overline{n}(c) \geq 1$ simplifies subsequent proofs w.l.o.g..

However, when basic human needs are concerned, we can assume that individuals will satisfy these first. As a consequence, low reservation fees result from limited payment abilities. Although there might be deviations from this suggested behavior, we postulate a strictly positive correlation between income and payment willingness for the good. The resulting demand curve, therefore, presumes equally intense consumption preferences across all individuals and solely reflects the wealth of applicants.

We further assume that each applicant intends to consume exactly one unit of the good and that each $n \in [0, \overline{n}(0)]$ indexes one individual with a specific disposable income. According to the previous argumentation, the index is negatively correlated to the individual's reservation fee and wealth, respectively. In other words, the higher the index n is, the lower is the individual's income. In particular, the individual $n = 0$ is able to pay the prohibitive price f_{max} whereas the poorest individual $n = \overline{n}(0)$ cannot afford to pay anything. At the same time, a specific element n likewise denotes the total quantity of individuals with a higher income than n. Hence, the term $\overline{n}(f)$ provides two important details. It shows the quantity of applicants for the good at a given user fee, and it simultaneously indexes the poorest individual being even able to afford this fee.

The social entrepreneur's non-price rationing instrument ensures that only the poorest applicants out of the quantity $\overline{n}(f)$ receive a unit of the good. This requires a direct or indirect detection of reservation prices. Given that the entrepreneur can directly observe reservation prices,[6] she can formulate adequate eligibility criteria and directly exclude wealthier applicants. Even in cases in which the entrepreneur cannot observe them, theory suggests that there are ways to indirectly exclude the wealthiest applicants, e.g. rationing by waiting. Therefore, we forego an explicit modeling of direct and indirect non-price allocation mechanisms by assuming that the entrepreneur has a general non-price tool at hand, which ensures the provision of the poorest applicants. The quantity of the wealthiest individuals being excluded from consumption is denoted by $\underline{n}(f)$, with $\underline{n} \in [0, \overline{n}(f)]$. This term likewise denotes the recipient with the highest income. The combined application of both rationing instruments determines the final quantity of recipients which is given by $\overline{n}(f) - \underline{n}(f)$.

In allocating the good to the needy, the social entrepreneur is restricted by a nonprofit-condition. With $F(f) = f \cdot [\overline{n}(f) - \underline{n}(f)]$ as total user-fee receipts, the constraint is given by

[6] Steinberg and Weisbrod (2005) give several arguments in favor of this assumption.

(1) $F(f)+D=c\cdot[\overline{n}(f)-\underline{n}(f)]$.

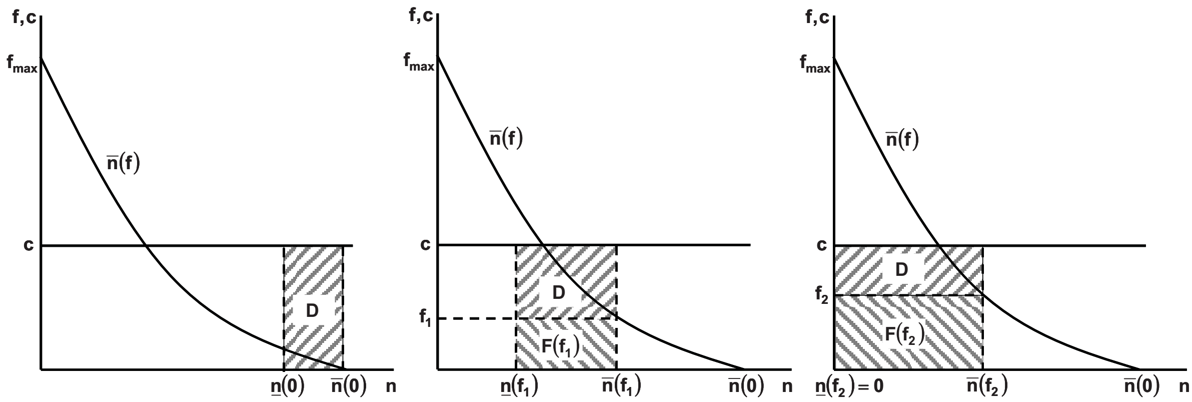
The nonprofit-condition requires the social entrepreneur to spend her total revenues completely on the supply of the good. By rearranging equation (1), one obtains $\underline{n}(f)=\overline{n}(f)-[D/(c-f)]$, which shows the endogenous determination of the wealthiest recipient for a given fee f. With the poorest individual able to afford the user fee given by $\overline{n}(f)$, a total of $\overline{n}(f)-\underline{n}(f)$ recipients can be served when the entire donations D are spent to finance the gap between marginal costs and individual contribution ($c-f$).

Figure 1 summarizes the impact of the entrepreneur's rationing mechanisms on the market. In panel (a) the entrepreneur allocates the good for free. All individuals of the target group are willing to purchase the good but, due to the limited donations, only the fraction $\overline{n}(0)-\underline{n}(0)$ is served and the wealthiest $\underline{n}(0)$ individuals are rationed by the non-price instrument. Since the entrepreneur's budget is not enlarged by additional user-fee revenues, the project shows the lowest possible volume. Panel (b) considers the combined use of both rationing instruments. The entrepreneur chooses the user fee f_1 which rations the poorest $\overline{n}(0)-\overline{n}(f_1)$ applicants who are unable to afford the good. Although this fee increases total revenues at first, the budget remains insufficient to provide all applying individuals ($[F(f_1)+D]/c<\overline{n}(f_1)$). Consequently, the entrepreneur excludes the wealthiest $\underline{n}(f_1)$ applicants by use of the non-price mechanism. In contrast, panel (c) considers the exclusive supply of the most solvent individuals. The entrepreneur chooses the user fee which maximizes her total revenues, subject to the nonprofit-condition. This ensures that the maximum quantity of applicants is served.

An additional effect of the nonprofit-constraint is the unique relationship between the user fee and total user-fee revenues. Inserting $F(f)=f\cdot[\overline{n}(f)-\underline{n}(f)]$ into equation (1) yields

(2) $$F(f)=\frac{f\cdot D}{c-f}.$$

According to this equation, the entrepreneur's choice of f determines her total receipts $F(f)$. Subsequently, we take advantage of this relationship and reverse it. We characterize the social entrepreneur's choice in terms of F instead of the individual fee. At a later stage, this allows for a direct derivation of the project size $F+D$ and, therefore, simplifies the analysis.



(a) (b) (c)

Figure 1: The allocative outcome of rationing by user fees and the non-price instrument.

Rearranging equation (2) yields the implicit function

$$(3) \quad \ell(F) = \frac{c \cdot F}{F + D} \; ,$$

with

$$\ell_F := \ell'(F) = \frac{c \cdot D}{(F + D)^2} > 0$$

and

$$\ell_{FF} := \ell''(F) = -\frac{2 \cdot c \cdot D}{(F + D)^3} < 0 \, .$$

Employing equation (3) into the demand function yields

$$(4) \quad \overline{n}(\ell(F)) = \overline{n}\left(\frac{c \cdot F}{F + D}\right) \; ,$$

with

$$(5) \quad \overline{n}_F := \overline{n}'(\ell(F)) = \overline{n}_f \cdot \ell_F < 0$$

and

$$(6) \quad \overline{n}_{FF} := \overline{n}''(\ell(F)) = \overline{n}_{ff} \cdot \ell_F^2 + \overline{n}_f \cdot \ell_{FF} > 0 \, .$$

Next, we assume that the entrepreneur draws a nonnegative level of utility from each allocated unit of the good to a target-group individual, which is specified by the value function

$$(7) \quad u(n) = n^{\alpha} \, .$$

Here, the parameter $\alpha \in R_+$ determines the constant elasticity of marginal utility $\varepsilon = \alpha - 1$ [7] and is likewise a measure for the curvature of the value function. Marginal utility is decreasing with $\alpha \in (0,1)$, constant with $\alpha = 1$, and increasing with $\alpha > 1$. As with the class of Cobb-Douglas utility functions, α characterizes the entrepreneur's preference intensity for recipients with different incomes and will be subsequently interpreted as the entrepreneurial poverty aversion. Specifically, for $\alpha = 0$ the entrepreneur shows no aversion and values the service of each individual the same.[8] However, given a positive level of poverty aversion $(\alpha > 0)$, the entrepreneur obtains a utility surplus from substituting the provision of a lower-income for a higher-income individual. This surplus increases as α grows and becomes infinite with $\alpha \to \infty$. As will be shown later, entrepreneurs with such extreme aversions are predetermined to serve only the poorest target group individuals.

The entrepreneur maximizes her aggregated utility of served individuals by implicitly choosing total user-fee revenues F. According to equation (3), this choice uniquely correlates to a specific price level for the good $(f = \ell(F))$. Individuals who cannot afford f are barred from consumption and, if total revenues are insufficient to serve the remaining applicants (i.e. $(F + D)/c < \overline{n}(\ell(F))$), the non-price allocation instrument is implemented to exclude the wealthiest individuals $\underline{n}(\ell(F))$ from consuming the good, because they provide the least value to the social entrepreneur. Finally, only the poorest applicants with the ability to pay f receive a unit. Consequently, the entrepreneur's maximization problem can be written as[9]

$$(8) \quad \begin{aligned} &\max_F \; U(F) = \int_{\underline{n}}^{\overline{n}} n^{\alpha} dn \\ &s.t. \; \underline{n} = \overline{n} - (F + D)/c \; . \end{aligned}$$

By employing the rearranged nonprofit-constraint into the utility function, one obtains the following first and second derivative:

$$(9) \quad \frac{dU(F;D)}{dF} = \left(\overline{n}^{\alpha} - \underline{n}^{\alpha}\right) \cdot \overline{n}_F + \underline{n}^{\alpha} \cdot \frac{1}{c} \begin{matrix} > \\ = \\ < \end{matrix} 0$$

and

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[7] The elasticity of marginal utility is defined as $\varepsilon = \frac{du'(n)}{dn} \cdot \frac{n}{u'(n)}$.

[8] With α = 0, the value of serving individual $n = 0$ is not defined. To simplify this case, we set $u(0) = 1$.

[9] In the maximization problem and subsequent derivations we simplify the explicit notation $\overline{n}(\ell(F))$ and $\underline{n}(\ell(F))$ by use of $\overline{n}$ and $\underline{n}$.

(10) $$\frac{d^2U(F;D)}{dF^2} = \left(\overline{n}^{\alpha} - \underline{n}^{\alpha}\right) \cdot \overline{n}_{FF} + \alpha \cdot \overline{n}_F^2 \cdot \left[\overline{n}^{\alpha-1} - \left(1 - \frac{1}{\overline{n}_F \cdot c}\right) \cdot \underline{n}^{\alpha-1}\right] + \frac{\alpha}{c} \cdot \underline{n}^{\alpha-1} \cdot \left(\overline{n}_F - \frac{1}{c}\right) \gtreqless 0 .$$

In the next section we prove the possibility of interior utility maxima and corner solutions.

3. Interior and Corner Solutions

It is important to keep in mind that the entrepreneur can solely enhance her user-fee revenues through an increase of the user-fee level. The unique quantitative relationship between both variables is given by equation (2). Although, this equation comprises additional parameters like the amount of donations or the marginal costs of producing the good as well, they are outside the entrepreneur's scope of influence.

We define the following terms. The optimal level of user-fee revenues will be denoted by F^* and the corresponding user fee by f^*. Furthermore, the maximum user-fee revenues will be denoted by F_{max} which is achieved if the entrepreneur's total income suffices to serve all applying needy. Consequently, $\underline{n} = 0$ and F_{max} fulfills the reduced nonprofit-condition (1), i.e.

(11) $$F_{max} + D = c \cdot \overline{n}(f(F_{max})) .$$

The entrepreneur's mission is achieved best if all individuals of the target group receive one unit of the good. Hence, a costless provision of beneficiaries is required to avoid a rationing of the poorest individuals. Consequently, the production costs of serving the total target group must be completely covered by donations $(D = \overline{n}(0) \cdot c)$. If donations are not available $(D = 0)$, i.e. the applicants' provision is not externally subsidized, the entrepreneur must refrain from the allocation of the good or, alternatively, serve only those individuals who can afford a cost covering user fee $(f = c)$. The dominance of the second option results from value function (7). Since any individual of the target group is assigned a nonnegative value $u(n)$, serving only individuals who can afford the good is preferred to non-provision. Total utility (equation (8)) is maximized if all applicants who show a payment ability of at least marginal production costs c are served.

Proposition 1. Given $D = 0$, the entrepreneur charges a cost covering user fee $\left(f^* = c\right)$ and serves all needy individuals that can afford to apply, $\overline{n}(c)$.

Proof. With $D = 0$, equation (3) yields $\ell(F) = c$. Substituting c for $\ell(F)$ in utility function (8) and differentiating with respect to F yields $dU(F;0)/dF = \underline{n}^{\alpha} \cdot (1/c) \geq 0$. Consequently, utility is maximized if all $\overline{n}(c)$ applicants are served. **Q.e.d.**

Now, suppose total donations amount to $\widetilde{D} \in (0, \overline{n}(0) \cdot c)$, which suffices to initially serve $\widetilde{D}/c < \overline{n}(0)$ applicants. Confronted with the resulting excess demand, the social entrepreneur determines her optimal level of user-fee revenues, which, again, is a choice of how many individuals are excluded by the user fee and how many are rationed by the non-price rationing instrument. According to the first derivative (9), the increase of user-fee receipts F is accompanied by two effects on the entrepreneur's utility. First, there is a non-positive *crowding-out effect* $\left(\overline{n}^{\alpha} - \underline{n}^{\alpha}\right) \cdot \overline{n}_F \leq 0$. Let revenues and, equivalently, the quantity of recipients be constant, then an increase in user fees cuts off the poorest from consumption and shifts the released units of the good to wealthier individuals. This effect is utility neutral only if the entrepreneur values all individuals equally. In contrast, given a positive level of poverty aversion α, the substitution of wealthier for poorer beneficiaries decreases her utility. The second term of equation (9) denotes the nonnegative *revenue effect* $\underline{n}^{\alpha} \cdot (1/c) \geq 0$. The additional user-fee receipts enable the entrepreneur to extend the quantity of recipients which increases her utility. The value of the revenue effect becomes zero if all applicants are served.

Dependent on the entrepreneur's poverty aversion, both interior and corner solutions are possible. If the crowding-out effect dominates the revenue effect for any level of user-fee revenues, the entrepreneur allocates the good for free $\left(F^* = 0\right)$ and rations applicants by the non-price instrument. Intuitively, the higher the poverty aversion is, the less the entrepreneur is willing to substitute wealthier for poorer individuals and the sooner she foregoes charging a user fee. On the other hand, if the revenue effect exceeds the crowding-out effect independent of the level of user-fee receipts, the entrepreneur generates maximum revenues $\left(F^* = F_{max}\right)$ and serves the maximum quantity of beneficiaries. This corner solution arises for a non-poverty averse entrepreneur for whom applicants are perfect substitutes. Finally, there are interior utility maxima for moderate levels of poverty aversion $\left(0 < F^* < F_{max}\right)$. The value of the initially dominant revenue effect is offset by the crowding-out effect at some positive level of user-fee revenues and overcompensated for higher levels. Consequently, as exemplarily depicted in figure 1 (b), the poorest applicants are rationed by the user fee and the wealthiest applicants are excluded by the non-price allocation mechanism. In the next three propositions, we show the possibility of interior and corner solutions.

Proposition 2. Given $D \in (0, c \cdot \overline{n}(0))$, there exists a finite poverty aversion level $\overline{\alpha}$ such that for all $\alpha \geq \overline{\alpha}$, $F^* = 0$.

Proof. For notational clarity, we temporarily expand the term $U(F;D)$ to $U(F;D,\alpha)$ to emphasize the influence of the entrepreneur's poverty aversion. Let $D \in (0, c \cdot \overline{n}(0))$. Since $\overline{n}_F < 0$ and $\overline{n} > \underline{n}$, there exists a finite $\overline{\alpha} \geq \left[ln\left(1 - \frac{1}{\overline{n}_F \cdot c}\right) \Big/ (ln\,\overline{n} - ln\,\underline{n}) \right]$ for all $F \in [0, F_{max}]$ which implies $-\left(\overline{n}^{\overline{\alpha}} - \underline{n}^{\overline{\alpha}}\right) \cdot \overline{n}_F \geq \underline{n}^{\overline{\alpha}} \cdot (1/c)$. Since the revenue effect does not exceed the crowding-out effect for all levels of user-fee revenues, an entrepreneur with the poverty aversion level $\overline{\alpha}$ chooses $F^* = 0$. Since, by definition, $\overline{n}(c) \geq 1$, $\frac{\partial^2 U(F;D,\overline{\alpha})}{\partial F \partial \alpha} = ln\,\overline{n} \cdot \overline{n}^{\overline{\alpha}} \cdot \overline{n}_F - ln\,\underline{n} \cdot \underline{n}^{\overline{\alpha}} \cdot \left(\overline{n}_F - \frac{1}{c}\right) < 0$ and the first derivative (9) is negative, given $F \in [0, F_{max}]$ and $\alpha > \overline{\alpha}$. Consequently, $F^* = 0$. **Q.e.d.**

According to proposition 2, any social entrepreneur with a level of poverty aversion equal or higher than a specific value $\overline{\alpha}$ does not wish to charge user fees.[10] For those entrepreneurs the first derivative of the utility function (equation (9)) is non-positive. This result is mainly driven by the utility difference between the poorest and the wealthiest marginal recipient, which is a component of the crowding-out effect. Since this difference increases with the entrepreneur's poverty aversion, there exists a specific level, above which the crowding-out effect dominates the revenue effect for all levels of user-fee revenues. Consequently, utility is maximized if the entrepreneur refrains from charging user fees and finances its allocation exclusively by donations.

Proposition 3. Given $D \in (0, c \cdot \overline{n}(0))$, there exists a positive poverty aversion level $\hat{\alpha} \leq \overline{\alpha}$ such that for all $\alpha < \hat{\alpha}$, $F^* > 0$.

[10] The same results arise with a consideration of price discrimination. Intuitively, since reservation prices, to some extent, are lower than marginal costs, recipients must be subsidized by donations or 'cash cows' (Steinberg and Weisbrod, 2005). If revenues are insufficient to allocate the good to all applicants, the entrepreneur must ration them and decide who and how many needy will be served. If she chooses the poorest applicants, this requires the highest individual subsidies and benefits the lowest quantity of recipients. In contrast, the maximum quantity of recipients follows from serving the wealthiest applicants. It is important to note that a change of quantity causes the same qualitative effects on the entrepreneur's utility: a non-positive crowding-out effect and a nonnegative revenue effect. For the same reason, interior and corner solutions are possible and depend on the entrepreneur's level of poverty aversion.

Proof. Let $D \in (0, c \cdot \overline{n}(0))$ and $\hat{\alpha} = \left[ln\left(1 - \frac{1}{\overline{n}_F \cdot c}\right) \Big/ (ln\,\overline{n} - ln\,\underline{n}) \right]\Bigg|_{F=0} > 0$. $\alpha < \hat{\alpha}$ then implies $\left[\underline{n}^{\alpha} \cdot (1/c) > -\left(\overline{n}^{\alpha} - \underline{n}^{\alpha}\right) \cdot \overline{n}_F \right]_{F=0}$, which is a necessary condition for the existence of a utility maximum with $F^* > 0$. **Q.e.d.**[11]

Proposition 3 claims that any social entrepreneur with sufficiently low poverty aversion chooses a positive level of user fees. Again, consider the entrepreneur's marginal utility (equation (9)) for the first unit of user-fee revenues. In line with the intuition of the previous proposition, with a poverty aversion below a specific level $\hat{\alpha}$ the utility difference between the poorest and the wealthiest marginal recipient and, hence, the crowding-out effect are sufficiently low. Consequently, the entrepreneur's marginal utility is positive and user fees are charged.

Proposition 4. Given $D \in (0, c \cdot \overline{n}(0))$, there exists a strict corner solution with $F^* = F_{max}$, if, and only if, $\alpha = 0$.

Proof. Consider the first derivative of the utility function (9). Let $D \in (0, c \cdot \overline{n}(0))$ and $\alpha > 0$. Since $\overline{n} > 0$ and, by definition, $\overline{n}_F < 0$, $\lim_{F \to F_{max}} \left(\overline{n}^{\alpha} - \underline{n}^{\alpha}\right) \cdot \overline{n}_F < 0$ and $\lim_{F \to F_{max}} \underline{n}^{\alpha} \cdot (1/c) = 0$. Hence, $\lim_{F \to F_{max}} dU(F;D)/dF < 0$ and $F^* < F_{max}$. In contrast, let $\alpha = 0$. Since, $\left(\overline{n}^{\alpha} - \underline{n}^{\alpha}\right) \cdot \overline{n}_F = 0$ and $\underline{n}^{\alpha} \cdot (1/c) \geq 0$, $dU(F;D)/dF \geq 0$ for all $F \in [0, F_{max}]$ and $F^* = F_{max}$. **Q.e.d.**

According to proposition 4, only non-poverty-averse entrepreneurs choose the corner solution with the maximum of user-fee revenues F_{max}. For a deeper understanding of the result consider again the entrepreneur's marginal utility (equation (9)). Given a positive level of poverty aversion, the crowding-out effect is strictly negative, since the substitution of lower-valued wealthier for higher-valued poorer individuals always entails a loss in utility. Concerning the revenue effect, on the other hand, the additional utility the entrepreneur gains from enlarging the group of recipients through additional user-fee receipts approaches zero since

[11] The set $[0, \hat{\alpha})$ is far from being complete. One can show that there are global utility maxima for higher levels of poverty aversion which start with a dominant crowding-out effect for the first unit of user fees $\left[\underline{n}^{\alpha} \cdot (1/c) \leq -\left[\overline{n}(F)^{\alpha} - \underline{n}^{\alpha}\right] \cdot \overline{n}'(F) \right]_{F=0}$. The increase of fees initially decreases utility to some minimum before the revenue effect overcompensates the utility loss and induces a global maximum. Since all important results can be proved without an extension to these special cases, we simplify the analysis by ignoring them.

the wealthiest recipient $(n=0)$ is of no value to the entrepreneur. As a result, there is a level of user-fee revenues at which both effects offset each other and, hence, $F^{*} < F_{max}$. In contrast, non-poverty-averse entrepreneurs assign equal value to each individual, which implies that there is no crowding-out effect. The marginal utility is characterized by a nonnegative revenue effect implying that the maximum user-fee revenues F_{max} are chosen.

The graphical characterization of propositions 2-4 is presented in figure 2. It contrasts total revenues $F+D$, also considered as *project volume*, and the entrepreneur's overall utility $U(F;D)$. As an important point of reference, the graph $U(0;D)$, with

$$U(0;D)=\int_{\overline{n}(0)-\frac{D}{c}}^{\overline{n}(0)} n^{\alpha}\,dn,$$

$$\frac{dU(0;D)}{dD}=\frac{1}{c}\cdot\left[\overline{n}(0)-\frac{D}{c}\right]^{\alpha}\geq 0,$$

and

$$\frac{d^{2}U(0;D)}{dD^{2}}=-\frac{\alpha}{c^{2}}\cdot\left[\overline{n}(0)-\frac{D}{c}\right]^{\alpha-1}\leq 0,$$

denotes the upper utility boundary for any given project volume. It considers utility as a pure function of donations D, which implies an allocation of the good free of charge. Its concave shape accounts for the impact of the entrepreneur's non-price rationing instrument on the sequence of the applicants' provision. A poorer individual with a likewise higher value is served prior to the next wealthier applicant. The entrepreneur's marginal utility of an additional recipient, therefore, is decreasing. Her aggregated utility reaches a maximum if all applicants are served through donations $(D=\overline{n}(0)\cdot c)$.

The lower boundary of the utility spectrum is given by $U(F;0)$, which presumes the non-availability of donations. According to equation (3), in this case, the social entrepreneur chooses a user fee equal to marginal costs and allocates the good to applicants successively. The user-fee revenues thereby increase with the quantity of served individuals. The corresponding utility function is given by

$$U(F;0)=\int_{\overline{n}(c)-\frac{F}{c}}^{\overline{n}(c)} n^{\alpha}\,dn,$$

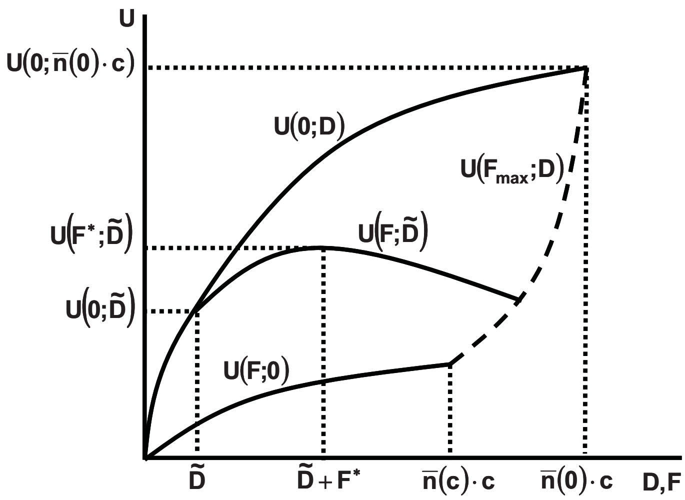


Figure 2: A utility function for a moderate level of poverty aversion and the interior optimum.

with

$$\frac{dU(F;0)}{dF}=\frac{1}{c}\cdot\left[\overline{n}(c)-\frac{F}{c}\right]^{\alpha}\geq 0$$

and

$$\frac{d^2U(F;0)}{dF^2}=-\frac{\alpha}{c^2}\cdot\left[\overline{n}(c)-\frac{F}{c}\right]^{\alpha-1}\leq 0\,.$$

The maximum project size is reached at $F_{max}=\overline{n}(c)\cdot c<\overline{n}(0)\cdot c$, i.e. a lower level compared to the maximum volume resulting from complete donation financing.

In figure 2, the right increasing dashed graph $U(F_{max};D)$ connects both elements. It depicts the entrepreneur's utility in dependence on the maximum project volume. Since a maximum project size implies $\underline{n}=0$, $U(F_{max};D)$ is obtained by rearranging the reduced nonprofit-condition (11) to $\overline{n}(\ell(F_{max}))=(F_{max}+D)/c$ and inserting it into the utility function:

$$U(F_{max};D)=\int_{0}^{\overline{n}(\ell(F_{max}))}n^{\alpha}\,dn\,,$$

with

$$\frac{dU(F_{max};D)}{d(F_{max}+D)}=\frac{1}{c}\cdot\left[\frac{F_{max}+D}{c}\right]^{\alpha}>0$$

and

$$\frac{d^2U(F_{max};D)}{d(F_{max}+D)^2}=\frac{\alpha}{c^2}\cdot\left[\frac{F_{max}+D}{c}\right]^{\alpha-1}\geq 0\,.$$

The curvature shown in figure 2 can be explained as follows. The larger the initial donation D is, the less user-fee revenues are needed to reach a certain project volume $F+D$ and, hence, the fewer applicants are excluded. Consequently, more individuals can be served by a further increase of the user fee which extends the maximum project volume.

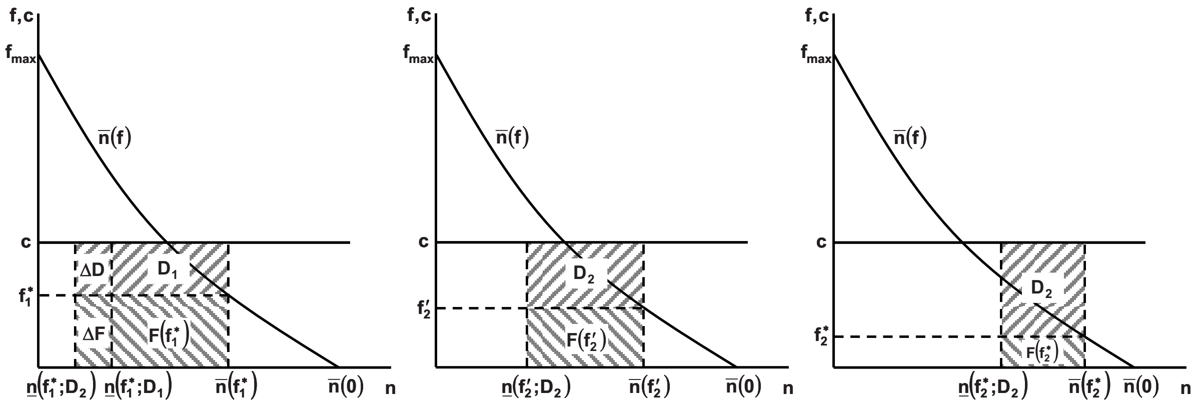
The three boundaries define the spectrum of possible utility functions. As an example, consider the graph $U(F;\tilde{D})$. At $\tilde{D}$ the entrepreneur charges no user fee and the service of the poorest $\tilde{D}/c$ individuals provides her with utility of $U(0;\tilde{D})$. The introduction of user fees initially enhances the entrepreneur's utility due to a dominating revenue effect. As the project volume reaches $F^*+\tilde{D}$ the crowding-out effect offsets the revenue effect, and an interior utility maximum results.

4. Variation in Donations

In figure 2, the social entrepreneur's donations amount to $\tilde{D}$ and the project volume $F^*+\tilde{D}$ is chosen. In this section, we analyze how the optimal choice of user-fee revenues and, hence, the optimal project volume change when donations increase. We argue that various results are possible and that their occurrence strongly depends on the entrepreneurs' level of poverty aversion and the status-quo level of donations. More specifically, given that the initial level of donations is sufficiently low, the project volume increases for relatively low levels of poverty aversion and it decreases for relatively high levels. Moreover, there is a specific value of α for which the optimal project size remains unchanged. However, given that the status-quo level of donations is relatively high, all entrepreneurs increase the project volume.

This section primarily focuses on the second entrepreneurial reaction, namely the reduction of the optimal project size, since this appears to be least intuitive. Figure 3 characterizes the change of the allocative outcome.[12]

[12] For notational clarity the terms $\underline{n}(f)$ in figure 3 and $F(f)$ are expanded to $\underline{n}(f;D)$ and $F(f;D)$ to emphasize the influence of donations.



(a) (b) (c)

Figure 3: A decrease of the optimal project volume as the highly poverty-averse entrepreneurial reaction.

Figure 3 shows the direct effect of the exogenous increase of donations and then decomposes the entrepreneur's reaction into two steps. Consider first panel (a). Given a constant user fee, an increase in donations additionally increases the user-fee revenues. This result is obtained by differentiating total entrepreneurial revenues with respect to donations, where user-fee revenues are given by equation (2),

$$(12) \quad \frac{d\left[F\left(f^*;D\right)+D\right]}{dD}=\frac{f^*}{c-f^*}+1=\frac{c}{c-f^*}.$$

As a consequence, the entrepreneur's marginal utility of charging user fees decreases:

$$(13) \quad \left.\frac{\partial^2 U(F;D)}{\partial F \partial D}\right|_{F=F^*}=\frac{\alpha}{c-f^*}\cdot \underline{n}^{\alpha-1}\cdot\left(\overline{n}_F-\frac{1}{c}\right)\leq 0.$$

Intuitively, consider the particular project volume $\overline{n}\left(f_1^*\right)-\underline{n}\left(f_1^*;D_1\right)$ at which the crowding-out effect $\left(\left(\overline{n}^\alpha-\underline{n}^\alpha\right)\cdot\overline{n}_F\leq 0\right)$ and the revenue effect $\left(\underline{n}^\alpha\cdot(1/c)\geq 0\right)$, as defined by equation (9), offset each other. Now, the increase in donations enables the entrepreneur to cover the difference between marginal costs and user fee for previously unconsidered applicants. Moreover, the fee paid by the new recipients additionally increases the entrepreneur's revenues. As a result, the value of the "new" wealthiest recipient $\underline{n}\left(f_1^*;D_2\right)$ is lower and the marginal utility of increasing the user fee becomes negative.

As a consequence, the entrepreneur wishes to reduce the user-fee receipts to readjust the crowding-out and the revenue effect. Given that this reduction does not compensate for the previous increase in total revenues, the optimal project volume rises compared to the status quo. On the other hand, the optimal project size decreases if the user fee reduction overcompensates the previous increase in total revenues. In this case, the absolute change of the entrepreneur's marginal utility (equation (9)) is larger for an increase of donations than for a decrease of user-fee revenues.

The analysis of the entrepreneur's reduction of user-fee receipts is decomposed into two steps illustrated by panels (b) and (c) in figure 3. In panel (b) we consider a first reduction such that the project volume reaches the status-quo level $\left([\overline{n}(f_2') - \underline{n}(f_2'; D_2)] = \left[\overline{n}\left(f_1^*\right) - \underline{n}\left(f_1^*; D_1\right)\right]\right)$. This partial adjustment provides an important result: The equally large reduction of user-fee revenues increases the revenue effect of equation (9) to the same extent as the marginal utility decreases due to the additional donations (equation (13)).[13] In other words, if we leave the crowding out of recipients unconsidered, any variation in revenues (i.e. donations or user fees) identically affects the entrepreneur's marginal utility. As a consequence, it suffices to analyze the impact of the considered user fee reduction on the non-positive crowding-out effect. Given that this effect decreases, the resulting total change of the revenue and crowding-out effect is, in absolute terms, larger for an increase of donations than for a decrease of user-fee revenues. We assume this scenario to be given in figure 3. Therefore, in panel (b), the entrepreneur's marginal utility of charging additional user fees is negative at the status-quo project volume $\overline{n}(f_2') - \underline{n}(f_2', D_2)$ and the entrepreneur is induced to further reduce user-fee revenues until the new optimal project volume $\overline{n}\left(f_2^*\right) - \underline{n}\left(f_2^*, D_2\right)$ is reached. This outcome is characterized in panel (c).

A sufficiently high level of poverty aversion, which exceeds the specific lower limit $\breve{\alpha}$ [14], causes a decreasing crowding-out effect for the following reason. In figure 3, panel (b), the values of the two marginal recipients $\overline{n}(f_2')$ and $\underline{n}(f_2'; D_2)$, as components of this effect, are strictly higher than in panel (a) $\left(\overline{n}\left(f_1^*\right)\right.$ and $\left.\underline{n}\left(f_1^*; D_1\right)\right)$. Since both values are weighted exponentially by the entrepreneur's level of poverty aversion, the utility difference between the marginal recipients is larger in panel (b). In other words, the utility loss of substituting the wealthiest for the poorest marginal recipient is c. p. larger, the poorer both individuals are, and, consequently, the lower the non-positive crowding-out effect is.

The effect of increasing donations on the social entrepreneur's utility function is depicted in figure 4. According to figure 3, the figure likewise characterizes a reduction of the optimal

[13] This result is shown within the next proof.

[14] The conditions specifying $\breve{\alpha}$ are presented within the next proof. For the current argumentation it suffices to set $\breve{\alpha} > 1$.

project volume. In the status quo, the entrepreneur receives the donations D_1 and chooses the optimal level of user-fee revenues F_1^*. Now, consider an increase in donations to D_2. Since the entrepreneur shows a relatively high level of poverty aversion, she reduces user-fee receipts to an even larger extent $\left(F_1^* - F_2^* > D_2 - D_1\right)$, which decreases the optimal project volume.

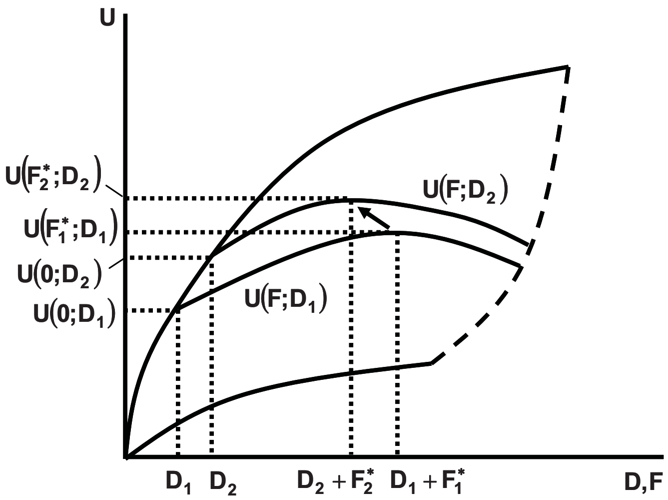


Figure 4: The shift of the utility function due to an increase in donations.

It is important to highlight again: The social entrepreneur's choice of a positive level of user-fee revenues in the status quo is a necessary precondition to the characterized result in figures 3 and 4. According to proposition 3, this choice requires that the level of poverty aversion falls short of a specific value $\hat{\alpha}$. However, the entrepreneur reduces the project volume in response to increased donations if her poverty aversion exceeds the lower limit $\breve{\alpha}$. Given that $\hat{\alpha}$ falls short of $\breve{\alpha}$, all entrepreneurs with a poverty aversion level below $\hat{\alpha}$ charge user fees but, given donations increase, all of them react with an enlargement of the project volume. In contrast, those entrepreneurs who, in principle, show the propensity to reduce the optimal project size $(\alpha \geq \breve{\alpha})$ do not charge user fees in the status quo. Instead, their project volume increases by the amount of the additionally obtained donations. Consequently, only if $\breve{\alpha} < \hat{\alpha}$, the predicted behavior occurs. The next proposition shows that a sufficiently low level of status-quo donations ensures that $\breve{\alpha} < \hat{\alpha}$. Moreover, it will be proven that an increase in donations leads to a reduction of the optimal project volume if the entrepreneur's poverty aversion falls between both parameter values.

Proposition 5. There exists a level of donations $D' < \overline{n}(0) \cdot c$ and a level of poverty aversion $\breve{\alpha}$, such that for all $D \in [0, D')$ and $\alpha \in (\breve{\alpha}, \hat{\alpha})$, an increase in donations leads to a reduction of the optimal project volume $F^* + D$.

Proof. See Appendix.

Corollary. Given $D \in [0, D')$ and let donations increase, then entrepreneurs with $\alpha = \breve{\alpha}$ do not change and entrepreneurs with $\alpha < \breve{\alpha}$ increase the optimal project volume $F^* + D$.

Proof. Consider again the proof of proposition 5. An increase in donations leads to a constant optimal project volume if $dF^*/dD = -c/(c - f^*)$ or, equivalently, if the value of equation (A.1), namely Ω, is zero. The proof showed that this is uniquely fulfilled for $\alpha = \breve{\alpha}$. On the other hand, an increase in the optimal project volume requires that $dF^*/dD > -c/(c - f^*)$ which gives a negative sign of Ω. The proof showed that this is fulfilled for all $\alpha < \breve{\alpha}$. **Q.e.d.**

Proposition 5 consists of two parts. First, it claims that social entrepreneurs reduce their project volume in response to increased donations if their level of poverty aversion exceeds the lower limit $\breve{\alpha}$. The intuition of the proposition follows the argumentation given previously in this section. Accordingly, for those high levels of α the non-positive crowding-out effect decreases if user-fee revenues are reduced. Moreover, as the corollary outlines, the crowding-out effect and, hence, the optimal project volume remain unchanged if $\alpha = \breve{\alpha}$ and increase if $\alpha < \breve{\alpha}$.

Second, a precondition to the result of the proposition's first part is the imposition of user fees in the status quo or, equivalently, a level of poverty aversion below $\hat{\alpha}$. Only if $\breve{\alpha}$ falls short of $\hat{\alpha}$ there is room for the existence of entrepreneurs decreasing the project volume. This requirement is fulfilled if the status-quo level of donations is sufficiently low. The corresponding intuition proceeds as follows. As the extreme case, consider an entrepreneur with an infinitesimal amount of donations. With these funds at hand she is restricted to serve only an insignificant quantity of individuals. Hence, the exponentially weighted value difference between the marginally poorest and the marginally wealthiest recipient, i.e. the crowding-out effect, is negligible for finite levels of poverty aversion. However, there exists a significant revenue effect because an increase of one unit of user-fee receipts enables the entrepreneur to considerably enlarge the group of recipients compared to the initial quantity. Therefore, the level of poverty aversion $\hat{\alpha}$ at which the negative (and insignificant) crowding-out effect outweighs the positive (and significant) revenue effect is infinitely large. On the other hand, consider an entrepreneur with initial donations sufficing to serve almost all individuals. Here, since the absolute value of the crowding-out effect reaches its maximum whereas the revenue effect becomes infinitesimal ($\underline{n} \rightarrow 0$), the level of poverty aversion at which the crowding-out effect dominates the revenue effect approaches zero. As a result, possible values of $\hat{\alpha}$ range from zero to infinity and are negatively correlated to the status-quo amount of donations.

In contrast, the parameter value $\breve{\alpha}$ is finite. Consider again the discrete project volume depicted in figure 3, panel (b). As argued previously, an entrepreneur with the poverty aversion $\breve{\alpha}$ is indifferent between the status-quo volume $\overline{n}(f_2') - \underline{n}(f_2', D_2)$ and a smaller one resulting from a marginal reduction of user fees. Since any of those comparisons always presumes a positive level of user-fee revenues, both project sizes are significant. Consequently, an entrepreneur valuing the two project volumes equally must have a finite level of poverty aversion $\breve{\alpha}$. Comparing this result with the argued range of $\hat{\alpha}$-values, it follows that $\breve{\alpha}$ falls below $\hat{\alpha}$ if the status-quo amount of donations is relatively low.

The results of this section can be summarized as follows. Given that the status-quo level of donations is sufficiently low, an increase in donations leads to mixed reactions of social entrepreneurs concerning their optimal choice of the project size (measured in total revenues $F^* + D$). Specifically, relatively low poverty-averse entrepreneurs increase the project volume while those with relatively high aversion decrease it. Moreover, entrepreneurs with a specific value $\breve{\alpha}$ do not change the volume at all. However, given that the status-quo level of donations is relatively high, all entrepreneurs increase the project volume. In the next section, we conclude with a discussion of these results.

5. Conclusion

Our objective in this paper was to develop a positive model of the pricing decision of a social entrepreneur in the light of other exogenous and limited third-party funds. Beside the user fee, we assumed the entrepreneur to handle congestion by applying a non-price rationing instrument. It enables the entrepreneur to provide the good to her most valued applicants that are able to pay the user fee. In line with the existing literature on the allocation of public goods, we assumed that a recipient is preferred more, the poorer the person is. Accordingly, we proposed a utility function which accounts for the entrepreneur's degree of poverty aversion. Subject to a nonprofit-condition, the entrepreneur maximizes the aggregated value of served individuals.

Concerning the optimal user fee and, correlated with it, the optimal project volume, we found three qualitatively different outcomes. First, given that the entrepreneur shows no poverty aversion and values all individuals equally, she decides for the user fee which maximizes the project size. Rationing arises exclusively for the poorest applicants who lack the necessary payment ability. Second, if poorer individuals receive a larger value than wealthier applicants, allocations arise, in which a moderate user fee is chosen and applicants on both ends of the income scale are rationed. Finally, given a sufficiently high poverty aversion, the good is allocated for free and the poorest individuals receive the good. In this case, the entrepreneur exclusively rations the wealthiest applicants by use of the non-price allocation mechanism.

As we have shown with our analysis, the introduction of a poverty aversion parameter into the entrepreneur's utility function enables us to explain observable nonprofit practices. There are social businesses being confronted with substantial congestion but, simultaneously, do not charge user fees at all, such as soup kitchens or homeless shelter. At the other extreme, there may be nonprofit businesses in similar situations charging sufficiently high prices to supply all applicants, such as university cafeterias or youth hostels. One can also observe organizations which set positive user fees and face excess demand. Consider micro health insurance schemes in India. Recipients pay relatively low insurance premiums but only certain population groups gain access.[15]

Our analysis additionally showed that an increase of donations might not necessarily lead to an increase of the project volume. Entrepreneurs with relatively high levels of poverty aversion will wish to reduce their user-fee revenues to an even larger extent, although this theoretical phenomenon has yet to be confirmed empirically. Nevertheless, the result should be of particular interest to lead donors, typically granting a significant and often the largest part of the initial financial need of social entrepreneurs.[16] If donor and entrepreneur disagree on the optimal quantity and composition of recipients, their regulation in form of a variation of the donation volume may have unintended effects which should be taken into account.

[15] See McCord et al. (2001).
[16] See Andreoni (1998, 2006).

Appendix

Proof of proposition 5. Let $D \in [0, \overline{n}(0) \cdot c)$ and $\alpha < \hat{\alpha}$ with

$$\hat{\alpha} = \left[ln\left(1 - \frac{1}{\overline{n}_F \cdot c}\right) \Big/ (ln\,\overline{n} - ln\,\underline{n}) \right]\Bigg|_{F=0} .$$

Then, according to proposition 3, $F^* > 0$.[17] Now, consider equation (12). With $f = f^* > 0$, an increase in donations enlarges the entrepreneur's total income by $c/(c - f^*)$. Consequently, an increase in donations leads to a decrease of the optimal project volume if $dF^*/dD < -c/(c - f^*)$. Applying the implicit function theorem to the first-order condition yields

$$\frac{dF^*}{dD} = -\frac{\partial^2 U(F;D)/\partial F \partial D}{\partial^2 U(F;D)/\partial F^2}\Bigg|_{F=F^*} < -\frac{c}{c - f^*} ,$$

which can be rearranged to

$$\left[\frac{\partial^2 U(F;D)}{\partial F^2} - \frac{c-f}{c} \cdot \frac{\partial^2 U(F;D)}{\partial F \partial D} \right]\Bigg|_{F=F^*} > 0 .$$

$\partial^2 U(F;D)/\partial F^2$ is given by equation (10) and $\left(\partial^2 U(F;D)/\partial F \partial D\right)\big|_{F=F^*}$ by equation (13). Hence, the optimal project volume decreases if

$$\text{(A.1)} \quad \Omega := \left[\frac{\partial^2 U(F;D)}{\partial F^2} - \frac{c-f}{c} \cdot \frac{\partial^2 U(F;D)}{\partial F \partial D} \right]\Bigg|_{F=F^*} = \Bigg[\left(\overline{n}^{\alpha} - \underline{n}^{\alpha}\right) \cdot \overline{n}_{FF} + \alpha \cdot \overline{n}_F^2 \cdot \left[\overline{n}^{\alpha-1} - \left(1 - \frac{1}{\overline{n}_F \cdot c}\right) \cdot \underline{n}^{\alpha-1} \right] \Bigg]\Bigg|_{F=F^*} > 0 .$$

The two terms of condition (A.1) characterize the change of the crowding-out effect due to an increase in user-fee revenues. The first term is positive by definition and the second term is nonnegative for all $\alpha \geq \alpha'$, with

$$\alpha' = 1 + \left[ln\left(1 - \frac{1}{\overline{n}_F \cdot c}\right) \Big/ (ln\,\overline{n} - ln\,\underline{n}) \right]\Bigg|_{F=F^*} .$$

---

[17] Although $\hat{\alpha}$ is not defined for $D = 0$, recall that, according to proposition 1, all entrepreneurs charge user fees.

Next, we show that a unique $\breve{\alpha} \in (0,\alpha')$ exists for which Ω is zero. Hence, Ω is positive for all $\alpha > \breve{\alpha}$ and negative for all $\alpha < \breve{\alpha}$.

Rearranging equation (A.1) yields

(A.2) $\widetilde{\Omega} := y(\alpha,\underline{n}) - z(\alpha,\underline{n})$,

with

$$y(\alpha,\underline{n}) := \left[\left(\overline{n}_{FF} + \alpha \cdot \overline{n}_F^2 \cdot \frac{1}{\overline{n}}\right) \cdot \left(\frac{\overline{n}}{\underline{n}}\right)^{\alpha}\right]\Bigg|_{F=F^*}$$

and

$$z(\alpha,\underline{n}) := \left[\overline{n}_{FF} + \alpha \cdot \overline{n}_F^2 \cdot \left(1 - \frac{1}{\overline{n}_F \cdot c}\right) \cdot \frac{1}{\underline{n}}\right]\Bigg|_{F=F^*}.$$

With $\overline{n} > \underline{n}$, $\overline{n}_F < 0$, and $\overline{n}_{FF} > 0$, $y(\alpha,\underline{n})$ is the product of a linear and a convex increasing function of α. Hence, $y(\alpha,\underline{n})$ is also increasing and convex in α. On the other hand, $z(\alpha,\underline{n})$ is linearly increasing in α. Consequently, the difference of both terms, $\widetilde{\Omega}$, has maximally two roots. Apparently, one is given for $\alpha = 0$.[18] There exists a second root for $\alpha = \breve{\alpha} > 0$ if and only if $(dy(\alpha,\underline{n})/d\alpha)|_{\alpha=0} < (dz(\alpha,\underline{n})/d\alpha)|_{\alpha=0}$, i.e.

$$y_\alpha := \left(\frac{dy(\alpha,\underline{n})}{d\alpha}\right)\Bigg|_{\alpha=0} = \overline{n}_F^2 \cdot \frac{1}{\overline{n}} + \overline{n}_{FF} \cdot ln\left(\frac{\overline{n}}{\underline{n}}\right) < \overline{n}_F^2 \cdot \left(1 - \frac{1}{\overline{n}_F \cdot c}\right) \cdot \frac{1}{\underline{n}} = \left(\frac{dz(\alpha,\underline{n})}{d\alpha}\right)\Bigg|_{\alpha=0} =: z_\alpha .$$

This condition holds since $\alpha \to 0$ implies that $F^* \to F_{max}$ and $\underline{n} \to 0$.[19] Although the limits of y_α and z_α are infinity for $\underline{n} \to 0$, the application of l'Hôpital's rule shows that y_α and z_α diverge and $z_\alpha > y_\alpha$ results:

[18] This technical result does not imply that non-poverty-averse entrepreneurs do not change their project size if donations increase. Rather, in line with proposition 4, non-poverty-averse entrepreneurs behave project-size maximizing. Consequently, their project volume increases with higher donations. The zero-value of equation (A.1) emanates from the fact, that a crowding-out effect does not exist for $\alpha = 0$ and, hence, does not change if user-fee revenues are increased.

[19] Rearranging the first-order condition (setting equation (9) zero) yields $\underline{n} = \overline{n} \cdot \left[1 - 1/\left(\overline{n}_F \cdot c\right)\right]^{-1/\alpha}$ with $\lim_{\alpha \to 0} = \overline{n} \cdot \left[1 - 1/\left(\overline{n}_F \cdot c\right)\right]^{-1/\alpha} = 0$.

$$\lim_{\underline{n}\to 0}\frac{y_\alpha}{z_\alpha}=\lim_{\underline{n}\to 0}\left[\frac{\left(\partial^2 y(\alpha,\underline{n})/\partial\alpha\partial\underline{n}\right)}{\left(\partial^2 z(\alpha,\underline{n})/\partial\alpha\partial\underline{n}\right)}\right]\Bigg|_{\alpha=0}=\frac{\overline{n}_{FF}\cdot\underline{n}}{\overline{n}_F^2\cdot[1-(1/(\overline{n}_F\cdot c))]}=0\,.$$

Consequently, there exists a unique $\breve{\alpha}\in(0,\alpha')$ for which the value of $\widetilde{\Omega}$, or respectively Ω, is zero.

Yet, we assumed that $\alpha<\hat{\alpha}$ and derived the requirement, that $\alpha>\breve{\alpha}$. Consequently, an increase in donations leads to a reduction of the optimal project volume if $\breve{\alpha}<\hat{\alpha}$ and $\alpha\in(\breve{\alpha},\hat{\alpha})$. However, $\breve{\alpha}<\hat{\alpha}$ requires a sufficiently low level of donations. For $D\to\overline{n}(0)\cdot c$, $ln\overline{n}-ln\underline{n}$, which determines $\hat{\alpha}$ and α', is infinitely large, such that $\hat{\alpha}\to 0$ and $\alpha'\to 1$. Since $\breve{\alpha}<\alpha'$, it must hold that $\breve{\alpha}\in(0,1)$ and, consequently, $\breve{\alpha}>\hat{\alpha}$. In other words, given that the amount of donations is relatively high, all entrepreneurs react with an enlargement of the project volume on an increase in donations. In contrast, for $D\to 0$, $(ln\overline{n}-ln\underline{n})|_{F=0}\to 0$ and, hence, $\hat{\alpha}\to\infty$. According to proposition 3, all entrepreneurs with $\alpha<\hat{\alpha}$ choose F^*>0. Consequently, $(ln\overline{n}-ln\underline{n})|_{F=F^*}>0$ and $\alpha'\in(1,\infty)$. Since $\breve{\alpha}<\alpha'$, it holds that $\breve{\alpha}<\hat{\alpha}$. As a result, there exists a specific level of donations D' such that $\breve{\alpha}=\hat{\alpha}$, if $D=D'$, and $\breve{\alpha}<\hat{\alpha}$, if $D\in[0,D')$. Hence, for all $D\in[0,D')$ an increase in donations leads to a reduction of the optimal project volume F^*+D, if $\alpha\in(\breve{\alpha},\hat{\alpha})$. **Q.e.d.**

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