WORKING PAPER SERIES

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Working Paper No. 6/2016

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG

FACULTY OF ECONOMICS AND MANAGEMENT

Impressum (§ 5 TMG)
Herausgeber:
Otto-von-Guericke-Universität Magdeburg
Fakultät für Wirtschaftswissenschaft
Der Dekan

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http://www.fww.ovgu.de/femm

Bezug über den Herausgeber
ISSN 1615-4274

An Exact Solution Approach to the Single-Picker Routing Problem in Warehouses with an Arbitrary Block Layout

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Abstract

The Single-Picker Routing Problem (SPRP) arises in warehouses when items have to be retrieved from their storage locations in order to satisfy a given demand. It deals with the determination of the sequence according to which the respective locations have to be visited. The storage locations in the warehouse are typically arranged in a specific way and constitute a so-called block layout. Using this structure, Scholz et al. (2016) proposed a model to the SPRP in a single-block layout whose size (in terms of number of variables and constraints) is independent of the number of locations to be visited. They briefly described how the model can be extended to deal with multiple blocks, but also stated that its size will drastically increase. In this paper, the extension of this formulation is considered and different scenarios are pointed out which can be used to significantly decrease the size of the model. By means of numerical experiments, it is demonstrated that the size of the formulation can be decreased by up to 60%, resulting in a reduction of computing time by up to 99.5%. Furthermore, it is shown that computing times do not increase with an increasing number of blocks, which is a major advantage of the model as no efficient solution approach to the SPRP is available able to deal with more than two blocks.

Keywords: Traveling Salesman, Order Picking, Picker Routing

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1 Introduction

Every day, a warehouse receives a high amount of items in large lot-sizes which have to be stored and redistributed in small volumes based on thousands of daily customer orders (Wäscher, 2004). The retrieval of requested items from their storage locations (order picking) accounts for up to 55% of the costs in a warehouse (Tompkins et al., 2010), which can be attributed to the fact that, in many warehouses, human operators (order pickers) are assigned to execute the picking process (de Koster et al., 2007). This process is mainly composed of traveling through the warehouse, searching for the respective items and picking them from their storage locations, while traveling consumes approximately 50% of the total working time of a picker (Tompkins et al., 2010). In order to reduce the travel time, different procedures can be applied which are improving the allocation of the articles in the warehouse (storage assignment), grouping customer orders into picking orders (order batching) and determining a sequence, in which the order picker can retrieve the items by covering only a short distance (picker routing).

The last-mentioned procedure is a part of the so-called Single-Picker Routing Problem (SPRP) which deals with finding a tour of minimum length including all storage locations of requested items (Ratliff & Rosenthal, 1983). The SPRP represents a special case of the well-known Traveling Salesman Problem (TSP) and, thus, solution approaches to the TSP can be applied to solve the SPRP. However, the storage locations in the warehouse typically constitute a block layout (Roodbergen, 2001) which is totally neglected when modeling the SPRP as a general TSP and mainly results in two problems. First, problem-specific solution approaches to the SPRP may outperform TSP approaches by far in terms of computing time. Ratliff & Rosenthal (1983) and Roodbergen & de Koster (2001) developed efficient exact algorithms for the SPRP in warehouses composed of one and two blocks. These algorithms are able to solve any practical-sized SPRP within fractions of a second which is not possible by using TSP algorithms. Second, problem-specific approaches may easier be adapted to further constraints arising in practical applications. For example, order pickers in practice seem to prefer simple routes which are easy to memorize (Petersen & Schmenner, 1999). Furthermore, some aisles may be very narrow making it impossible for the order picker to change his moving direction within the aisle which has to be considered when constructing a tour. The possibility of depositing items at the end of each picking aisle represents another modification which often arises in practical applications (de Koster & van der Poort, 1998). All of these constraints can easily be integrated in problem-specific approaches to the SPRP but may quite difficult be taken into account by TSP algorithms (Scholz et al., 2016).

Apart from approaches of Ratliff & Rosenthal (1983) and Roodbergen & de Koster (2001), no efficient algorithm is available to the SPRP. In particular, no efficient algorithm exists which can deal with an arbitrary number of blocks (Roodbergen, 2001). Scholz et al. (2016), therefore, designed a problem-specific model formulation to the SPRP which can be applied to any block layout. They presented the model for a single-block layout and then demonstrated how to extend it to multiple blocks. However, the extension of this formulation leads to a model whose size is multiplied by the number of blocks, which may result in increasing computing times and a restricted applicability of the model to SPRPs in warehouses with a larger number of blocks. Furthermore, especially when applied to problems in which only few items have to be picked in a relatively large warehouse, the model formulation of Scholz et al. (2016) leads to quite unsatisfactory results.

In order to improve the performance of this approach and to make it applicable to any arbitrary block layout, several possibilities to reduce the size of the mathematical model are considered in this paper. This is done by reducing the size of the graph the model formulation is based on. First, a so-called pyramid structure is introduced in order to cut off the components of the graph representing parts of the warehouse which do not have to be included in an optimal tour. In a next step, different scenarios according to the distribution of the storage locations to be visited are considered in order to further reduce the number of vertices and arcs in the graph. By means of numerical experiments, it is shown that these considerations lead to a drastic improvement regarding the number of variables and constraints in the model, resulting in a significant reduction of computing time. Furthermore, the results from the experiments indicate that computing times do not increase with an increasing number of blocks, implying that the formulation can be applied to SPRPs in warehouses with an arbitrary number of blocks.

The remainder of this paper is organized as follows: The SPRP is introduced and the related literature is reviewed in the next section. As it is the basis for the considerations in this paper, Section 3 contains a brief review of the graph construction according to Scholz et al. (2016) and the different components of the corresponding model. Different circumstances are then considered under which the size of the graph can be reduced significantly (Section 4). For this purpose, a pyramid structure is introduced to the graph and several special cases of the distribution of storage locations are taken into account. Section 5 comprises the design of the numerical experiments and the results obtained from them. The paper concludes with a summary and an outlook on further research given in Section 6.

2 Single-Picker Routing Problem

2.1 Problem Description

The SPRP consists of finding a tour through the warehouse which starts and ends at the depot while all requested items are retrieved. In a warehouse, the items are stored on pallets or racks which typically constitute a block layout (Roodbergen, 2001). According to this layout, two different types of aisles have to be distinguished, namely picking and cross aisles. Picking aisles are arranged parallel to each other and include the storage locations of the items. Thus, for retrieving an item, the corresponding picking aisle has to be entered. In contrast, cross aisles do not contain any storage locations. They are required for changing over from one picking aisle to another. Furthermore, cross aisles divide the picking area of the warehouse into blocks and picking aisles into subaisles. A block is the part of the picking area located between two adjacent cross aisles, whereas a subaisle is defined as the part of a picking aisle corresponding to the same block. A warehouse with $q+1$ cross aisles and m picking aisles consists of q blocks and $q \cdot m$ subaisles. The corresponding layout is called a q-block layout.

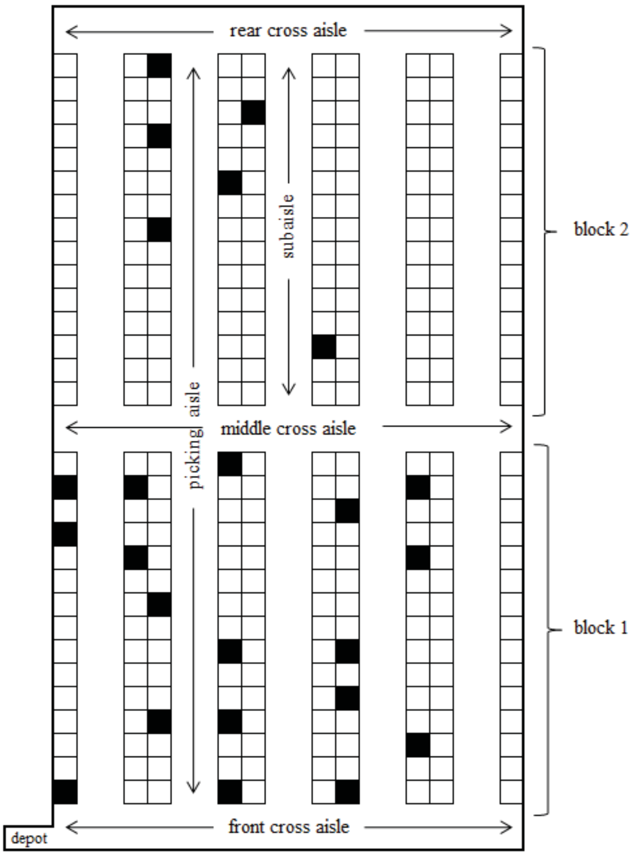

Fig. 1: Two-block layout

In Fig. 1, a picking area with two blocks and 5 picking aisles is depicted. The rectangles depict the storage locations and the black rectangles are the locations of the requested items (pick locations). The depot is situated in front of the leftmost picking aisle. As the picking area is divided into two blocks, three cross aisles exist, namely the front, the rear and a middle cross aisle, which can be used to switch between picking aisles. The front (rear) cross aisle represents the cross aisle nearest to (farthest from) the depot. In order to change over from one block to another, the middle cross aisle has to be crossed.

The retrieval of requested items from their storage locations is carried out by an order picker who walks or drives through the warehouse using a picking device. Among all operations required for retrieving items and returning them to the depot, traveling consumes a major part of the working time of an order picker (Tompkins et al., 2010). Furthermore, other components (such as setup times at the depot or searching and picking times at the racks) can considered to be constant (Caron et al., 2000), as they are independent of the sequence according to which the items are to be picked. Minimizing the total travel time is, therefore, a common objective when dealing with the SPRP. Assuming a constant travel velocity, the travel time is a linearly increasing function of the travel distance (Jarvis & McDowell, 1991), implying that minimizing the travel time is equivalent to finding a tour of minimum length.

The SPRP can then be defined as follows (Ratliff & Rosenthal, 1983; Scholz et al., 2016): Given a set of items to be picked from known storage locations, in which sequence should the locations be visited such that the total length of the corresponding tour is minimized?

2.2 Literature review

The SPRP deals with the determination of a tour of minimum length including the depot and the respective pick locations. Thus, it represents a special case of the well-known TSP which means that general TSP formulations may be appropriate for modeling the SPRP. For the first time, Dantzig et al. (1954) proposed a model formulation to the TSP. However, the number of constraints required for excluding subtours exponentially increases with the number of pick locations as it is the case for several other mathematical programming formulations to the TSP (Gouveia & Pires, 2001). Due to memory restrictions, these formulations cannot be applied to large SPRPs.

In compact formulations, the number of variables and constraints only increases polynomially with the number of pick locations which allows for explicitly representing larger SPRPs. A variety of compact formulations to the TSP exists (Öncan et al., 2009). A major drawback of these formulations is the quality of the lower bound obtained by solving the corresponding LP relaxation which may be much weaker than the lower bound generated by application of the Dantzig formulation (Padberg & Sung, 1991). Due to this reason, compact TSP formulations may not be able to deal with larger SPRP instances either. This has also been demonstrated by Scholz et al. (2016) who tested the performance of different TSP formulations applied to the SPRP.

A more appropriate way to model the SPRP has been presented by Burkard et al. (1998) who formulated the SPRP as a Steiner TSP. In a Steiner TSP, the set of vertices V can be divided into a subset P and a subset $V \setminus P$ (Steiner points). A Steiner tour has then to include all vertices of the subset P. Steiner points may but do not have to be visited. Furthermore, vertices and edges are allowed to be visited and used more than once. The Steiner TSP consists of finding a Steiner tour of minimum length (Burkard et al., 1998).

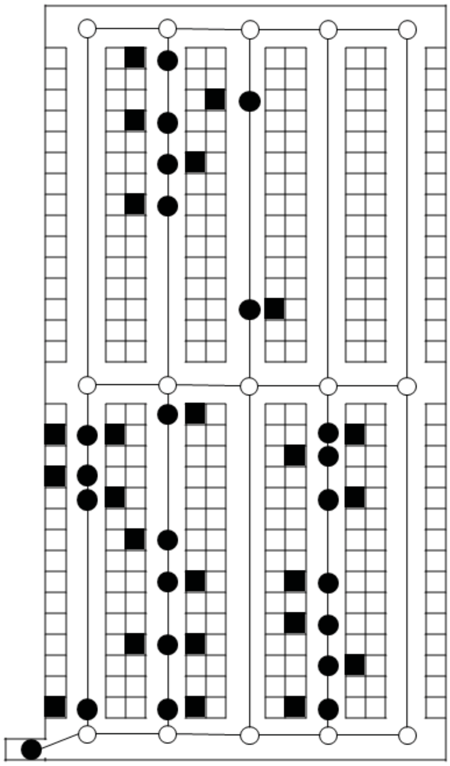

Fig. 2: Illustration of a Steiner TSP

In Fig. 2, a representation of the SPRP as a Steiner TSP is given. The black vertices represent the pick locations as well as the location of the depot and, thus, have to be visited. They constitute the subset P. As for the SPRP, the Steiner points (white vertices) are the intersections between a cross aisle and a picking aisle which enable the order picker to switch between subaisles. The same intersection may be used to enter and leave a certain subaisle. Furthermore, not all subaisles necessarily have to be visited and, therefore, some of the intersections may not be included in the tour.

As can be seen in Fig. 2, the representation of the SPRP as a Steiner TSP results in a quite sparse graph, as the maximum degree of a vertex amounts to 4. Thus, this kind of representation requires much less edges than the corresponding TSP graph would include. Since the number of variables is only dependent on the number of edges in many TSP formulations, the consideration of a sparse graph can be expected to result in model formulations of smaller size. Due to this fact, Letchford et al. (2013) developed compact formulations to the Steiner TSP. They demonstrated that the application of these formulations outperforms general TSP formulations in terms of computing time if the number of Steiner points is large compared to the total number of vertices. Scholz et al. (2016) applied the most promising Steiner TSP formulation to SPRP instances and showed that this model formulation is much more appropriate for solving the SPRP than general TSP formulations. However, the results from the numerical experiments also indicated the limitations of the application of the Steiner TSP formulation. Since the size of this formulation is dependent on both the number of pick locations and the number of intersections, the model does not seem to be suitable for large warehouses with many items to be retrieved. Furthermore, Steiner TSP as well as general TSP solution approaches do not allow for problem-specific modifications, such as the construction of simple tours in which the number of changes in direction is limited or the consideration of multiple locations where retrieved items can be deposited (Scholz et al., 2016). For the integration of such aspects, the application of problem-specific approaches to the SPRP is inevitable.

For warehouses with one or two blocks, efficient problem-specific solution approaches exist. Ratliff & Rosenthal (1983) proposed an exact algorithm for the SPRP in a single-block layout which is based on dynamic programming. The computational effort of the algorithm increases linearly with the number of pick locations and picking aisles and, therefore, can be used to solve any practical-sized instance within fractions of a second. Roodbergen & de Koster (2001) extended this algorithm to the two-block case. However, they stated that it would be very difficult to further extend this approach to layouts with more than two blocks. Up to now, no efficient approach to the SPRP with an arbitrary block layout exists (Roodbergen, 2001).

The model formulation of Scholz et al. (2016) is the only problem-specific exact solution approach to the SPRP which can deal with an arbitrary number of blocks. Based on the representation of the SPRP as a Steiner TSP and some characteristics of optimal SPRP solutions, a graph is first constructed whose size is independent of the number of pick locations. A model formulation is then obtained by applying a TSP formulation to the graph. The construction of the graph as well as the components of the model formulation are explained in greater detail in Section 3. Scholz et al. (2016) presented the formulation for the single-block case and demonstrated that general and Steiner TSP formulations are outperformed by far in terms of computing time. They also explained how the model can be extended to multi-block layouts. However, in this case, the size of the graph (in terms of vertices and arcs) is multiplied by the number of blocks which also leads to a significant increase of the size of the resulting model formulation.

Thus, it can be expected that this formulation is not suitable for SPRPs in warehouses with a larger number of blocks. We, therefore, consider modifications to the graph by which its size is drastically reduced. These modifications include the introduction of a so-called pyramid structure as well as the investigation of different cases of item distribution and are dealt with in Section 4.

3 Model formulation of Scholz et al. (2016)

3.1 Graph construction

As mentioned before, the underlying graph for the model formulation of Scholz et al. (2016) is based on the Steiner TSP representation of the SPRP (see Fig. 2). In the formulation, a directed graph is considered, i.e. each edge is replaced by two reverse arcs representing possible movements of an order picker. Arcs between Steiner points relate to movements within cross aisles, whereas the other arcs (except for arcs incident to the vertice representing the depot) stand for movements in subaisles. The latter can be further restricted by considering the structure of optimal solutions to the SPRP. According to Ratliff & Rosenthal (1983) only six possible combinations of arcs within a subaisle have to be taken into account for constructing an optimal tour (see Fig. 3).

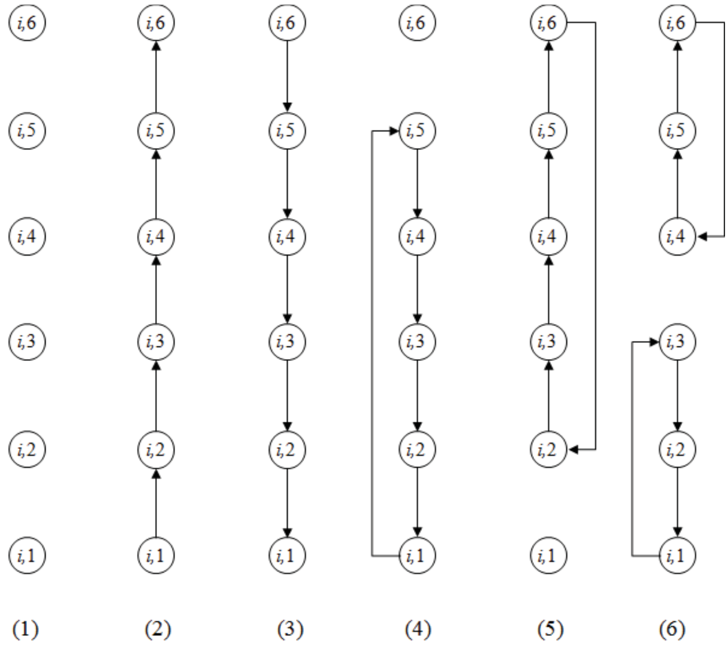

Fig. 3: Movements within a subaisle to be considered for constructing an optimal tour

In an optimal tour, each subaisle is visited at most twice. Entering a subaisle is not necessary if no requested items are located in the subaisle (see Fig. 3 (1)). If a subaisle is visited exactly once, then it is either entered from a cross aisle and left via the other adjacent cross aisle or the same cross aisle is used for entering and leaving the subaisle. In the first case, the items are picked while traversing the subaisle (see Fig. 3 (2) and (3)). In the second case, the order picker returns at the pick locations farthest from the cross aisle from which the subaisle has been entered (see Fig. 3 (4) and (5)). If a subaisle is visited twice, this can be done in two different ways. First, the subaisle may be traversed two times, once in each direction. This case may occur if the particular subaisle is required for switching from one block to another in such a way that the total tour length is minimized. No additional arc combination has to be considered for representing this kind of movements as it can be obtained by combining the arcs depicted in Fig. 3 (2) and (3). Second, a subaisle can be visited twice by entering and leaving the subaisle via the same cross aisle, once for each adjacent cross aisle. If the tour is optimal, this is done in such a way that the largest distance between two adjacent pick locations or a pick location and the adjacent cross aisle (called largest gap) is maximized. From each adjacent cross aisle, the subaisle is then entered up to its largest gap (see Fig. 3 (6)).

Only six vertices are required for representing a subaisle (Scholz et al., 2016). These vertices correspond to (see Fig. 3):

(a) the intersection between subaisle i and an adjacent cross aisle (vertices $[i,f]$ and $[i,b]$),

(b) the pick locations defining the largest gap (vertices $[i,2]$ and $[i,3]$) and

(c) the pick locations nearest to an adjacent cross aisle (vertices $[i,1]$ and $[i,4]$).

By introducing these vertices and the arcs depicted in Fig. 3 for each subaisle as well as arcs representing movements required to switch between picking aisles, Scholz et al. (2016) constructed a graph whose size is independent of the number of pick locations. In order to obtain a model formulation to the SPRP, they decided to apply a TSP formulation to the graph. Since these formulations require that a vertex is visited at most once, Scholz et al. (2016) modified the graph by copying the vertices based on the maximum number of visits in an optimal tour. This results in one copy for each vertex representing a pick location, two copies for front and rear cross aisle vertices and three copies for vertices which represent intersections with the middle cross aisle. The resulting graph is depicted in Fig. 4.

The denotation of the vertices is as follows: The first entry indicates the direction in which the tour is proceeded after visiting the vertex, where "r" and "l" symbolize that the next step will be a movement to the right and to the left, respectively. Movements towards the rear and the front cross aisle are indicated by "u" ("up") and "d" ("down"). The second component stands for the number of the block, if the vertex represents a pick location, or the number of the cross aisle, otherwise. Cross aisles are enumerated from 1 to $p+1$, where p is the number of blocks and cross aisle 1 is the cross aisle nearest to the depot.

The third component characterizes the number of the corresponding picking aisle. Picking aisles are enumerated in ascending order from left to right, i.e. picking aisle 1 is the leftmost picking aisle, while m denotes the rightmost aisle. Furthermore, vertices representing a pick location have an additional fourth component indicating the position of the vertex in the corresponding subaisle. Finally, vertex "0" represents the location of the depot.

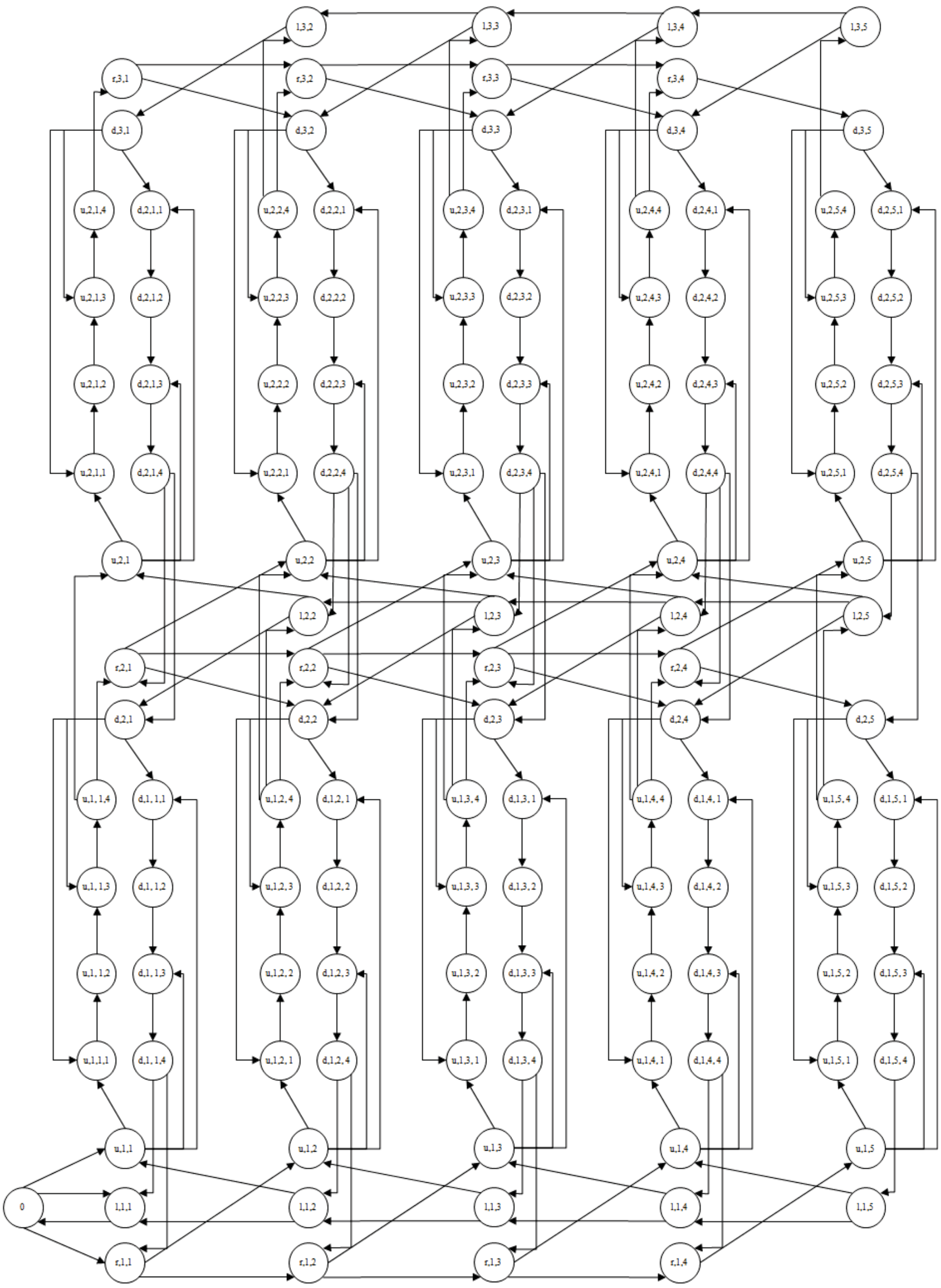

Fig. 4: Graph for a SPRP with two blocks and five picking aisles

3.2 Components of the model formulation

The graph to the SPRP depicted in Fig. 4 allows for claiming that each vertex is visited at most once in an optimal order picking tour as it is the case for a general TSP. The only difference consists in the fact that some vertices may not be included in the tour. Therefore, Scholz et al. (2016) modified a compact TSP formulation and applied it to the graph in order to obtain a problem-specific formulation to the SPRP. The resulting SPRP formulation contains the following components:

- Degree constraints: Each vertex visited has to be left afterwards.
- Subtour elimination constraints: The tour has to be connected.
- Depot inclusion constraint: The depot has to be a part of the tour.
- Pick location inclusion constraints: Each pick location has to be visited for retrieving all items.

In the model formulation two different types of variables are used. A binary variable is introduced for each arc of the graph indicating whether the arc is included in the tour or not. Using these variables the degree constraints can be formulated by claiming that, for each vertex, its indegree has to be equal to its outdegree. The second type of variables is required for excluding subtours, which is achieved using so-called single commodity flow constraints (Gavish & Graves, 1978). The general concept of these constraints consists in assuming the existence of a single commodity type of which one unit has to be delivered to each location included in the tour. For this reason, a non-negative variable is introduced for each arc indicating the amount of the commodity passing the arc. In this way, vertices are enumerated according to their appearance in the tour excluding subtours.

Since not all vertices have to be visited, the introduction of additional constraints is necessary in order to ensure that the depot is included in the tour and all items are retrieved. The inclusion of the depot can be guaranteed by requiring the outdegree of the corresponding vertex "0" not to be smaller than 1. For the pick location inclusion constraints, the construction of the graph has to be considered as not all pick locations are represented by the graph. Instead, a constant number of vertices is used to represent a subaisle. These vertices correspond to the pick locations adjacent to a cross aisle and the locations defining the largest gap (see Fig. 3 and Fig. 4). All requested items are located between the two pick locations adjacent to a cross aisle. By definition, no pick location can be situated between the two locations specifying the largest gap. It can then be ensured that all requested items are retrieved in a subaisle i of a block q by requiring that either arc $([u,q,i,1],[u,q,i,2])$ or $([d,q,i,3],[u,q,i,4])$ and either arc $([u,q,i,3],[u,q,i,4])$ or $([d,q,i,1],[d,q,i,2])$ are included in the tour.

Each arc in the graph results in two variables in the model formulation. Furthermore, one degree and one subtour elimination constraint is introduced for each vertex, while the exclusion of subtours requires an additional constraint for each arc. Thus, as it is the case for the graph, the size of the resulting model formulation increases linearly with the number of subaisles and is independent of the number of pick locations. This kind of representation is a great advantage if a lot of items have to be collected in a subaisle. Scholz et al. (2016) demonstrated that this model formulation clearly outperforms general or Steiner TSP formulations in terms of computing time required to solve the model, if the ratio $n/(m \cdot p)$ (where n is the number of pick locations, m denotes the number of picking aisles and p stands for the number of blocks) is not too small. However, in warehouses with multiple blocks only a small number of locations have to be visited in a subaisle due to the increasing number of subaisles. In the graph constructed by Scholz et al. (2016), each subaisle is represented by a constant number of vertices regardless of the number of pick locations in that aisle. Even if the subaisle does not contain any pick location, eight vertices are introduced for the representation of the subaisle. Thus, a large number of vertices may be required for representing only a small number of pick locations. Therefore, it can be concluded that the graph will not be appropriate for providing the basis of a model formulation to SPRPs in warehouses with several blocks as such warehouses tend to contain a quite large number of subaisles.

4 Considerations to reduce the size of the graph

4.1 Pyramid structure

As can be seen from the statements above, the main drawback of the graph consists in its representation of subaisles containing only few or even no pick locations. In order to reduce the size of the graph, cases are first identified in which subaisles do not have to be represented at all.

Large warehouses containing a high number of racks (and subaisles) can be used to store many different articles. However, due to the size of the warehouse, order pickers may cover great distances when retrieving requested items. In order to reduce the distance to be covered by order pickers, articles are assigned to storage locations in a specific way. For example, articles may be assigned according to the class-based storage assignment procedure. When applying this procedure, articles with a high expected demand are stored near to the depot (Petersen & Schmenner, 1999). Many SPRPs arising in those settings will then be characterized by not containing pick locations in subaisles very far away from the depot. For the determination of an optimal solution to such a SPRP, some subaisles may be excluded from the graph since an order picker will never visit the corresponding part of the warehouse. In the following, it is pointed out which criteria have to be fulfilled for allowing a subaisle to be removed from the graph while guaranteeing that an optimal solution can still be found.

Let $B = \{1, \ldots, p\}$ be the set of blocks and $\overline{\mathscr{M}}_q$ the rightmost subaisle of block $q \in B$ containing at least one requested item. Furthermore, $\overline{m}_q$ denotes the rightmost subaisle of block $q \in B$ to be included in the graph in order to construct an optimal order picking tour. Obviously, it must hold $\overline{m}_q \geq \overline{\mathscr{M}}_q$ for each block $q \in B$ since each subaisle containing a requested item has to be visited. Another reason for visiting a subaisle is to change over to another cross aisle in order to reach an adjacent block. Due to this fact, simply removing all subaisles containing no pick location from the graph could result in longer tours. Even if no subaisle with pick locations exists on the right of the same block, the removal of an empty subaisle may lead to the exclusion of an optimal solution (e.g. if this subaisle is required for reaching a subaisle of another block in a way which minimizes the tour length). Thus, it is necessary to also take into account the characteristics of other blocks, as a subaisle of a block $q \in B$ must not be removed if both, the adjacent lower and upper block, include subaisles which have to be considered and are located further on the right, i.e. if it holds $\overline{\mathscr{M}}_{q-1} > \overline{\mathscr{M}}_q$ and $\overline{\mathscr{M}}_{q+1} > \overline{\mathscr{M}}_q$.

The index $\overline{m}_q$ of the rightmost subaisle of block $q \in B$ to be included in the graph can then be determined by solving the following mathematical program. (Note that $\overline{m}_{p+1}$ is defined to be 0.)

$$\min \ \overline{m}_q \tag{1}$$

$$\overline{m}_q \geq \overline{\mathscr{M}}_q \tag{2}$$

$$\overline{m}_q \geq \overline{m}_{q+1} \tag{3}$$

The objective function minimizes the index of the rightmost subaisle to be considered in block q. Constraint (2) guarantees that no subaisle of block q containing at least one requested item is removed from the graph. Constraint (3) ensures that the index of the rightmost subaisle to be considered is not larger for block $q+1$ than for block q. Due to this constraint, the resulting structure of the graph is referred to as a pyramid structure. The constraint guarantees that no optimal solution is excluded by removing subaisles of block q from the graph.

Since $\overline{m}_{p+1} = 0$, the optimal solution for block p is $\overline{m}_p = \overline{\mathscr{M}}_p$. Then $\overline{m}_q$ can be determined successively for the remaining blocks $q \in B \setminus \{p\}$:

$$\overline{m}_q = \max\left\{\overline{\mathscr{M}}_q;\, \overline{m}_{q+1}\right\}. \tag{4}$$

An analogue procedure can be applied to subaisles located in the first (leftmost) subaisles of a block. Let $\underline{\mathscr{M}}_q$ be the leftmost subaisle of block $q \in B$ containing a pick location and $\underline{m}_q$ the leftmost subaisle of block $q \in B$ which has to be considered for constructing an optimal tour. With the same line of argumentation as above, it results $\underline{m}_p = \underline{\mathscr{M}}_p$ and for each block $q \in B \setminus \{1, p\}$, $\underline{m}_q$ can be determined as follows:

$$\underline{m}_q = \min\left\{\underline{\mathscr{M}}_q;\ \underline{m}_{q+1}\right\}. \tag{5}$$

The only difference between these two procedures can be seen in the first block. As for the determination of $\overline{m}_1$, formula (4) is applied, formula (5) cannot be used to compute $\underline{m}_1$. Since the depot is located in front of the leftmost picking aisle, subaisle 1 of the first block has to be considered ensuring a connection to the depot. Even if the first picking aisle does not contain any pick locations, $\underline{m}_1$ has to be set to 1.

For each block $q \in B$, all subaisles located further to the left than subaisle $\underline{m}_q$ or in the right of subaisle $\overline{m}_q$ are then removed from the graph. When removing a subaisle i of a block q, all of the eight vertices representing the pick locations are deleted. Since this aisle will not be entered in an optimal tour, vertices $[u, i, q]$ and $[d, i, q+1]$ can be removed as well. Furthermore, vertices $[r, i-1, q+1]$ and $[l, i, q+1]$, required for switching over to the subaisle, can also be deleted. In conjunction with these vertices, incident arcs are removed. In total, depending on the location of the subaisle, a removal of a single subaisle will reduce the size of the graph by up to 12 vertices and 30 arcs.

4.2 Special cases of item distribution

In the graph of Scholz et al. (2016), the pick locations in each subaisle are represented by 8 vertices, while 18 arcs are required to specify the possible movements within the subaisle (see Fig. 4). The number of vertices and arcs is independent of the number of pick locations. If the subaisle contains only few or even no pick locations, Scholz et al. (2016) introduce dummy vertices and arcs. When dealing with a single-block layout, this procedure does not lead to any problems as the number of pick locations per subaisle is usually sufficiently large. However, in case of multiple blocks, the number of subaisles is multiplied by the number of blocks, resulting in a quite small ratio between the number of pick locations and subaisles. Thus, lots of dummy vertices are to be introduced when dealing with multiple blocks significantly increasing the size of the graph as well as the size of the resulting model.

In the following, it is demonstrated how the number of vertices and arcs for representing locations and movements in a subaisle can be reduced. No dummy vertices will be introduced. Furthermore, special cases of item distribution are identified in which less vertices and arcs are required for the representation of a subaisle. For the sake of simplicity of exposition, a subaisle i of block $q \in B \setminus \{1, p\}$ with $\underline{m}_{q+1} < i < \overline{m}_{q+1}$ is considered.

The largest reduction can be observed when a subaisle does not contain any pick locations (denoted by special case 1). In this case, arcs are only required in order to ensure that this subaisle can be used to switch over to an adjacent cross aisle. On the left hand side of Fig. 5, the subaisle including the storage locations is depicted while the corresponding part of the graph is shown on the right hand side. For entering the subaisle, either vertex $[u,q,i]$ or $[d,q+1,i]$ has to be visited. The order picker can then proceed his tour by going to the left, to the right or by entering a subaisle of an adjacent block. Within the subaisle, no locations to be represented exist, which is why no vertices are to be introduced. The representation of the above mentioned possible movements requires 6 arcs. Thus, this kind of representation gives a reduction of 8 vertices (100%) and 12 arcs (67%) per subaisle.

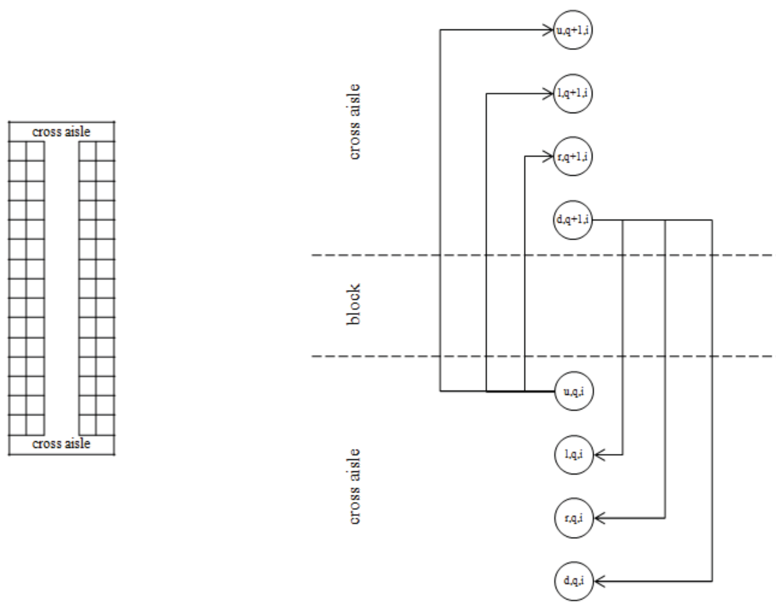

Fig. 5: Representation of a subaisle containing no pick locations

Another considerable size reduction can be obtained if the largest gap lies between cross aisle q and the adjacent pick location (special case 2). Vertex $[u,q,i]$ and vertex pair $([u,q,i,2],[d,q,i,3])$ will then represent the same location (see Fig. 4). In this case, applying a move regarding the largest gap strategy (see Fig. 3 (6)) is the same as performing a return strategy from cross aisle $q+1$ (see Fig. 3 (5)). Vertex $[u,q,i,1]$ represents the pick location nearest to cross aisle q which also defines "the end" of the largest gap. The location of the requested item farthest from cross aisle q is given by vertex $[d,q,i,1]$ (see Fig. 6). In this case, we have single vertices and no vertex pair which means that the distance to vertex $[u,q,i]$ may be different for both vertices. For representing the movements in such a subaisle, 2 instead of 8 vertices (75% reduction) and 10 instead of 18 arcs are required (44% reduction). The same line of argumentation holds if the largest gap lies between cross aisle $q+1$ and the adjacent pick location.

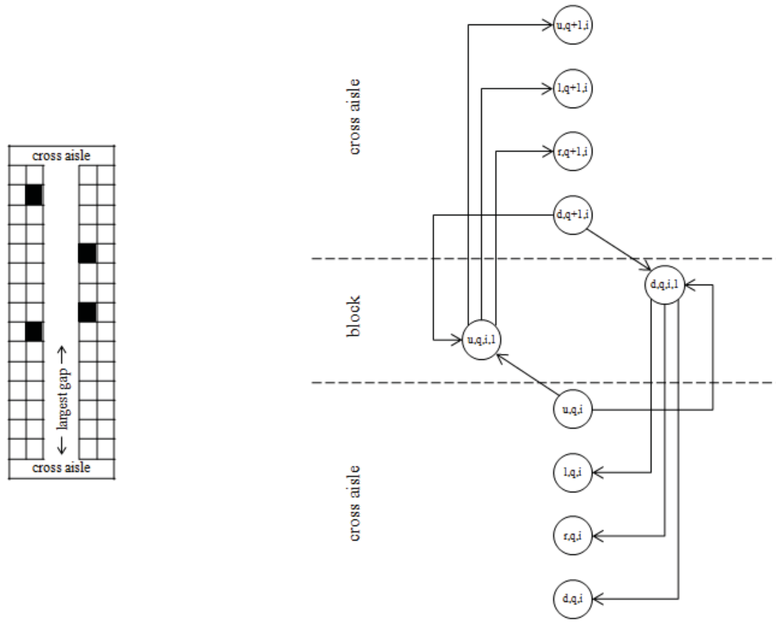

Fig. 6: Representation of a subaisle with largest gap located between a pick location and the adjacent cross aisle

A very simple possibility to reduce the size of the graph arises when only two pick locations are contained in a subaisle while the largest gap is situated between these locations (special case 3). A vertex pair is then introduced for each pick location and the arcs are chosen in such a way that all strategies included in Fig. 3 can be performed. 4 vertices (50% reduction) and 14 arcs (22% reduction) are needed to represent the locations and the movements in the subaisle.

If the pick location adjacent to cross aisle q defines "the beginning" of the largest gap, a slight size reduction can be achieved as the vertex pairs $([u,q,i,1],[d,q,i,4])$ and $([u,q,i,2],[d,q,i,3])$ define the same location (special case 4). Therefore, one vertex pair can be removed from the graph, resulting in a reduction of 2 vertices (25%) and 2 arcs (11%). Analogously, two vertices and arcs can be removed when the pick location adjacent to cross aisle $q+1$ defines "the end" of the largest gap (special case 5).

4.3 Resulting changes regarding the model formulation

As can be seen in Fig. 4, the graph to the SPRP depicted in Fig. 1 includes 125 vertices and 217 arcs. By applying the considerations from above, its size is significantly reduced.

First, a size reduction according to the pyramid structure is realized. In block 2, only the second and the third subaisle contain requested items which results in $\underline{\mathscr{M}}_2 = 2$ and $\overline{\mathscr{M}}_2 = 3$. In the first block, the fourth subaisle is the rightmost subaisle with pick locations resulting in $\overline{\mathscr{M}}_1 = 4$. Since the depot is located in front of picking aisle 1, $\underline{\mathscr{M}}_1$ is equal to 1. Applying formulas (4) and (5) then leads to $\underline{m}_1 = 1$,

$\underline{m}_2 = 2$, $\overline{m}_1 = 4$ and $\overline{m}_2 = 3$, implying that subaisle 5 of block 1 and subaisles 1, 4 and 5 of block 2 can be removed from the graph.

In the second step, according to the special cases of item distribution, further removable vertices and arcs are identified. Starting with block 1, the largest gap in the leftmost subaisle is situated between the pick location nearest to the front cross aisle and an adjacent pick location (special case 4). The analogue case (special case 5) can be observed in subaisle 2. The next subaisle does not contain any requested items (special case 1). The rightmost subaisle to be considered in this block follows the standard case in which the pick locations defining the largest gap and the pick locations nearest to the cross aisles represent different locations, implying that no vertices and arcs can be removed here. In block 2, only subaisles 2 and 3 are to be considered after the introduction of the pyramid structure. In subaisle 2, the largest gap is located between a pick location and the adjacent cross aisle (special case 2), whereas only two pick locations exist with the largest gap situated between them in subaisle 3 (special case 3).

The introduction of the pyramid structure and the consideration of the special cases of item distribution result in a graph still representing the same instance of the SPRP but including 54 vertices and 98 arcs only. Thus, both the number of vertices as well as the number of arcs are reduced by approximately 55%.

As the graph has been modified, changes regarding the model formulation also have to be made. First, variables corresponding to removed arcs have to be deleted. Second, the different components of the system of constraints have to be adjusted. Degree and subtour elimination constraints have to be removed for vertices and arcs which do not exist anymore. The depot inclusion constraint remains unchanged. However, the pick location inclusion constraints are now to be formulated dependent on the storage locations of requested items in a subaisle. No such constraints are required for subaisles corresponding to special case 1 as no requested items are located in these subaisles. For the remaining subaisles, the vertices to be visited are identified and it is then guaranteed that these vertices are included in the tour by claiming their indegree or outdegree not to be smaller than 1, as has been done for subaisles with a standard distribution of requested items.

For a subaisle i of block q, in which the items to be picked are distributed according to special case 2, all pick locations are situated between the locations defined by vertex $[u,q,i,1]$ and $[d,q,i,1]$. The arcs are arranged in such a way that visiting and leaving one of these vertices corresponds to a traversal of the subaisle or to moving to the pick location farthest away from an adjacent cross aisle and then returning to this cross aisle. In both cases, all requested items are retrieved in the subaisle. Thus, it has to be ensured that either vertex $[u,q,i,1]$ or $[d,q,i,1]$ is included in the tour. Subaisles assigned to special case 3 only contain two pick locations. All requested items are collected if both locations are visited. This can be guaranteed if for each pick location at least one vertex of the corresponding vertex pair is contained in the tour. For subaisles belonging to special cases 4 or 5, it is sufficient to ensure that the two locations defining the largest gap are visited. Due to the degree constraints, all requested items are then retrieved in this subaisle. This can be obtained by guaranteeing that at least one vertex of the vertex pair representing such a location is included in the tour.

The model formulation itself is not presented in this paper as it includes more than 500 different types of constraints. Instead, we refer to the corresponding working paper of Ruberg & Scholz (2016) in which the pick location inclusion constraints are presented in detail and the model formulation is contained in the appendix. The reason for this huge amount of different constraints consists in the introduction of the pyramid structure as well as the consideration of the special cases of item distribution as they make the graph dependent on the problem data. The size of the graph is not only defined by the number of pick locations or subaisles but rather it is specified by the distribution of the pick locations over the subaisles. In the resulting model formulation, therefore, a lot of distinction of cases have to be integrated in order to be able to deal with all possible problem instances to the SPRP. However, only a few of the constraints will be included in an actual model formulation to a specific SPRP instance.

5 Numerical experiments

5.1 Setup

In order to evaluate the impact of the procedures for the reduction of the size of the graph, numerical experiments are performed. First, it is investigated to which extent the size of the model of Scholz et al. (2016) can be reduced. In the second part of the experiments, instances from various problem classes are solved by means of a commercial IP-solver using the formulation of Scholz et al. (2016) and the reduced model, respectively. The model formulations are then compared with respect to the number of optimal solutions obtained within a given time limit, the optimality gap if no optimal solution has been found as well as the computing time required for generating an optimal solution.

The settings for the numerical experiments are adapted from the experiments of Scholz et al. (2016). They considered the SPRP in a single-block layout and demonstrated that their model formulation was able to provide optimal solutions within a small amount of computing time. Due to this good performance, the application of procedures for the size reduction is not necessary, which is why we do not use these instances. Instead, we consider warehouses with 2 and 3 blocks. As done by Scholz et al. (2016), the number of picking aisles is set to 5, 10, 15, 20, 25 and 30, i.e. the number of subaisles varies between 10 (2 blocks and 5 picking aisles) and 90 (3 blocks and 30 picking aisles). Each subaisle consists of 50 storage locations uniformly arranged on both sides of the subaisle. The length of a single storage location is equal to 1 length unit (LU). The distance between an adjacent cross aisle and the nearest storage location of the subaisle amounts to 1 LU as well. Thus, 26 LUs are to be covered in order to traverse a subaisle. The distance between two adjacent picking aisles is 5 LUs, and the depot is located in front of the leftmost picking aisle.

The number of stops an order picker has to perform during the tour is defined by the number of pick locations. In the experiments, instances with 30, 45, 60, 75 and 90 pick locations are considered. The articles are assigned to storage locations according to a class-based storage assignment policy. As done by Henn & Wäscher (2012), depending on the demand frequency, articles are divided into three classes A, B and C. Class A consists of 10% of all articles which possess the highest demand frequency and represent up to 52% of the total demand. 30% of all articles are assigned to class B. These articles are responsible for 36% of the total demand. The remaining articles are assigned to class C and have quite low demand frequencies. For each class, subaisles are determined based on the distance to the depot. 10 % of all subaisles with the shortest distance to the depot are assigned to class A, while 60% of all subaisles farthest away from the depot correspond to class *C*. The remaining subaisles belong to class B. Each article from a class is then randomly assigned to a storage location of a corresponding subaisle.

As can be seen, the number of blocks, picking aisles and pick locations is varied in the experiments. Combination of the parameters leads to 60 different problem classes. For each problem class, 30 instances have been generated, resulting in 1800 instances in total. The formulation of Scholz et al. (2016) as well as the reduced formulation have been implemented and solved by CPLEX 12.6.3 on a desktop PC with a 3.4 GHz Pentium processor with 8 GB RAM. The computing time for solving an instance by using a formation has been limited to 30 minutes.

5.2 Results

Before considering the performance of the model formulations when solved by means of a commercial IP-solver, the formulations are compared regarding their size in terms of number of variables and constraints. In Table 1 the number of variables (#var) as well as the number of constraints (#cons) are depicted for the model formulation of Scholz et al. (2016) (SHSW) and the reduced formulation (S). Furthermore, the amount of reduction (in %) is given which is obtained by applying the procedures described in Section 4 in order to receive the S formulation. In this table, a three-block layout is considered, while the number of picking aisles m and the number of pick locations n are varied.

Table 1: Size of the model formulations in case of a three-block layout

m	n	**SHSW**		**S**		**reduction [%]**	
		#var	#cons	#var	#cons	#var	#cons
5	30	652	855	398.7	538.3	38.85	37.04
5	45	652	855	415.3	557.3	36.30	34.82
5	60	652	855	450.0	597.7	30.98	30.09
5	75	652	855	470.8	624.0	27.79	27.02
5	90	652	855	477.6	631.4	26.75	26.15
10	30	1372	1715	706.8	905.5	48.48	47.20
10	45	1372	1715	756.4	964.8	44.87	43.74
10	60	1372	1715	823.6	1036.4	39.97	39.57
10	75	1372	1715	850.4	1072.0	38.02	37.49
10	90	1372	1715	905.1	1128.4	34.03	34.20
15	30	2092	2575	1305.9	1594.5	37.58	38.08
15	45	2092	2575	1131.5	1395.4	45.91	45.81
15	60	2092	2575	1200.7	1475.2	42.61	42.71
15	75	2092	2575	1267.1	1545.7	39.43	39.97
15	90	2092	2575	1305.9	1594.5	37.58	38.08
20	30	2812	3435	1234.9	1490.2	56.12	56.62
20	45	2812	3435	1422.0	1713.3	49.43	50.12
20	60	2812	3435	1469.2	1791.5	47.75	47.85
20	75	2812	3435	1577.9	1901.9	43.89	44.63
20	90	2812	3435	1679.7	2018.8	40.27	41.23
25	30	3532	4295	1598.1	1871.9	54.75	56.42
25	45	3532	4295	1679.3	2015.8	52.45	53.07
25	60	3532	4295	1846.7	2215.8	47.72	48.41
25	75	3532	4295	1958.0	2335.7	44.56	45.62
25	90	3532	4295	2040.0	2423.4	42.24	43.58
30	30	4252	5155	1764.7	2071.3	58.50	59.82
30	45	4252	5155	2056.1	2397.2	51.64	53.50
30	60	4252	5155	2235.2	2624.6	47.43	49.09
30	75	4252	5155	2270.1	2682.2	46.61	47.97
30	90	4252	5155	2452.9	2874.3	42.31	44.24

As mentioned before, it can be seen that the size of the SHSW formulation depends on the number of picking aisles only and is independent of both the number of pick locations and where requested items are located in the warehouse. It further can be observed that the number of variables and constraints linearly increase with the number of picking aisles. In contrast to the SHSW formulation, the size of the reduced model is dependent on both m and n. The size of the model is even dependent on the actual problem data, i.e. the locations of the items to be picked, which is why each instance may result in a model formulation of a different size. Therefore, for the S formulation, the average number of variables and constraints is depicted in Table 1. As expected, the size of the model increases with an increasing number of picking aisles and pick locations. However, compared to the SHSW formulation, far less variables and constraints are required for modeling an instance to the SPRP. The amount of the reduction achieved regarding the number of variables ranges from 26.75% ($m = 5$, $n = 90$) to 58.5% ($m = 30$, $n = 30$). The amount of reduction is similar when considering the number of constraints. The procedures for reducing the size of the model have the largest impact for instances characterized by a large number of picking aisles and a small number of pick locations. In these cases, only a few pick locations have to be visited in a subaisle, which means that lots of dummy vertices are introduced when constructing the graph to the SHSW formulation. These dummy vertices are removed by applying the reduction procedure regarding the distribution of pick locations. Furthermore, if the number of subaisles is large compared to the number of pick locations, many subaisles do not contain any requested items and, thus, do not have to be visited. Due to this fact, the introduction of the pyramid structure leads to considerable size reductions. With an increasing number of pick locations the amount of reduction decreases since less subaisles can be removed from the underlying graph. Analogously, the effect of the procedures for reducing the size fades when SPRP instances with a small number of subaisles are to be dealt with.

As the S formulation includes considerable less variables and constraints, while having the same components as the SHSW formulation, it can be expected that the application of an IP-solver using this formulation leads to better results regarding the computing time required for determining an optimal solution. This expectation is verified by the results from the numerical experiments. In Table 2, the number of instances solved to optimality within 30 minutes of computing time is depicted for both formulations. If a number is equal to 30, all instances from the problem class have been solved optimally by using the respective formulation.

Considering the SHSW formulation, it can be observed that its performance is independent of the number of pick locations, which can be deduced to the fact that each subaisle is represented by a constant number of vertices in the underlying graph. However, two parameters can be identified which have an impact on the number of optimal solutions obtained within the time limit. First, an increasing number of picking aisles results in a decreasing number of optimal solutions. Using the SHSW formulation, all instances from problem classes with 5 or 10 picking aisles can be solved to proven optimality. However, this number rapidly decreases when the warehouse includes more picking aisles. For problem classes with 20 picking aisles, an optimal solution can be determined for approximately half of all instances,

Table 2: Number of optimally solved instances (out of 30) within 30 minutes of computing time

		2 blocks		3 blocks					2 blocks		3 blocks	
m	n	**SHSW**	**S**	**SHSW**	**S**		m	n	**SHSW**	**S**	**SHSW**	**S**
5	30	30	30	30	30		20	30	17	30	8	30
5	45	30	30	30	30		20	45	16	30	10	30
5	60	30	30	30	30		20	60	15	30	6	30
5	75	30	30	30	30		20	75	17	30	11	29
5	90	30	30	30	30		20	90	17	30	11	29
10	30	30	30	30	30		25	30	2	30	0	30
10	45	30	30	30	30		25	45	2	30	0	30
10	60	30	30	30	30		25	60	7	30	0	30
10	75	30	30	30	30		25	75	5	30	1	30
10	90	30	30	30	30		25	90	8	28	0	30
15	30	27	30	22	30		30	30	0	30	0	30
15	45	29	30	21	30		30	45	1	29	0	30
15	60	29	30	26	30		30	60	0	27	0	29
15	75	30	30	20	30		30	75	0	25	0	29
15	90	27	30	19	30		30	90	0	20	0	27

whereas only a single instance with 30 picking aisles has been solved. This performance matches with the results from the experiments of Scholz et al. (2016) as well as with the observations from Table 1, as the size of the SHSW formulation increases with an increasing number of picking aisles. The second important parameter is the number of blocks. An increasing number of blocks leads to less optimal solutions obtained. For 2 blocks, nearly all instances with 15 picking aisles or less can be solved to optimality. However, for 3 blocks, only approximately two third of the instances with 15 picking aisles can be optimally solved using the SHSW formulation. Furthermore, an optimal solution was found for a single instance with $m \geq 25$ only. This performance was also expected as the size of the formulation is multiplied by the number of blocks and leads to the conclusion that the SHSW formulation is not suitable for SPRPs in warehouses with multiple blocks.

Using the S formulation, most of the instances can be solved to optimality. For all instances with 15 picking aisles or less, optimal solutions have been found. When the number of picking aisles is increased to 20 or 25, only two instances have not been solved to optimality, respectively. If m gets very large ($m = 30$), several instances exist to which no optimal solution has been determined. However, even in the most difficult problem class (2 blocks, 30 picking aisles, 90 pick locations), 66% (20 out of 30) of the instances can be solved, which demonstrates that the application of the S formulation outperforms the usage of the SHSW formulation by far. In contrast to the SHSW formulation, the performance of the S formulation is also dependent on the number of pick locations. This can be expected since more vertices can be removed from the underlying graph when less pick locations have to be represented. A further difference to the SHSW formulation consists in the performance regarding an increasing number of blocks. Whereas the number of optimal solutions obtained significantly decreases for the SHSW formulation, a larger number of blocks does not seem to have a negative impact on the performance of the S formulation. On the contrary, a larger proportion of the instances can be solved to optimality when three blocks are considered. This can be deduced to the reduction procedures of the graph. The pyramid structure ensures that the size of the graph is kept at a reasonable level as lots of subaisle are removed. In the second step, lots of vertices in the remaining subaisles can be removed since the pick locations are distributed over a large number of subaisles, which means that only few pick locations exist in a subaisle. Thus, it can be concluded that the model formulation is applicable to any multi-block layout, which is a main advantage as no efficient solution approach exists able to deal with more than two blocks.

Besides the number of optimal solution obtained, the quality of solutions is investigated if no optimal solution has been found, i.e. if the solution process has been terminated after 30 minutes of computing time. In Table 3, the maximum optimality gaps are depicted for problem classes with a larger number of picking aisles ($m \geq 20$). If the maximum gap amounts to 0.00%, all instances from the corresponding class have been solved to optimality.

Table 3: Maximum optimality gaps [%]

		2 blocks		3 blocks	
m	n	**SHSW**	**S**	**SHSW**	**S**
20	30	7.28	0.00	8.10	0.00
20	45	3.98	0.00	7.54	0.00
20	60	4.66	0.00	6.03	0.00
20	75	3.36	0.00	3.74	0.62
20	90	5.24	0.00	5.63	1.05
25	30	8.62	0.00	12.99	0.00
25	45	7.98	0.00	8.89	0.00
25	60	9.55	0.00	10.40	0.00
25	75	5.71	0.00	6.80	0.00
25	90	4.79	1.02	7.08	0.00
30	30	16.41	0.00	20.62	0.00
30	45	10.60	0.20	16.04	0.00
30	60	8.89	4.11	10.05	0.43
30	75	7.36	1.63	8.34	1.37
30	90	8.26	4.50	7.84	2.83

As expected, for the SHSW formulation, the maximum optimality gap increases with an increasing number of picking aisles as well as an increasing number of blocks. It can be observed that solutions found result in a tour length up to 20% higher than the length of an optimal tour (3 blocks, 30 picking aisles, 30 pick locations) and, therefore, it is again concluded that this formulation should not be applied to SPRPs when dealing with multiple blocks. When the number of pick locations is increased, the maximum optimality gap tends to decrease, which can be explained by the fact that the minimum tour length increases when more pick locations have to be visited. Thus, small changes to the tour do not have such a large impact on the relative deviation to the optimal tour length.

The maximum optimality gaps obtained by application of the S formulation are much smaller compared to the SHSW formulation. As it is the case for the number of optimal solutions found, the performance deteriorates for an increasing number of picking aisles and pick locations, whereas it does not seem to depend on the number of blocks. The problem class containing instances with 2 blocks, 30 picking aisles and 90 pick locations has the largest optimality gap. However, all gaps are not larger than 4.50%, which means that even for the most difficult instances near optimal solutions have been found.

The average computing time required for determining an optimal solution by application of the respective model formulation is depicted in Table 4. If no optimal solution has been found within the predefined time interval, a computing time of 30 minutes has been reported. Thus, an average computing time of 1800.00 seconds means that no instance from the corresponding problem class has been solved to optimality.

Table 4: Computing times [sec]

		2 blocks		3 blocks					2 blocks		3 blocks	
m	n	**SHSW**	**S**	**SHSW**	**S**		m	n	**SHSW**	**S**	**SHSW**	**S**
5	30	0.78	0.44	0.99	0.37		20	30	1000.09	6.91	1390.47	7.25
5	45	0.71	0.47	0.98	0.40		20	45	1010.82	16.44	1606.71	17.06
5	60	0.67	0.46	0.84	0.44		20	60	1110.61	53.12	1704.86	66.37
5	75	0.74	0.52	1.14	0.47		20	75	1111.41	113.47	1309.55	100.81
5	90	0.78	0.55	1.21	0.56		20	90	1015.24	168.85	1277.25	108.43
10	30	14.29	1.03	20.71	1.57		25	30	1695.72	20.02	1800.00	18.47
10	45	12.17	1.42	35.52	1.83		25	45	1727.68	44.54	1800.00	22.50
10	60	13.69	1.38	21.11	1.79		25	60	1517.52	162.40	1800.00	93.28
10	75	10.85	1.58	30.20	2.31		25	75	1654.45	270.24	1766.42	170.96
10	90	10.03	1.42	29.93	3.16		25	90	1529.55	317.64	1800.00	205.47
15	30	428.07	6.08	525.43	14.00		30	30	1800.00	59.00	1800.00	71.38
15	45	351.52	6.54	428.78	6.44		30	45	1752.28	184.08	1800.00	58.65
15	60	355.98	19.50	397.78	7.20		30	60	1800.00	330.98	1800.00	232.28
15	75	271.85	13.10	593.61	17.79		30	75	1800.00	581.39	1800.00	155.85
15	90	478.34	47.94	525.43	14.00		30	90	1800.00	904.34	1800.00	339.70

The number of picking aisles and the number of blocks have an impact on the computing time required for applying the SHSW formulation, whereas the computing time to be spent does not depend on the number of pick locations. The number of picking aisles has the largest impact as the amount of computing time rapidly increases when m gets larger. While instances with up to 10 picking aisles can be solved within a few seconds on average, 6 to 9 minutes are required for $m = 15$ and even 10 to 28 minutes when 20 picking aisles are considered. Computing times for instances with more than 20 picking aisles are not meaningful as not many optimal solutions have been found in these cases.

For the S formulation, the results from Table 4 also match with the observations made based on Table 1. Both the number of picking aisles and the number of pick locations have an impact on its performance. Regarding the number of picking aisles, all instances with $m \leq 15$ can be solved within less than one minute on average. If m is further increased, the impact of the number of pick locations n gets significant. While the maximum average computing time amounts to 1.2 minutes (3 blocks, 30 picking aisles) for instances with a small number of pick locations ($n = 30$), on average up to 15 minutes are required for solving instances with the same number of picking aisles but 90 pick locations. When considering the number of blocks, it can be observed that computing times tend to decrease with an increasing number of blocks. In particular, this is true for instances difficult to solve, i.e. instances with a large number of picking aisles or pick locations. The largest reduction can be seen in the problem classes with 30 picking aisles and 75 pick locations, where the average computing time decreases by 73% (from 581.39 sec to 155.85 sec).

Compared to the SHSW formulation, application of the S formulation consumes far less computing time. It can be observed that the impact of the number of picking aisles on the computing time is much smaller for the S formulation. This particularly holds if a small number of pick locations ($n = 30$) is considered since in this case lots of vertices can be removed from the underlying graph. The reduction of the size of the graph results in a reduction of the average computing time by up to 99.5% (3 blocks, 20 picking aisles). When more pick locations are considered, the computing time required for applying the S formulation increases. However, even for a very large number of pick locations ($n = 90$), reductions of up to 91.5% (3 blocks, 20 picking aisles) are achieved. The maximum amount of improvement would be even higher if computing times were not limited to 30 minutes. The advantage of the S formulation gets very large when considering a large number of blocks. Whereas computing times significantly increase for the SHSW formulation with an increasing number of blocks, computing times remain unchanged or even decrease for the S formulation. This performance demonstrates that the reduction procedures are pivotal in order to obtain a problem-specific model formulation able to deal with SPRPs in multi-block layouts.

6 Conclusion and Outlook

In this article, the Single-Picker Routing Problem in warehouses with an arbitrary number of blocks has been considered. This problem represents a special case of the Traveling Salesman Problem. However, Scholz et al. (2016) demonstrated that problem-specific solution approaches lead to better results and can easier be adapted to changes arising in practical applications. They proposed a mathematical programming formulation in order to deal with SPRPs in a single-block layout. This model formulation is extended to the case of multiple blocks in this paper. A so-called pyramid structure is introduced and special cases of item distribution are identified in order to significantly reduce the size of the formulation.

By means of numerical experiments, the proposed reduced model formulation is evaluated and compared to the formulation of Scholz et al. (2016). It is demonstrated that the reduced model formulation outperforms the basic formulation by far in terms of optimal solutions found, optimality gaps and computing times. By application of a commercial IP-solver to the new model, nearly all instances from the experiments have been solved to optimality within the given time limit, while the basic model was only able to solve instances with a small number of picking aisles. Average computing times can be reduced by up to 99.5% by using the new formulation. Furthermore, it is pointed out that computing times do not increase with an increasing number of blocks. Thus, it can be concluded that this formulation is able to determine optimal or at least near optimal solutions to SPRPs with an arbitrary number of blocks within a reasonable amount of computing time. This is a major advantage of the formulation as no efficient algorithm exists which can deal with more than two blocks.

The model formulation has been designed for warehouses following a block layout. However, a recent trend is to design the layout of a warehouse without using parallel picking and cross aisles. Instead, non-conventional layouts such as fishbone layouts are applied (Çelik & Süral, 2014). The construction of problem-specific formulations to those layouts would be a promising area for future research.

Further research could also concentrate on picker blocking aspects. In this paper, wide aisles are assumed enabling order pickers to pass each other. Without losing generality, it can be assumed that only one picker is available when dealing with the routing problem as routes for different pickers can be determined independent of each other. This is not the case if blocking aspects are taken into account. Thus, it would be interesting whether the model can be extended to multiple order pickers while considering blocking aspects.

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ISSN 1615-4274