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Some Game Theoretic Aspects of Brexit

Roland Kirstein

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Roland Kirstein
Otto-von-Guericke-Universität Magdeburg
Fakultät für Wirtschaftswissenschaft
Postfach 4120
39016 Magdeburg
Germany

http://www.fww.ovgu.de/femm

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Prof. Dr. Roland Kirstein*

ORCID: 0000-0002-1375-5862

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Abstract

The paper initially explains some fundamentals of interactive decision-making ("game theory") and then applies different approaches of game theory to different aspects of Brexit. The first analysis perceives the 2016 referendum as a "simple voting game" and challenges the view that the observed outcome of about 52% percent in favor of Brexit have to be interpreted that the "vox populi" (and, thus, also the "vox dei") is in favor of a "no-deal" Brexit. Rather, there seem to have existed three camps among the voters, of whom 25% have actually opted for a no-deal Brexit, whereas 27% seem to have approved Brexit in the expectation of a sensible deal. Hence, the 48% who favored "remain" have been by far the largest homogeneous group, although they fall short of an absolute majority. Social choice theory shows that, in a situation without an option supported by a clear majority, no aggregation procedure – such as majority voting or pairwise binary voting – exists that guarantees collective rationality (Arrow theorem) or satisfies some desirable properties (Gibbard-Satterthwaite theorem).

The next analysis scrutinizes the hypothesis that the observed outcome of the referendum was due to the "remainers'" failure to participate. The economic theory of voter participation explains intermediate participation rates as a mixed strategy equilibrium. For the two or three groups mentioned above, the incentives to participate were different. The third model section takes a closer look at the negotiations between the UK and the EU, focusing on the transition from Theresa May to Boris Johnson. A simple Nash bargaining model demonstrates that the bargaining outcome may depend on the preferences of the delegate who negotiates on behalf of the represented party. Switching from one delegate to another, hence, may lead to a more favorable outcome. A final section discusses existing literature on game theoretic analysis of Brexit, which essentially deals with various non-cooperative bargaining models.

* Economics of Business and Law, Faculty of Management and Economics, Otto-von-Guericke Univ. Magdeburg. I am indebted to Joshua Gomersall for excellent assistance in preparing this manuscript.

1 Introduction

Microeconomics is a theory of human decision-making under scarcity. Robinson Crusoe, on his deserted island, had to decide how to make the best use of his time and the few items he could save after the shipwreck. Consumers have to allocate their limited budgets to products. Firms have to decide which products to produce and which mix of input factors to employ.

Game theory is the theory of interactive decisions. What a firm's best decision actually is may depend on its competitors' and its deliverers' decisions, and vice versa. Robinson, after having met Friday, had to decide how to share the daily chores on the island that suddenly appeared not so deserted anymore. Consumers' best decisions may depend on what decisions the producers or other consumers make.

Game theory provides powerful tools for analyzing social institutions. Positive analysis scrutinizes how institutions influence the interactive decisions of human actors ("players"). Normative analysis identifies inefficiencies in their interactive behavior and derives proposals which institutions to implement (or how to modify existing institutions). Two branches of game theory exist: non-cooperative and cooperative. Non-cooperative game theory assumes that players have specific options at hand. Their task is to choose an option that they like best, anticipating the decisions of the other players. A player's plan for "playing" out the whole "game" between him and the other players is called a "strategy" of this player. Positive theory aims at predicting which strategies the players choose.

Strategy combinations lead to consequences for each player. If player 1 may choose between strategies A and B, and player 2 may choose between C and D, then the combined choice of, say, B (by player 1) and D (by 2) may lead to other consequences, both for 1 and 2, than the combined choice of A and D or of B and C. The consequences of a strategy combination that a player expects for himself are called his "payoff". His payoffs may represent the player's preferences over strategy combinations, or it may reflect the monetary consequences for a player that are induced by this strategy combination. Example: If a buyer of a rare book pays € 1,000 to the seller, but the seller does not deliver the book, the monetary consequence for the buyer is -1,000 and for the seller +1,000.

In social sciences, the second approach has a considerable advantage: The monetary consequences of interactions, influenced by institutions, can often be determined in an objective manner. The preferences of a player over a set of strategy combinations, however, are difficult to determine without actually playing the game. Under the assumption that players seek to maximize their (expected) monetary income, then the two approaches are equivalent. If players pursue other goals (instead of or in addition to money), then the two approaches may differ.

A non-cooperative model to analyze interactive behavior makes assumptions regarding the identity of the players, their strategies, and their preferences over strategy combinations. Moreover, the information available to the respective player can be crucial: Does a player know the aforementioned elements of the game (players, strategies, players' payoffs)? If not, the game is one of "incomplete information." A related problem is whether a player, when having to make his own decision(s), knows what the other players have previously done. In chess, the other player's past behavior is always observable, but in poker, players may keep as private information which cards they have initially received and which of these they decide to keep. If a player exist who, when making own decisions, is unable to observe some other players' past behavior, then the game is one of "imperfect" information.

In addition to the non-cooperative theory, another branch of game theory exists: cooperative games. Cooperative games rest on the assumption that players can form "coalitions." Such coalitions consist of an expressive agreement, or can have a rather informal character. Two bargaining parties may come to an agreement, some voters vote for the same candidate, or some members of a supervisory board choose the same option. All of these formal or informal agreements would lead to consequences, in other words: have a value for the players involved. If the group of voters is large enough, their candidate is elected, which these voters appreciate, whereas other voters are perhaps less excited). Cooperative games do not look at the actual casting of the vote, or the interactive decision-making of the voters, but only at the value of the feasible coalitions. The value of coalitions may depend on the institutions or rules under which the players form such coalitions. For example, under the absolute majority rule, a group of 51 voters who vote for the same candidate will see this candidate prevail. Under a 2/3-majority rule, the same group will be unsuccessful.

A sub-branch of cooperative game theory is Nash bargaining. Two players may share a "pie" between them if they close an agreement, and they receive an "outside option" if they fail to close such an agreement. If the size of the pie exceeds the sum of outside options, then it is collectively beneficial to make an agreement, but a conflict exists between the parties as to which agreement they bring about. Hence, bargaining games consist of a cooperative and a non-cooperative element: The parties have a mutual interest in closing an agreement, but they differ with regard to sharing the agreement rent. The Nash bargaining theory predicts how the bargaining result may depend on the size of the parties' respective outside option, on the shape of their individual utility functions, or on their (im)patience.

This paper uses concepts from cooperative and non-cooperative game theory to analyze three different aspects of the development that eventually led to Brexit. The second section uses social choice theory and cooperative game theory to analyze the referendum of 2016. In this referendum, the voters had to decide between two options: remain or leave. In the subsequent years, however, it became apparent that, in fact, three options existed: remain, leave with a deal, or leave without a deal.

Is it possible to evaluate the outcome of (or set the rules for) such a referendum to determine, in an unambiguous manner, the true preferences of the people, the "vox populi"? Or is it possible that the referendum results perhaps did not fully reflect the true preferences of the people?

A deviation between referendum outcome and "true preferences" is even more likely if voters have to option to abstain. Section 3 uses the most prominent solution concept for non-cooperative games, the Nash equilibrium, to analyze why voters participate, or do not participate, in elections. The model may shed light on why remainers have chosen abstention to a higher degree than their fellow Brexiteers. Thus, it may contribute to the explanation of the referendum result.

Section 4 uses the Nash bargaining theory to analyze a later stage of the Brexit negotiations between the UK and the EU, when the UK replaced their representative, Prime Minister Theresa May, by Boris Johnson. This step was criticized, as it made the conclusion of a "deal" more unlikely. It is easy to understand that those who were not in favor of any deal supported Johnson. The Nash bargaining model helps to understand why it may have been beneficial to pursue this course, even for those who wanted a deal, preferably a better one.

2 Social choice: Did the referendum reveal the “vox populi”?

Social choice theory analyzes the aggregation of individual preferences into a group decision. The application of the majority rule is trivial if only two options are feasible and the option to abstain does not exist: If the majority of the voters supports one of the options, then only a minority supports the other option. Therefore, it appears reasonable to call the majority option the “social decision.”

Things become less trivial if more than two options are available. In French or US presidential elections, sometimes more than twelve candidates initially run for office. The average support for all these candidates amounts to only about 8% of the voters. Under the simple majority rule, the option prevails which the highest number of voters supports. Therefore, 9% could be enough to prevail under this rule. It is highly questionable whether an option that is supported by 9%, but opposed by 91%, should be declared the “social decision.” For an option to become the social decision, it would be desirable that an absolute majority of voters (more than 50%) supports it. However, the larger the number of options, the higher the likelihood that no absolute majority for one of the options exists. If three or more options are feasible, then there is no guarantee that an absolute majority exists. Hence, it can be crucial for the legitimacy of the outcome whether, in a social choice situation, just two options are feasible, or three or more.

2.1 The Brexit vote: Is “vox populi” equal to “vox dei”?

During the Brexit referendum on June 23, 2016, the people were asked whether the UK should remain in the EU or leave. As it is well known, the outcome was 51.9% in favor of “leave.” At first glance, only two options existed. Should, therefore, the option supported by the majority be considered the legitimate social decision?

The negotiations between the UK government and the EU in 2019 shed some doubts on whether the referendum question represented the actual decision problem. During the negotiations, three options were truly at stake: remain, leave based on a negotiated treaty between the UK and the EU, or leave without such a deal.

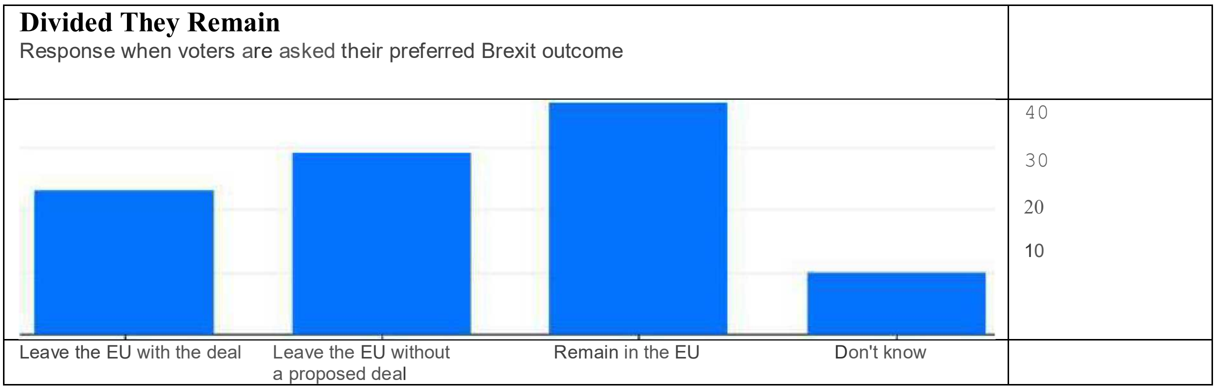

Figure 1: Poll results with three options. Taken from Gongloff (2019)

Gongloff (2019) reports a poll performed by “Number Cruncher Politics,” according to which the prevailing option actually finds little support if the respondents face three options instead of two: Only

25% are in favor of "no deal," whereas 44% would rather remain. 21% would have preferred to leave the EU only with a deal, and about 10% of the respondents were undecided. Hence, among the 90% who picked one of the three aforementioned options, the 46% Brexiteers formed a small majority over the 44% remainers. This accurately reflects the result of the referendum. However, with three options to choose from, the referendum would clearly have led to another social decision. Under the simple majority rule, it would have been "remain." Under the absolute majority rule, the referendum would have had no result.

2.2 Social decision rules

A social decision rule, to be effective and acceptable, should satisfy some requirements. Among the most important requirements are the following. A social decision rule should be

- decisive
- resolute
- non-manipulable.

"Decisive" means that the decision rule leads to some decision. No scenario is possible in which the decision rule fails to select one or more feasible options as the social decision. "Resolute" means that if the rule leads to a social decision, then it is unique. A non-resolute rule selects more than one option as the social decision, between which society is indifferent. Finally, a social decision rule is "non-manipulable" if it induces all voters to cast their vote according to their true preferences.

Consider a scenario with three voters (1, 2, 3) and three feasible options (A, B, C).The absolute majority rule turns an option into the social decision if it is supported by two or three voters. Hence, the absolute majority rule is not decisive: It is possible that, for example, voter 1 prefers option A, whereas 2 prefers B, and 3 prefers C. The absolute majority rule, however, is resolute, because it is impossible for more than one option to receive the absolute majority of votes. If voters 1 and 2 support option A, then neither option B nor C can beat A. Similarly, the 2/3-majority rule and the unanimity rules are not decisive, but resolute.

The simple majority, on the other hand, is decisive, but not resolute. In the scenario in which the three voters all prefer different options, the social decision would be the whole set of options, i.e., {A; B; C}. Hence, there is no guarantee that the social decision is unique.

A "dictatorship" rule is a social decision rule that stipulates that, regardless of what the other voters would favor, only one of the voters determines the social decision. If voter 1 is the "dictator," then the social preference (according to this decision rule) is identical to 1's preferences. Such a dictatorship rule is decisive, and it is resolute if the dictator is able to identify a unique optimum. An innocuous example for such a "dictatorship" social decision rule is the concept of private property, which endows single owners with the right to make decisions regarding the usage of specific resources.

The absolute majority rule is attractive for its resoluteness. An attempt to avoid its lack of decisiveness is the following modification of the majority rule used in French presidential elections: The elections take place in two rounds; during the first round, voters can decide among all candidates ("options"). Only the two leading candidates make it to the second round; hence, this is a modification of the simple majority rule. During the second round, voters may decide only between these two options, and the one that receives the higher number of votes is the social decision. This is an application of the absolute majority rule. If no ties and abstentions occur, then this two-stage procedure ensures decisiveness (it is guaranteed that there will be two leading candidates in round 1) and resoluteness as, in round 2, one of the two candidates automatically receives the absolute majority of the recognized votes.

However, this two-stage rule with absolute majority is manipulable. Here is a simple scenario that demonstrates that voters may have an incentive to vote strategically instead of according to their actual preferences. Assume that, if all voters vote honestly, then options A and B will make it to the second round, in which A will beat B. Assume furthermore that, if option C would make it to the second round, then C would attract more voters than A. The last assumption is that voters exist who would individually rank the three options the following way: B is better than C is better than A. These voters would have an incentive not to vote for B in the first round, even though B is their most preferred option Instead, they are better off if they vote for C in the first round, which is only their second best option, because if C makes it to the second round, it will beat A. By voting for C in the first round, these voters conceal their actual preferences – if they could alone determine the social outcome, they happily would choose B.

Strategic voting, as an alternative to honest voting, is motivated by the insight that one's own best option will not prevail in the end anyway. To prevent the case in which the most preferred option prevails in the first round, only to be defeated by the worst option in the second round, it is better to choose the second-best option in the first round, which then will prevail against the least desirable option during the second round. A voter who understands that he will not get his first choice anyway might be tempted to conceal his true preferences during the first round so as to turn his second-best option into the social decision (and thereby prevent the worst option).

A social decision rule that is manipulable may, thus, fail to motivate the voters to conceal their true preferences, and rather induce them to vote for the option they only like second-best. To elicit the true preferences of the people, it would therefore be desirable to have a social decision rule that is strategy-proof, or non-manipulable. Moreover, it would be desirable for the social decision rule to be decisive and resolute. However:

- Simple majority, which is decisive, but not resolute, is also manipulable. It may pay to support the second option in one's own preference ranking, so as to prevent the victory of the worst option, if the own favorite option has insufficient support anyway.
- Absolute majority is resolute, but not decisive – but at least it is non-manipulable. The attempt to fix the drawbacks of these two rules by combining them into a two round procedure reaches resoluteness and decisiveness, but it does not avoid manipulability.
- The two-stage voting rule used in French presidential elections is decisive and resolute, but manipulable.
- A dictatorship rule, according to which one member of society can fully determine the social outcome, is decisive, resolute, and non-manipulable, because no player can improve the social decision by concealing their actual individual preferences.

This leads to the question whether (other) social decision rules exist that are decisive, resolute, and non-manipulable. Gibbard (1973) and Satterthwaite (1975) have analyzed the relation between these three properties in different contributions that were published simultaneously. Therefore, their result is called the "Gibbard-Satterthwaite-Theorem":

> *If three or more voters face three or more options, then any social decision rule that is decisive, resolute, and non-manipulable is dictatorship.*

In other words: If society wants the social decision rule to be decisive, resolute, and non-manipulable, then it had to transfer the decision right to a single individual, whose preferences then are identical to the social preferences. None of the genuine "social" decision rules, which let the social decision depend on several (or even all) voters' preferences, satisfies all three of these requirements. For instance, if a social decision rule is decisive and resolute, then it must be manipulable. The Gibbard-Satterthwaite theorem has the same structure as a similar, and very famous, theorem derived by the Nobel laureate of 1972, Kenneth Arrow (1951). For an attempt to combine these two theorems, see Reny (2001).

2.3 Conclusion

The two lines of argument presented in this section shed a sad light on the outcome of the 2016 Brexit vote. The majority for Brexit may have been small, but it was an absolute majority, nevertheless. Does the outcome, therefore, unambiguously express the "vox populi" and, thus, the "vox dei"?

First of all, the reduction to just two options (Brexit or remain) has concealed that the "Brexit camp" should be divided into two subsets, both of which are far from being supported by an absolute majority. In fact, three options existed: Brexit without a deal, Brexit with a deal, or remain. However, as soon as three or more voters have to decide among three or more options, the Gibbard-Satterthwaite-theorem applies. It would be desirable for the social decision rule to be resolute, decisive, and non-manipulable.

The absolute majority has only been decisive because the option set has been reduced to two options by merging "unconditional Brexit" and "Brexit, but with a deal." With three options, the absolute majority is not decisive anymore (and the poll figures insinuate this). The simple majority rule is decisive, but manipulable. Other rules that are decisive and resolute are manipulable, according to the theorem. If the applied social decision rule is manipulable, then the outcome of the vote does not necessarily represent the "vox populi," because some voters may have voted strategically (against their true preferences) instead of honestly, which means: in accordance with their actual preferences.

Hence, it is naïve to interpret a social decision of more than three voters among more than three options as "vox populi." To interpret a social outcome as the "vox populi," the social decision rule needs to be non-manipulable – it should avoid setting incentives for strategic voting. To implement a non-manipulable rule, however, requires accepting other drawbacks, such as a lack of resoluteness or a lack of decisiveness.

3 Nash equilibrium: Why participate in elections and votings?

3.1 What is the real puzzle: abstention or participation?

Sometimes, commentators bemoan the frequently high rates of abstention from public votes and elections. There are several possible reasons: Voters are perhaps uninterested in the issues at stake, face "opportunity cost" of participating, or have the feeling that their single vote makes no difference. All of these reasons may even "threaten" democracy; this is the reason why some countries (like Belgium) have made participation in votes and elections mandatory.

In case of the Brexit referendum, it was observed that younger and well-educated voters had abstained to a larger degree than other groups. Polls have uncovered that the majority of these voters were in favor of "remain." Hence, a greater participation of this group might have changed the outcome.

With regard to Brexit, the first of the three reasons mentioned above should have played a minor role. Nearly everyone was involved in the fierce public discussion that divided even families and friends. It was certainly not easy to declare Brexit a matter of little relevance. The second line of reasoning postulates that abstention rates increase with better weather (or heavy rain). However, in the case of Brexit, the weather conditions were about the same for all voter groups, so this would at best explain the overall abstention rate. It provides no explanation for the phenomenon that different groups have behaved differently.

The third reason appears entirely reasonable at first glance. If I have just one vote, among millions, then my influence is negligible. If my vote has no effect, then why cast it? Following this line of argument, the "voting puzzle" that social sciences really has to explain is not a high abstention rate. The true puzzle is why the participation rates are, repeatedly, considerably higher than zero.

However, game theory renders this line of argument fallacious. Imagine that all other voters think like this and abstain. Then the vote of a single voter would suddenly be decisive. Hence, any single voter would have a strong incentive to participate, if all others abstain. The best reply to a high abstention rate is, therefore, participation.

However, if all others follow the same idea, then all voters would participate. The line of argument started with the idea that the influence of one voter is negligible if all voters take part, so the best response to general participation would be for all voters to abstain. However, if all (other) voters abstain, then it is best to take part (for every voter). And so on.

Obviously, a vicious cycle emerges: The best individual reply to general participation is abstention; the best individual reply to general abstention is, however, participation. This is an interactive decision situation (a "game") in which an actors' best behavior depends on the behavior of the other voters. In the language of game theory: A player's best strategy choice depends on the strategies he expects the other players to choose. A "strategy" of a player is a plan how to behave throughout the whole game. If a player has to make only one decision, then his strategy set contains exactly the actions available at this point of the game. Games exist in which players have to make several decisions, perhaps interchangeably. Then the strategy of a player would consist of combinations of the actions from which this player can choose whenever he has to make a decision.

3.2 Elements of a non-cooperative game

A (non-cooperative) game is defined by four elements:

- A set of players (decision-makers whose choices may influence the outcome of the game);
- their strategy sets, each of which contains the actions available to this player (in our example: each player may choose between "abstain" and "vote");
- the payoffs, i.e., the utility at which each player evaluates the possible combinations of strategies (the possible "outcomes") of the game.
- Finally, the information available to the players when making their decisions is of relevance. Sometimes, other players' previous choices are observable (like in chess) when making the own next move, sometimes not (like in poker). If no player can observe the choices of the other players, then the game is called "simultaneous."

Formally, the "strategic form" or "normal form" of a game is a triple $<N, \Sigma, \pi>$, where:

- N={A; B; C....} is a set of players (each player has a strategy set Σ_i, $i \in N$, that consists of two or more strategies);
- $\Sigma = \Sigma_A \times \Sigma_B \times \Sigma_C$.... is the set of strategy combinations;

- each player $i \in N$ has a payoff function $\pi_i : \Sigma \rightarrow IR$ that attaches some value to any feasible strategy combination and $\pi = \pi_A \times \pi_B \times \pi_C$.... is the set of payoff combinations.

Hence, if N={A; B}, $\Sigma_A=\{a_1; a_2\}$ and $\Sigma_B=\{b_1; b_2\}$ then $\Sigma=\{(a_1,b_1); (a_2,b_1); (a_1,b_2); (a_2,b_2)\}$.The information is disregarded in the notion of a normal-form game. A very elegant way to write down such a 2-person game in strategic or normal form is the "bi-matrix":

B A	b_1	b_2
a_1	$\pi_B(a_1,b_1)$ $\pi_A(a_1,b_1)$	$\pi_B(a_1,b_2)$ $\pi_A(a_1,b_2)$
a_2	$\pi_B(a_2,b_1)$ $\pi_A(a_2,b_1)$	$\pi_B(a_2,b_2)$ $\pi_A(a_2,b_2)$

Figure 2: A 2x2 game in strategic form

The game is called a "2x2 game" as both players have only two strategies (if, for example, player B had eight strategies, this would be a 2x8 game). The bi-matrix contains all three elements of a normal form: The top left cell indicates the two players, the first column shows player A's strategies, the first row those of player B. The four cells contain the payoff combinations at which the players evaluate the respective outcomes.

In a series of very short and path breaking articles, the Nobel laureate John F. Nash (1950b, 1951) developed a solution concept for such an interactive decision situation that became the standard tool of game theory: the Nash equilibrium. Essentially, a Nash equilibrium is a strategy combination (one strategy per player), the elements of which are best replies to each other. In our 2x2 game above, the strategy combination (a_2, b_1) would be a Nash equilibrium if a_2 is a best reply to b_1 and, simultaneously, b_1 is a best reply to a_2.

Game theory pursues two goals: prediction of the players' behavior (positive theory) and derivation of recommendations (normative theory). Hence, a positive "solution" should allow for a prediction of the players' behavior. The Nash equilibrium is a powerful tool to pursue this positive goal, or to "solve" games. If a strategy is part of a Nash equilibrium, then it is a best reply to the other players' choices. If this is the case, then the player under scrutiny has no incentive to deviate from this strategy combination. As this holds for all players, the equilibrium provides a prediction of their behavior.

3.3 Problems and limits of non-cooperative game theory

However, two problems arise: the existence and the uniqueness of Nash equilibria. The 2x2 game depicted above, for example, may have two Nash equilibria, one, or zero (if no player attaches identical utility values to different strategy combinations). Assume, without loss of generality, that a_1 is the best reply to b_1. Either a_2 or a_1 is the best reply to b_2. If it is also a_1, then the strategy a_1 is dominant for A, and the game will have one equilibrium because, whenever in a 2x2 game at least one player has a dominant strategy, the game will have only one equilibrium.

If, however, a_2 is the best reply to b_2, and B also has no dominant strategy, then two scenarios are possible: b_1 is B's best reply to a_1, and b_2 is his best reply to a_2, then the game has two equilibria. On the other hand, if b_2 is the best reply to a_1, and b_1 the best reply to a_2, then no equilibrium can be identified (at least for now).

If a unique equilibrium exists, the analysis provides a sensible prediction of the players' behavior in the interactive decision-situation under scrutiny. Such a prediction is falsifiable if it can be compared to real-world observations: If, in reality, actual players in such a situation have chosen another strategy combination, this rejects the theoretical prediction.

The latter two scenarios, with two or zero equilibria, are problematic. Two equilibria means that the analysis is ambiguous, and the researcher does not know which one of the equilibria the players will actually carry out. Observing a single game, therefore, does not allow for a falsification of the theoretical prediction of which strategy a player chooses in equilibrium. If no equilibrium exists, then no prediction is possible at all.

3.4 Stylized examples of 2-person games

Such problems may occur in many situations. It is insightful to distinguish between three stylized examples. In the famous prisoners' dilemma, a prosecutor interrogates two prisoners separately. If one prisoner blows the whistle, he goes free (and his payoff is normalized to 0) whereas the other one is punished severely (assume his payoff amounts to -10). If both are silent, then each receives only a light sentence (payoff -1). If both confess, then they receive only a reduction of their sanction (payoff -7). Even if these interrogations take place one after the other, the game between the prisoners is technically a "simultaneous" game, because there is no information available to the second prisoner on what strategy the other has chosen.

In this situation, to confess is the dominant strategy for both players. Putting two prisoners into the described dilemma situation by offering them a reduction of their sanction in exchange for a confession has the same effect as torture: it elicits confessions, whether they are true or false. Torturing prisoners with game theory does not necessarily reveal the truth, as Holm (1995) has pointed out. Regardless of this, the normative evaluation of the unique equilibrium shows that both players were better off by remaining silent. Economists would call the Nash equilibrium "Pareto-inefficient."

B A	confess	silent
confess	-7 -7	-10 0
silent	0 -10	-1 -1

Figure 3: The prisoners' dilemma game, unique Nash equilibrium

A strategy combination is called "Pareto-inefficient" if switching to another strategy combination improves the outcome for at least one player without making any player worse off. If such a "Pareto-improvement" is not feasible, then the strategy combination is "Pareto-efficient" or a "Pareto-optimum."

The next example is a game with multiple Nash equilibria: Anna and Bill, still at their workplaces, consider spontaneously going out during the evening. This may get them stuck in a game called "Battle of the Sexes" (BOS game). Suppose Bill is an opera fan, whereas Anna would prefer they watch a soccer game (lucky enough, there is no third option). Both of them fully agree that the worst that could happen is not to pursue a common activity (payoff 0). Assume there is no communication between the two before they meet. If a player pursues the activity he prefers most, and the other one shows up as well, then "winner" gets a payoff of 3, and the other player gets 1. Obviously, the game has two Nash equilibria, which are asymmetric with regard to the activity and the obtained payoffs. Multiple Nash equilibria do not allow for a clear prediction (and recommendation) what the two players will (and should) do.

B A	opera	soccer
opera	1 3	0 0
soccer	0 0	3 1

Figure 4: "Battle of the sexes" game with multiple Nash equilibria

The third stylized scenario relates to a security guard who has to decide whether to actually walk around (choosing the strategy "active," which costs him C) or remain in his shack during the night shift (strategy lazy). His opponent is a thief who may carry out his plans (strategy "enter") or stay at home (strategy "not"). Subsequently, "he" shall refer to the security guard and "she" to the thief.

If the security guard is active, then the thief gets caught and has to pay a sanction S; the security guard then receives a reward R > C. If, on the other hand, the thief is successful, she enjoys her bounty worth B. The inspection game between these two parties is depicted in **Figure 5**.

thief security guard	enter	not
active	-S R-C	0 -C
lazy	B 0	0 0

Figure 5: Inspection game, no Nash equilibria

The security guard's best reply to "enter" is "active," whereas the best reply to "not" is "lazy." For the thief, the best reply to "active" is "not" and the best reply to "lazy" is "enter." So far, the absence of a Nash equilibrium makes it impossible to predict the players' behavior. The following subsection presents an equilibrium concept that allows us to derive a prediction even for this class of games.

3.5 Mixed strategy Nash equilibrium

One of the main results of Nash (1950b, 1951) was the assertion that each "finite" game has at least one Nash equilibrium. Essentially, a game is finite if all player's strategy sets are countable, like above, or form a closed interval, like "pick a number from between 0 and 1". That would contradict the observation made in the inspection game above.

Nash's idea leads to an extension of the normal-form game presented above. In the inspection game, the security guard has two "pure" strategies: active or lazy. Assume that he carries out his pure strategy active with a probability of $\alpha \in [0,1]$. This implies that the other pure strategy, "lazy," will be carried out with probability $(1-\alpha)$. Choosing "active" for certain is the same as choosing $\alpha = 1$.

Assume furthermore that the thief chooses "enter" with probability $\eta \in [0,1]$ and, therefore, "not" with probability $(1-\eta)$. Then, the normal-form game can be rewritten as $<N,\Sigma',\pi'>$, where $\Sigma'=[0,1]x[0,1]$ and $\pi'_i : \Sigma' \rightarrow IR$.

The probabilities α and η are called "mixed" strategies. Choosing a mixed strategy means choosing a probability distribution over the own set of pure strategies. Nash's assertion relates to mixed strategies: Any finite game has at least one Nash equilibrium in mixed strategies.

A combination of mixed strategies is an equilibrium if every player chooses their mixed strategy such that the other players are indifferent between their pure strategies. If the other players were not indifferent, then they would prefer one of their pure strategies. This cannot be part of an equilibrium in the inspection game; however, as we already know that this game does not have an equilibrium in pure strategies.

If the security guard chooses a mixed strategy α, then the thief receives, when playing her pure strategy "enter," an expected payoff of $[(1-\alpha)B-\alpha S]$. If the thief plays "not," she gets 0 for certain. Hence, if the security guard chooses α such that $(1-\alpha)B-\alpha S = 0$, which is equivalent to

$$\alpha^*=B/(B+S),$$

he makes the thief indifferent between her pure strategies. If the thief chooses a mixed strategy η, then the security guard receives an expected payoff of $(\eta R-C)$ if playing "active," and 0 if "lazy." Thus, the thief makes the security guard indifferent between his pure strategies if she chooses

$$\eta^*=C/R.$$

The unique Nash equilibrium of the above game is the mixed strategy combination (α^*, η^*). If both players choose these mixed strategies, then no one has an incentive to deviate. If one player would deviate, then the opponent would strictly prefer to carry out one of the pure strategies.

With regard to 2x2 games, Nash's insight is very helpful in the case of the inspection game, which has no Nash equilibrium in pure strategies. The existence of a unique Nash equilibrium in mixed strategies provides at least a stochastic prediction: The players choose their behavior with the equilibrium probabilities. If the game is played many times, then we would expect the pure strategy combination (active, enter) to occur with a probability of ($\alpha^*\eta^*$). This prediction is a falsifiable claim.

For a single observation, however, the mixed strategy equilibrium provides no sensible prediction: If the security guard is predicted to choose "active" with some probability α (where $0 < \alpha < 1$) and the thief chooses "enter" with η (where $0 < \eta < 1$) then all four combinations of pure strategies may occur with positive probability. No combination of pure strategies is impossible to occur. Therefore, any single observation, for example, (lazy, not), could neither confirm nor falsify such a prediction. Only if a researcher observes many of such games, it is possible to decide whether the observed frequencies of both players' behavior are close enough to the theoretical prediction.

In the case of the inspection game (without Nash equilibrium in pure strategies), Nash's discovery makes it possible to derive an equilibrium, which is even unique. With regard to the "Battle of the Sexes" game, however, the problem of multiple equilibria becomes even worse: Besides the two Nash equilibria in pure strategies, a third Nash equilibrium exists in mixed strategies. Only the games that have unique Nash equilibria in pure strategies are unaffected by the introduction of mixed strategies (the concept might, however, be useful for proving the existence of a unique equilibrium).

3.6 Application: abstention and participation

Above, we have looked at three classes of highly stylized games in normal form, with zero, one, and two equilibria in pure strategies. It depends on the exact description of the scenario to which of these categories a specific interaction between voters belongs. If, for example, voters were in general lacking incentives to participate, then abstention would be their dominant strategy, and the game would have a unique Nash equilibrium. This is the idea of an early contribution to the game theoretic analysis of voter behavior in Riker and Ordeshook (1968).

The idea of their model is as follows: Let $C > 0$ denote the cost of voting participation. $B > C$ is the benefit from winning. q is the probability that an additional vote changes the outcome; A voter is incentivized to take part if the condition $qB > C$ holds, which is equivalent to $q > C/B$.

What determines the probability q? It certainly depends on the size of the group of voters, denoted n. An intuitive approach would be to assume $q=1/n$; if a specific voter is one out of 100, then his influence is 1/100. However, this approach is mathematically unsound. Correct is the following derivation: Assume that the group of voters consist of three decision-makers who have to vote simultaneously either "yes" (Y) or "no" (N), and the social decision rule is absolute majority (hence, n should be an odd number to avoid ties).

By casting her vote, voter 3 changes the outcome if, and only if, voters 1 and 2 disagree. If, however, voters 1 and 2 vote identically, then it is completely irrelevant whether player 3 adds her vote or abstains. Assume that voter 3 is uninformed about her peers' individual preferences, so she assumes that both may vote Y or N with probability 1/2. Four cases are equally likely: Both vote Y, both N, one Y and one N. In the first two cases, which occur with probability 1/4 each, 3's vote is irrelevant. Hence, 3's vote influences the social decision with a probability of 50%.

If the number of player is higher, the resulting probability with which an individual vote makes a difference shrinks dramatically: If the voter under scrutiny is one out of

- 5 voters, then the probability of being decisive (2 of the other voters vote Y, the other two vote N) is 37.5%;
- 11 voters, then the probability of being decisive is 24.609%;
- 101 voters, then it is 7.966%;
- 500,001 voters: 1.128%
- 50,000,001 voters: 0,0113%
- 324,000,001 voters: 0,0044%

Apparently, q exceeds 1/n by far: If an individual voter in German elections (about 50,000,000 voters) were uncertain with regard to the preferences of the other voters, then he would expect to cast the decisive vote with a probability of 1.1%, so his perceived influence is much greater than 1/50,000,000. The reason is that the number of Y-and N-votes follows a binomial distribution. Many constellations exist in which exactly the half of the other voters choose Y, the other half N. Hence, the event that the other voters are split evenly may occur with a surprisingly high probability.

Nevertheless, q strongly decreases with n. For large groups, q is very small. If then C/B is non-negligible, it may occur that the condition q > C/B is not satisfied. In that case, the voter under scrutiny would abstain. In case of homogeneous voters, all of them will abstain. This is, however, not in line with the empirical observation that, typically, between 20 and 80 percent of voters do participate.

3.7 Abstention and participation as a mixed strategy equilibrium

A "Battle of the Sexes" game may also be possible, as shown in **Figure 6**: Assume that two players decide whether to vote or to abstain. If only one party votes, it prevails. The cost of voting C is smaller than the benefit from prevailing B: B > C > 0. If both vote, then they prevail with a probability of 50%. Assume C > B/2 > 0.

B A	vote	abstain
vote	-C+B/2 -C+B/2	0 B-C
abstain	B-C 0	0 0

Figure 6: 2-person voting as a "Battle-of-the-Sexes" game

This game has two Nash equilibria in pure strategies (in which one player votes, the other abstains). Moreover, it has a third Nash equilibrium in mixed strategies, in which both players choose vote with a probability of

$$2(B-C)/B.$$

The abstract BOS game depicted in **Figure 4** has a third Nash equilibria in mixed strategies as well, in which both parties choose their respective most preferred pure strategy with a probability of 3/4, and the least preferred with 1/4. As **Figure 7** below shows, the two parties would meet at the soccer stadium or at the opera with a probability of 3/16, respectively. Thus, with a probability of 3/8 they will pursue identical activities (and generate positive payoffs). This implies, however, that they will end up in different locations with a probability of 5/8.

With a probability of 9/16, player A will go to the opera and B to the soccer stadium. Both parties choose their most preferred activity and, hence, fail to meet. Moreover, with a probability of 1/16, player A chooses soccer while B goes to the opera, and the players would spend the evening in separate locations. The combined probability of not going out together is, therefore, 10/16 or 5/8. In these cases, both players' payoff is zero by assumption.

B A	Opera 1/4	Soccer 3/4
Opera 3/4	3/16	9/16
Soccer 1/4	1/16	3/16

Figure 7: "Battle-of-the-Sexes" game, mixed strategy equilibrium

Due to the existence of multiple equilibria, and due to the mixed strategy equilibrium, any combination of actions is part of the prediction. In other words, no observed combination of actions actually carried out by the parties in a single interaction is inconsistent with the theoretical predictions. This is certainly a weakness of the Nash equilibrium concept. On the other hand, both the existence of multiple equilibria and of a mixed strategy equilibrium offer an explanatory approach as to why positive participation rates are observable in reality.

The vicious cycle described above points at a game without pure strategy equilibria: If all other voters abstain, then a single voter's vote is immediately decisive. In the notation used above to present the Riker and Ordeshook model, the value of q depends on the participation rate p.

If p is close to zero, then the probability of being decisive is one: $q(0)=1$. If, however, the participation rate p goes towards one, then the problem outlined by Riker and Ordeshook applies: The probability of being decisive would be too small or, formally, $q(1) < C/B$ with $C/B > 0$.

The verbal description of the function q(p) insinuates that the probability of being decisive q is a declining function of the participation rate p. In their paper, Palfrey and Rosenthal (1983) have argued that, if this function is continuous (a straight line, without interruptions), then there must be a point at which q(p) equals $C/B > 0$. This indicates the participation rate that makes a voter indifferent between his pure strategies "participate" and "abstain."

Figure 8 illustrates this line of argument. On the vertical axis, it shows the probability with which a single voter can make a difference as a function of the participation rate (on the horizontal axis). For the qualitative argument, the exact shape of the function q(p) is not relevant. Relevant are the following three properties of q(p):

- q(0)=1 > C/B
- q(1) < C/B
- q(p) is continuous and monotonically decreasing.

If these three properties hold, then a participation rate p* exists, which is determined by

$$p^*=q^{-1}(C/B),$$

at which the voters are indifferent between participation and abstention. On the morning of the Election Day, if the anticipated participation rate falls short of p*, then all the voters who consider abstention would be inclined to reconsider their decision. On the other hand, if the perceived participation rate exceeds p* then the voters who consider voting will switch to abstention. Only a perceived participation rate of p* keeps all voters from altering their individual plans.

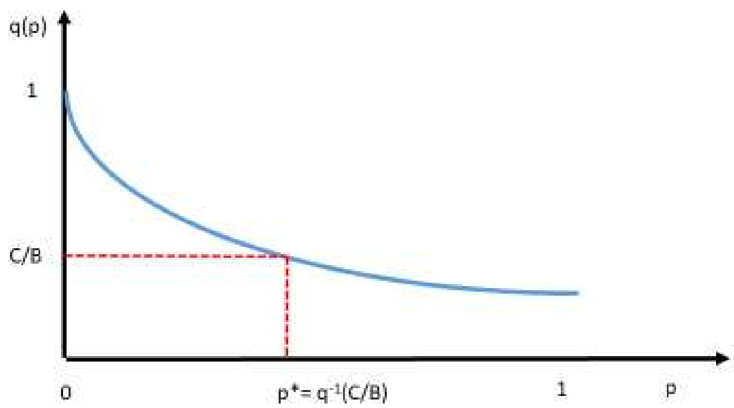

Figure 8: Indifference threshold of q(p)

This equilibrium result can be interpreted in two ways: Either, the voters are assumed to carry out their pure strategy "vote" with probability p* (and abstain with 1-p*), as if they were rolling an appropriately calibrated die.

The alternative interpretation stipulates that a share p* of the whole population makes the decision to vote, whereas a share 1-p* is determined to abstain. The two interpretations lead to the same result: The model predicts a participation rate of p* among homogeneous voters.

The equilibrium result of the Palfrey and Rosenthal model reacts to changes of the parameters. The lower the participation cost C or the higher the benefit upon prevailing, the higher the participation rate within a homogeneous voter group. If the ratio C/B is given, then the equilibrium participation rate would be lower if the function q(p) is initially steeper, which would be the case if the voters were more pessimistic as to whether their vote actually makes a difference.

The Palfrey and Rosenthal model would insinuate that the younger voters have faced higher opportunity cost of participation, or expected a lower perceived benefit B of avoiding Brexit. Moreover, their perception as to whether their individual vote would actually make a difference was perhaps more pessimistic. In **Figure 9**, this voter group is represented by the higher C_1/B_1 ratio and the steeper $q_1(p)$ curve.

Brexit supporters, on the other hand, were apparently more convinced that their vote would have an impact on the result, have seen a greater benefit prevailing, and perhaps suffered from lower opportunity cost of participation on the day of the poll. In **Figure 9**, this is represented by the flatter curve $q_2(p)$ and the lower ratio C_2/B_2. In equilibrium, the participation rate for group 1 is lower than for group 2. To determine the exact decline of the participation rate, it would be necessary to know the function q(p) exactly. However, the exact levels of p_1 and p_2 are of little interest. More relevant is to derive the qualitative impact that the slope of q(p) and the parameters B and C have on the equilibrium value of p*.

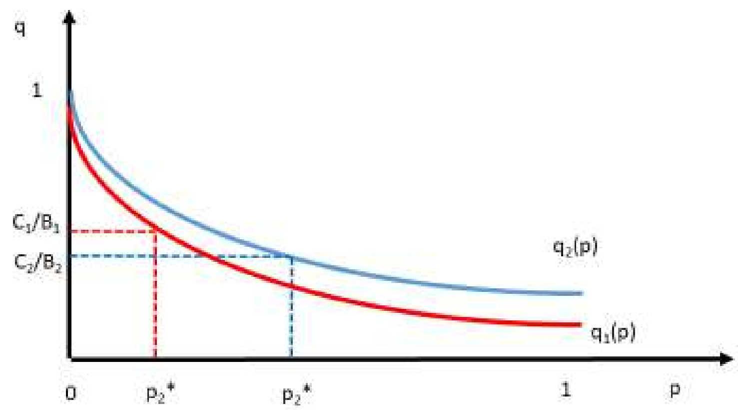

Figure 9: Indifference thresholds of two different voter groups

The model of Palfrey and Rosenthal may provide a qualitative explanation why different voter groups have shown different participation rates. However, this is only a stylized derivation of theoretical hypotheses – the contribution of game theory. Becker, Fetzer, and Novy (2017) have empirically analyzed the motives of different voter groups in the Brexit referendum. Essentially, they identified the following key drivers of the propensity to vote for Brexit: lower education, stronger historical dependence on manufacturing employment, lower income and higher unemployment.

4 Nash bargaining: Is it clever to have Boris instead of Theresa?

4.1 Non-cooperative vs. cooperative bargaining

Schwuchow and Pitsoulis (2018) sets out to analyze systematically "strategic irrationality" in the Brexit negotiations. They proposed a non-cooperative model, a game tree of the bargaining process. A non-cooperative model describes the players, their feasible actions ("strategies"), and the utilities the players attach to combinations of these actions. The problem with the no-cooperative approach is that a model is adequate (and its results can be interpreted in a meaningful way) only if it correctly describes the actual rules of the bargaining procedure under scrutiny. As an example, the authors make assumptions on which player is permitted to submit offers to the other party. After two stages of allowed proposals, i.e., one per player, their game tree ends. In reality, however, the players (denoted EU and UK), however, were not confined to such a bargaining protocol. Rather, bargaining could go on after the exchange of unaccepted proposals. Hence, it can well be the case that the positive results derived from a non-cooperative model have little or nothing to do with actual behavior, and the normative implications derived using these positive results have, therefore, limited relevance.

The alternative to non-cooperative bargaining models would be cooperative or axiomatic bargaining. This approach is adequate if no strict bargaining protocol exists, i.e., if bargaining occurs in an unstructured manner. John Nash, who received the Nobel prize in economics 43 years later, described a solution concept for cooperative bargaining in his first publication, Nash (1950a), focusing on two-person negotiations. The cooperative approach does not describe the rules (when the players are allowed to make what offers), but it instead begins with properties of a "sensible" solution. These properties are called "axioms" and the method, thus, is called "axiomatic approach."

Bargaining theory deals, in general, with the combination of (and the tension between) two problems: Bargaining parties simultaneously have a common interest and face a conflict. A typical (and highly stylized) bargaining situation involves two parties A and B who may share a specific sum of money among them. Let π denote the value of this bargaining "pie." If they close an agreement, then A receives his share x, and B receives (π-x).

If, however, the parties fail to agree, then they receive their "outside options," denoted α for A, and β for B. Their joint disagreement payoff, thus, amounts to ($\alpha+\beta$). The "bargaining rent" or "agreement rent" is positive if the joint agreement payoff exceeds the joint disagreement payoff: $(\alpha+\beta) < \pi$. The bargaining rent amounts to $(\pi-\alpha-\beta) > 0$.

The stylized example makes clear that it is in the common interest of the parties to come to an agreement, thereby creating a bargaining rent – if it is positive (if it is negative, then it is in the parties' common interest not to close an agreement and, instead, take their outside options). The conflict part of the ambivalent relation between the parties concerns the distribution of the bargaining rent. In other words, which precise agreement (x, π-x) should the parties close?

The higher x, the greater is A's share of bargaining rent; the lower x, the greater is B's share. This tension is similar to that in the BOS game described above: In that game the parties have a common interest to pursue the same activity. An agreement creates a rent, because the parties receive 3+1 instead of 0 as their joint payoff. However, the parties' interests in the two activities are divergent. If they coordinate on the one activity, then one party receives the bigger share of the cooperation rent; if they jointly carry out the other activity, then the other party gets the bigger share.

Three crucial differences exist between the BOS game and bargaining problems exist:

- In the non-cooperative game, coordination (or lack thereof) occurs spontaneously, without communication. Bargaining rests on the assumption that the parties communicate and, in the case of closing an agreement, the contract between them is enforceable.
- The rules of a non-cooperative game describe the interaction between the parties in detail, whereas in a bargaining problem, the parties interact without a specified protocol. Hence, the Nash equilibrium as the solution concept for non-cooperative games is of no value for the analysis of an unstructured bargaining situation.
- In the BOS game, the payoffs are not divisible and not transferable. If the parties meet at the opera, A receives 3 and B receives 1. Side-payments are ruled out (they could be introduced, but this would extend the strategy sets of the parties and, thereby, alter the non-cooperative game). In a bargaining problem, it is assumed that the joint payoff can be split up into infinitesimally small units and is transferable between the parties. Any sharing scheme x IR could, therefore, be implemented.

Recall the bargaining problem described above: Two parties A and B have to share π euros, and in the case of a non-agreement, A receives α euros, whereas B receives β euros, with (α+β) < π. It is natural to assume that neither party is interested in leaving crumbs on the table when splitting up their bargaining pie. Hence, if A receives x, then B receives the whole remainder, namely π-x. This is the "Pareto-optimality" axiom.

Moreover, it can be assumed that, in the case of an agreement x (denoting A's share), each party demands a payment that is at least as high as the respective outside option. In other words, no player would agree to an agreement if makes him worse off. Hence, the following two conditions both have to be satisfied for a sensible bargaining solution:

$$x \geq \alpha$$

$$(\pi - x) \geq \beta$$

These conditions can be summarized to α ≤ x ≤ (π-β), called the "individual rationality axiom." If two parties, when closing an agreement, obey these two axioms, then the predictions regarding their agreement x is limited to the interval [α, (π-β)]. Any x with α ≤ x ≤ (π-β) would be a valid prediction. The interval is called "bargaining range;" it is non-empty if α ≤ (π-β), which can be rearranged to

$$\pi \geq \alpha + \beta.$$

It is a necessary condition for a bargaining solution to exist that the bargaining range is non-empty. The problem with the bargaining problem is, however, that an infinite number of predictable agreements x exists which obey the two axioms. For the reasons already discussed above, it would be desirable to limit the number of predictions to just one.

4.2 The Nash bargaining solution

Nash (1950a) demonstrated that, under two more reasonable, but rather technical assumptions, the number of possible solutions of the bargaining problem becomes even smaller. The Symmetric Nash Bargaining Solution (SNBS) even leads to a unique prediction of the bargaining parties' behavior. The generalized Nash Bargaining Solution is the agreement that maximizes the "Nash product." The Nash product equals the multiplication of the net advantages that the bargaining parties derive from closing an agreement.

Assume that party i values a payment y at $U_i(y)$, with $i \in \{A; B\}$. Then A's net advantage from closing an agreement x amounts to $[U_A(x)-U_A(\alpha)]$, and the Nash product is

$$[U_A(x)-U_A(\alpha)]^{\delta}\,[U_B(\pi-x)-U_B(\beta)]^{(1-\delta)}.$$

Setting the first derivative of this Nash product with respect to x equal to zero provides the condition for an internal optimum. If the Nash product is concave in x, and if no corner solution exists, then the internal optimum is the value of x that maximizes the Nash product. The solution may depend on the parameter δ.

The parameter δ denotes the parties' relative bargaining skills: If A is the more experienced bargainer, then δ is closer to 1; if B is more skillful, then δ is closer to 0. If both parties have identical bargaining skills, then $\delta=0.5$ and the parameter would vanish from the solution. This is the case treated by the SNBS. Therefore, $x(\delta)$ is unique for any value of δ, and if δ is known, then the model provides a unique solution.

Three factors influence the NBS *ceteris paribus*: A party reaches a better bargaining result if

- the other party is more risk-averse (risk-aversion means that its utility function Ui(x) shows a stronger curvature);
- its bargaining skills are greater;
- its outside option is higher (as long as $\alpha+\beta \leq \pi$) or if the opponent's outside option is lower.

Assume, for simplification, that both parties are risk-neutral, i.e. their utility functions are linear. The identity $U_i(y)=y$ is as good as any other specification of a linear utility function. Moreover, assume for the parties to have identical bargaining skills. Then, our stylized bargaining problem lead to the following simplified Nash product:

$$[x-\alpha][(\pi-x)-\beta]$$

The first-order condition for an internal maximum then is $\pi-x-\beta-(x-\alpha)=0$, which can be rearranged to $2x=\pi+\alpha-\beta$. This leads to the SNBS

$$x^* = (\pi+\alpha-\beta)/2.$$

For a given pie size π, the SNBS x* is increasing in α and decreasing in β, just as postulated above for the general case. **Figure 10** demonstrates graphically the derivation of the SNBS for two risk-neutral parties.

The vertical axis shows A's payoff, and the horizontal axis shows B's payoff. The maximum payoff that a party could obtain is π, if negative shares are impossible. All sharing schemes that are Pareto-optimal (because they completely distribute the whole π) are on the diagonal line that connects the maximum payoffs, called the Pareto frontier. If negative shares were allowed, then the line had to be extended beyond these intersections, but this is irrelevant in our context.

The parameters α and β are assumed to be positive (one or both could be negative as well) and are represented by the dashed lines. Their intersection is the vector of outside options, the situation in which the parties are if they fail to close an agreement. An agreement brings them onto the Pareto frontier. Bargaining results that obey both the Pareto axiom and individual rationality are on the segment of the Pareto frontier between the two dashed lines.

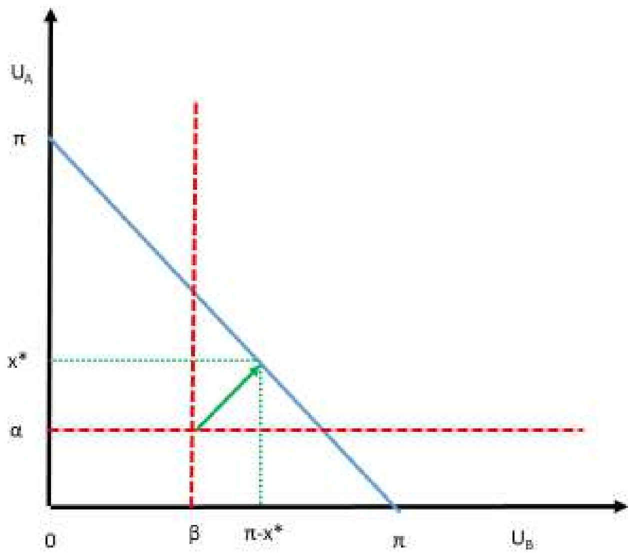

Figure 10: SNBS between two risk-neutral bargainers

For two parties of identical bargaining skill, the utility combination that maximizes the Nash product is found, starting in the vector of outside options, by following a 45-degree arrow towards the Pareto frontier. The dotted lines indicate the SNBS: A obtains x*, whereas B receives π-x*. The SNBS gives both parties an identical net gain from closing an agreement x*; hence x*-α = π-x*-β.

Technically, the bargaining solution is determined as the tangent point of the Pareto frontier and a hyperbolic curve that represents constant levels of the Nash product. However, for the line of argument pursued here, the idea of an arrow from the outside options towards the Pareto frontier is sufficient. For negotiators equipped with unequal bargaining skills, the slope of this arrow would be steeper (if A is more skilled) or flatter (if B is more skilled). If one party (or both) were not risk-neutral, then the Pareto frontier would be concave instead of linear.

4.3 Application to Brexit: Was it clever or just “madness” to switch to Boris?

During the negotiations between the UK and the EU over a Brexit agreement, it was often unclear whether a deal could be closed. As Muthoo and Benita (2017) predicted, the EU was reluctant to negotiate a trade agreement with the UK “until progress is made on their three ‘divorce issues’: EU citizens’ rights, the border with Northern Ireland, and the UK’s settlement of its exit bill.” One of these core issues was how to treat movement of goods and citizens across the border between the Republic of Ireland and Northern Ireland. Before Brexit, this border was intra-EU and, therefore, of diminished importance after the successful peace talks between the Northern Irish terror organization IRA and the UK. After Brexit, however, this line became a border between the EU and the UK. However, given the history of civil war and separation of the island, it was not in the interest of Ireland to go back to border control only because of the UK’s wish to leave the EU.

Indeed, Brexit was accomplished on January 31, 2020, without a definite agreement on these issues, which will certainly be an obstacle for the ultimate negotiations that shall be completed until the end of 2020. In the title of their paper, Schwuchow and Pitsoulis (2019) asked whether the way in which the UK have treated the negotiations with the EU was "madness." Even though it should clearly have been in the interest of both sides to close some deal, some actions taken made it less likely to accomplish this.

One event that took place during the pre-Brexit negotiations, in the summer of 2019, only created an additional obstacle for a quick agreement: Prime Minister Theresa May lost her office to Boris Johnson (who was confirmed in the public elections held in late 2019). Thereby, the UK chose to switch its representative for the Brexit negotiations with the EU. The May government was active in the Brexit negotiations for two years and appeared interested in a deal. Nevertheless, the early negotiations did not come to a result, as predicted by many authors, e.g. "The Balance of Trade" (2017). Already at the time of the referendum, Johnson was notorious for opposing a "bad" deal and favoring Brexit even without a deal. Was it, therefore, just "madness" for the UK to replace its most important representative?

This section examines whether it could make sense to switch to a hardliner representative, who threatens to leave the negotiations without a deal, in order to obtain a better deal. The idea behind this analysis is that it may influence negotiations if a delegate carries out the bargaining on behalf of a principal. If the bargaining protocol is well specified, it is possible to employ non-cooperative game theory and take into account the delegate's strategy set, which may differ from that of the principal. If delegation leads to a more favorable equilibrium outcome, and the additional payoff exceeds the cost of delegation, then it makes sense for the principal to send the delegate rather than bargain in person.

If, however, no specified bargaining protocol exists, then an axiomatic solution concept like the Nash bargaining solution is more adequate than non-cooperative game theory. The delegate may influence the Nash bargaining solution on three different channels. The delegate may

- have superior bargaining abilities (such as greater knowledge or better persuasion skills) in comparison to the principal;
- be less risk-averse than the principal;
- have a higher threat point than the principal or a lower marginal valuation of the bargaining outcome. Kirstein (2009) analyzes these two factors systematically.

The following analysis focuses on the third aspect of delegation. The parties' bargaining skills are, therefore, left out of the picture, and the players are all assumed to be risk-neutral. To simplify the line of argument, only three possible outcomes are assumed to exist:

- no deal (denoted ND),
- the "deal on the table" (denoted DOT),
- a deal that is "more favorable" for the UK (denoted MFD).

Three players are involved: the EU, the government of former Prime Minister May (TM), and the current government led by Prime Minister Boris Johnson (BJ).

The following table displays the assumed preference rankings of the three players:

	EU	TM	BJ
best	DOT	MFD	MFD
medium	MFD	DOT	ND
worst	ND	ND	DOT

For the EU, the least desirable scenario would clearly have been a Brexit without a deal. This would have enormously increased the transaction cost of future trade between the UK and the EU. Any deal would have been preferable. The May government was not opposed to this notion but had a different opinion regarding the optimal deal. Whereas the EU favored the DOT, player TM would have preferred the MFD. Nevertheless, the May government always appeared to be in favor of closing some sort of deal, rather than having no deal. While May was still in office, Fairchild (2019) described her main goal: "...to maximize the probability of a deal: thus, she has felt the need to compromise between the demands of the EU and the British people, in an attempt to gain acceptance from both sides." For both players, the EU and TM, both possible deals were inside the interval that is not only Pareto-optimal, but also individually rational, as depicted in the left hand side diagram of **Figure 11**.

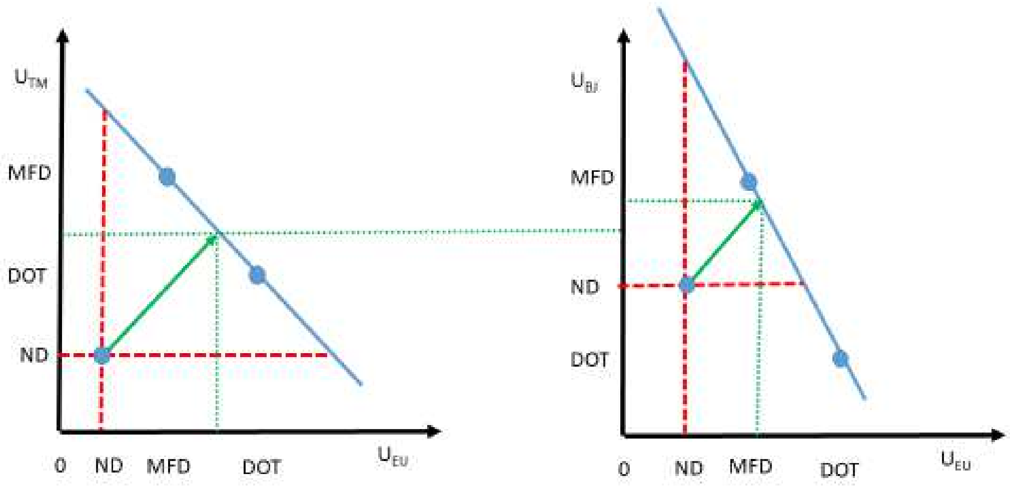

Figure 11: SNBS between EU and TM (left) and between EU and BJ (right)

This diagram shows the bargaining situation between the EU and the May government. The horizontal axis shows the preference ranking of the EU: DOT is optimal and MFD is better than ND. TM's ranking is on the vertical axis: MFD would be optimal, but DOT is still better than ND. The vector of outside options is located where the dashed lines at ND intersect. The Pareto frontier connects the DOT- and MFD-vectors. The exact shape of this frontier is of little relevance, as long as it connects these two vectors and is strictly downwards sloped. Moreover, the Pareto frontier should be non-convex (linear for risk-neutral players, or concave if one or more player is risk-averse).

As in the stylized bargaining problem discussed above, the SNBS is the intersection of the 45-degree arrow starting in the vector of outside options with the Pareto frontier. According to this model, both types of deal (DOT and MFD) were located in the area that is Pareto-optimal and individually rational, and the SNBS could have been expected to be a compromise between these two (not so) extreme positions.

The bargaining situation after the switch to the BJ government is displayed in the right hand side part of **Figure 11**. Johnson made it ultimately clear that his method of handling the Brexit negotiations started with the idea that "no deal is better than a bad deal," a position that has been heavily criticized among others, in Corbett (2017). Hence, the positions of the valuations of ND and DOT are inter-changed, whereas the UK's valuation of MFD has not changed, just as the three valuations of the EU. The effect of the switch to Johnson on the bargaining situation is an upwards shift of the vector of outside options. This is the point in which the solution arrow starts. Consequently, this shifts the SNBS upwards as well.

Note that the position of the Pareto frontier also has changed. The long dashed line that comes out of the left-hand-side diagram was the UK's negotiation result under May, whereas the shorter horizontal dashed line is the UK's bargaining result under Johnson. The position of the UK improves when sending a delegate whose valuations of the lesser preferred options (DOT and ND) are reversed.

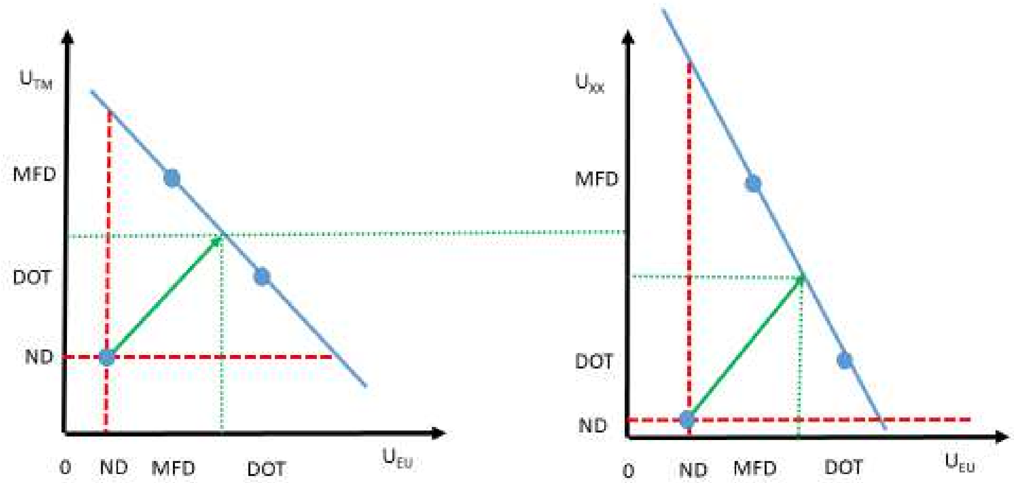

Figure 12: Impact of a decreased outside option on the SNBS

One caveat must be addressed: It makes no sense to switch to a delegate who has identical valuations for DOT and MFD and a much lower valuation of the option ND as the only difference in his ranking. To appoint such a delegate would have weakened the bargaining position of the UK. This would have led to a deterioration of the outcome for the UK, as it is shown in **Figure 12** (the left hand part of which is identical to that of **Figure 11**). In **Figure 12**, the qualitative ranking of the hypothetical delegate "XX" is the same as that of BJ, with the difference that the utility value of ND is now below that of DOT.

Decreasing the own valuation of the non-agreement decreases *ceteris paribus* the own share of the bargaining pie. An improvement of the own strategic position in negotiations would require, to the contrary, an increase of the own valuation of the outside option (as it has been discussed above). The most secure way to obtain such a strategic advantage would have been to find a new delegate (BJ) whose rankings for MFD and DOT were the same as that of the previous one (TM), but whose valuation of ND was higher than that of DOT.

5 Discussion of some of the existing literature

So far, only a few papers have, using game theory, attempted to analyze the processes and decisions that eventually lead to Brexit. This section presents a non-exhaustive selection and a brief discussion of some contributions.

5.1 Chu (2017): A chicken game

Chu (2017) has dubbed the negotiations between the UK and the EU a "game of chicken." The title of the paper describes this chicken game as one "between a juggernaut and a Mini – and Britain is the Mini." In the absence of a formal representation, the contribution is limited to a verbal description. The categorization as a chicken game is justified because it is assumed that, during the negotiations, both players are better off by playing "tough" to force the other player to make concessions. If this description were correct, then this would indeed leads to a "chicken" game. The term originates from the James Dean movie "Rebel Without a Cause," in which Dean is engaged in a peculiar duel: The players ride their cars towards a cliff, and the one who jumps last is the winner.

The game theoretic modeling of the original game is difficult, but here is a simplified version. It involves the two players, Jim and Biff, who drive on a narrow lane towards each other. The player who swerves left (it makes sense for the players to agree upon one direction beforehand, so as to avoid a stressful situation later) is the loser, whereas the one who goes straight ahead is the winner. If both stay on the road, they both die.

Assume that Biff values winning at W, losing at L, and being killed at D, with W > 0 > L > D. Jim values prevailing at π, losing at λ, and being killed at δ, with $\pi > 0 > \lambda > \delta$. **Figure 13** displays the strategic situation between the two boys.

Biff Jim	swerve left	stay straight ahead
swerve left	0 0	W > 0 $\lambda < 0$
stay straight ahead	L<0 $\pi > 0$	D << 0 $\delta << 0$

Figure 13: A "chicken game"

The game has two Nash equilibria in pure strategies, namely (turn, stay) and (stay, turn), and a third one in mixed strategies just as the BOS game discussed above, see Rasmusen (2001, p. 71). Therefore, it closely resembles the BOS game discussed earlier, but with a crucial difference regarding the payoffs assigned to the non-equilibrium strategy combinations. In the case of the BOS game, the parties would receive zero if they fail to coordinate on a Nash equilibrium strategy combination; they both would prefer to avoid this. In the chicken game, only one of the non-equilibrium strategy combinations brings zero for both players, whereas the other one provides a negative payoff for both. This difference in the payoff structures is important for the derivation of the mixed strategy equilibrium.

In the mixed strategy equilibrium, player Jim chooses to swerve with a probability of $W/(W+L-D)$. Because $D < L < 0 < W$ was assumed, $(L-D) > 0$ and, thus, the equilibrium probability of swerving is between 0 and 1. Biff's equilibrium probability of choosing to swerve is $\pi/(\omega+\lambda-\delta)$, also between 0 and 1.

Thus, each player chooses to drive straight ahead with probability $(L-D)/(W+L-D)$ and $(\lambda-\delta)/(\pi+\lambda-\delta)$, and the probability of the fatal outcome (stay, stay) is $(L-D)(\lambda-\delta)/(\pi-\lambda+\delta)(W+L-D)$. This result highlights the economic problem of a chicken game: In the mixed strategy equilibrium, the strategy combination (stay, stay) occurs with positive probability, but both players were better off by playing (swerve, swerve) instead. In other words, (stay, stay) is Pareto-dominated by (swerve, swerve).

5.2 Exton (2016): non-cooperative bargaining over two core issues

Exton (2016) set up a simple game in which two parties, "Britain" and "Brussels," carry out non-cooperative negotiations about two core issues. Britain may insist on denying freedom of movement to EU citizens, or it may relax its position and allow for such freedom of movement. Brussels may deny free trade with Britain, or it may allow for it. The author assumes that Brussels values free trade and freedom of movement at one utility unit each, whereas Britain attaches a utility unit to free trade and to no freedom of movement, respectively, whereas having to accept the respective other outcome brings a utility of -1. Hence, a player receives +2 if achieving both of his goals, zero if only one goal is reached, and -2 in case of total failure. The normal form of the game is displayed as a bi-matrix in **Figure 14**.

Brussels Britain	no free trade	free trade
no freedom (of movement)	-2 0	0 2
freedom (of movement)	0 -2	2 0

Figure 14: 2-person game from Exton (2016)

Apparently, both players have a dominant strategy: For Brussels, it is optimal to pursue the "free trade" strategy, regardless of Britain's choice. For Britain, "no freedom" is optimal, regardless of Brussel's choice. Hence, the game has a unique Nash equilibrium in dominant strategies, namely (no freedom, free trade), just like the famous "Prisoners' dilemma." The difference between the Prisoners' dilemma and Exton's game is, however, that the former has a Pareto-inefficient Nash equilibrium, whereas the Nash equilibrium in the latter is Pareto-efficient.

Exton comes to the same equilibrium result, but continues by calling this, a bit surprisingly, a "chicken game." The chicken game, as discussed in the previous subsection, is a specific 2-person game with two Nash equilibria in pure strategies and a third equilibrium in mixed strategies.

The other conceptual flaw of Exton's analysis is that "no free trade" or "no freedom of movement" are not strategies that players can pursue, but rather outcomes of the interaction. A strategy is a behavior in the game, such as "make a concession," "agree to something," or "refuse to agree." Combinations of strategies lead to outcomes such as "no free trade agreement" (is closed). The non-cooperative game of Exton, thus, is wrongly specified. If two issues are at stake, free movement (M) and free trade (T), then each player can agree to both issues, to just one, or reject both. Each player, therefore, has four strategies at hand: (T,M), (NT,M), (T,NM), (NT,NM), where the N indicates a "not."

Assume that closing an agreement requires the agreement of both players. Hence, if one player rejects the issue, then this agreement is not closed. 16 combinations of these four strategies exist, as it is shown in **Figure 15**. If both players choose (T,M), then both agreements are closed. If one player chooses a strategy that contains NT, this blocks the free trade agreement and implements NT. If one player chooses a strategy containing NM, then there is no agreement on free movement. The lower-case "n" in some cells of **Figure 15** denotes that no agreement at all will be closed.

Brussels Britain	(T,M)	(NT, M)	(T,NM)	(NT,NM)
(T,M)	TM	M	T	n
(NT,M)	M	M	n	n
(T,NM)	T	n	T	n
(NT,NM)	n	n	n	n

Figure 15: Possible outcomes in an 4x4 game, inspired by Exton (2016)

According to the assumptions made by Exton, the player's valuations differ: Player Brussels values two agreements at +2, only one agreement at 0, and no agreement at -2. Player Britain attaches +2 to the outcome T. Britain's payoff is 0 either if no agreement is closed ("n") or two agreements are closed (outcome TM) and -2 if free movement is possible without free trade (outcome M).

Note that no outcome is possible which grants both players their maximal payoff simultaneously. The best possible outcome of the game, thus, is one in which one player receives 2 and the other 0. It is not Pareto-optimal for one player to receive -2 while the other player receives 0 (in this case, both players could be made better off by switching to an outcome that brings both players 2 utility units more).

The correct model setup leads to a game in which no player has a dominant strategy. If, for example, player Brussels chose (T,M), then player Britain's best reply would be (T,NM), bringing about an individual payoff of 2. Should Brussels choose (NT,NM), then Britain would receive 0 regardless of which strategy it chooses; therefore, Britain would be indifferent between its pure strategies. The respective best replies are underlined and printed in bold in **Figure 16**. Any cell that contains a pair of best replies (two underlined, bold payoffs) is a Nash equilibrium in pure strategies.

The correctly modelled game has 4 Nash equilibria in pure strategies:

- (T,NM) by Britain, and (T,M) by Brussels, leading to a trade agreement (T);
- (T,NM) vs. (T,NM), leading to T as well;
- (NT,NM) vs. (NT, M), leading to none of the agreements (n);
- (NT,NM) vs. (NT,NM), leading to n.

The assumptions made by Exton regarding the preferences of the two players and the coarse description of their interaction (focusing on just two issues, and assuming non-cooperative bargaining) lead to a 4x4 game the Nash equilibria of which are characterized by Britain insisting on NM. The two strategies that entail "M" are never part of a Nash equilibrium. For Brussels, on the other side of the table, all four feasible strategies can be equilibrium strategies: If Britain chooses (T,NM), then Brussels should choose either (T,M) or (T,NM), thereby closing the T agreement (whereas Brussels' position on M is already irrelevant). If Britain chooses (NT,NM), then Brussels is indifferent between its strategies.

Brussels Britain	(T,M)	(NT,M)	(T,NM)	(NT,NM)
(T,M)	**2** 0	0 -2	0 **2**	-2 **0**
(NT,M)	**0** -2	**0** -2	-2 0	-2 **0**
(T,NM)	**0** **2**	-2 **0**	**0** **2**	-2 **0**
(NT,NM)	**-2** 0	**-2** **0**	**-2** 0	**-2** **0**

Figure 16: 2-person non-cooperative negotiations as a 4x4 game, inspired by Exton (2016)

To summarize, Britain can fully determine the equilibrium outcome of the game: If it chooses its strategy (T,NM), then Brussels happily closes at least the trade agreement. The two players receive 2 (for Britain) and 0 for Brussels. If, alternatively, Britain chooses its strategy (NT,NM), then it receives 0 and Brussels -2, which is Pareto-inferior.

Both outcomes are, from the viewpoint of positive analysis, Nash equilibria. Therefore, they are fully justified as predictions of the players' behavior. It would be a normative fallacy (or wishful thinking) to argue that the players would select the Pareto-superior of these possible outcomes. However, as Britain can fully determine the outcome and will individually benefit from committing to (T,NM), it can be expected to choose this strategy – in the framework of this game. Thus, it can be expected for the two players to close a trade agreement but no agreement on free movement.

An asymmetry exists with regard to the goals pursued by the two players in this game: For Britain, it is easier to achieve both goals, T and NM, whereas Brussels can be grateful if it achieves one goal, T, and the M goal is far out of reach (whereas the “chicken game” is fully symmetric!). The reason for this asymmetry is the assumption according to which it is easier to reach a non-agreement than an agreement. For a non-agreement, a player’s individual veto is sufficient, whereas any agreement requires both players’ consent. The two players in the Exton (2016) game have a common interest in closing the trade agreement and divergent interest with respect to migration. Thus, Britain is in a very comfortable position: It can lure Brussels into the trade agreement and unilaterally block the migration agreement.

5.3 Busch et al. (2016): Playing “tough” or “easy”?

The paper by Busch, Diermeier, Goecke, and Hüther (2016) also analyzes a very simple 2x2 game, in which the two players, UK and EU, negotiate their future trade relations and are assumed to have two strategies at hand: “tough” and “easy.” If both play “tough,” then Britain is, after Brexit, just another WTO member, without close ties to the EU. If the EU plays tough and Britain easy, then Britain would receive a status similar to that of Norway (no political participation, but free market access). If the UK plays it tough, and the EU chooses easy, then Britain would be allowed to “cherry-pick” its most favorable contract clauses, such as free market access without free movement, as it was discussed in Exton (2016). If both players choose easy, then the outcome is dubbed “Norway+” (similar to Switzerland’s status). These outcomes are explained on p. 7f. of the paper in greater detail. The authors assume two different preference rankings for the two players, one considering the long-run effects and the other one with regard to the short run effects of Brexit (see p. 9f. and p. 12f.).

Figure 17 depicts the game when considering long-run consequences. In their game forms, the authors use qualitative symbols (like --, -, 0 or +) in place of numerical payoffs. In **Figure 17**, these symbols have been replaced by ordinal numbers, where the higher figure denotes the more preferred outcome (and identical figures indicate identical preference ranks). The long-run game has a unique Nash equilibrium in dominant strategies: player UK chooses “easy” and player EU “tough,” which would lead to the “Norway” outcome.

EU UK	tough	easy
tough	**3** 1	1 1
easy	**4** **2**	2 **3**

Figure 17: 2-person game (long run) in Busch et al. (2016)

This equilibrium is Pareto-efficient. The player UK could only obtain a better payoff (3 instead of 2) if the outcome of player EU would be diminished (2 instead of 3). Hence, there does not seem to be an efficiency problem involved in the long-run version of this game.

In the analysis of the short-run effects, the rankings differ only slightly. This, however, would have a crucial effect on the strategic situation, which is displayed in **Figure 18**. The game now has two Nash equilibria in pure strategies, namely (tough, easy) and (easy, tough). The interests of the two parties in these Nash equilibria are strictly opposed to each other. Moreover, the two strategy combinations that are not Nash equilibria would bring very different results for the players. The short-run game is, thus, a variation of the chicken game outlined above. As already explained, it would have a third equilibrium, one in mixed strategies. To determine the equilibrium probability choices of the two players, however, would require assuming cardinal payoffs instead of ordinal ones (numbers are cardinal if the zero value and the distance between two values – the unit – are well defined).

The predictive value of the two model variations is, however, limited. The second model has multiple Nash equilibria anyway. If the players manage to coordinate on a pure-strategy equilibrium, then each player would have an interest in picking the one in which he alone chooses the easy strategy. The tough strategy is a good choice only if the other player chooses easy. The best reply to the other player's choice of "tough" is to choose "easy". This is perhaps not an adequate description of the strategic situation during the actual Brexit negotiations.

Moreover, in both games the Nash equilibria in pure strategies are Pareto-efficient, so it is questionable whether an economic or institutional problem exists. The coordination problem embodied in the short-run game may invite institutional design that facilitates successful coordination. Introducing side-payments would extend the players' strategy sets to allow them to reach a more satisfying solution.

EU UK	tough	easy
tough	1 1	**3** **4**
easy	**4** **2**	2 3

Figure 18: 2-person game, short-run version, in Busch et al. (2016)

5.4 McCulloch (2017): Nash equilibrium and subgame perfect equilibrium.

This paper initially looks at a 2x2 game, just as most of the papers discussed here. The EU's strategies are to offer a "bad" deal or a "good" deal. In the framework of Nash bargaining, calling a deal "good" may have two meanings: Either a Pareto-efficient outcome or a point on the Pareto frontier that is more favorable for one of the parties. In the framework of a non-cooperative game, however, the term "good deal" should refer to a strategy, a proposal made by the EU, and not to the outcome.

McCulloch assumes that Brexit will lead to mutual losses, because even with a deal gains from trade will forgo. A "good" deal is one that leads to smaller losses than a "bad" one. Under a bad deal, the UK (and perhaps the EU as well) will have to bear higher losses. Britain is initially assumed to have two strategies: accept the offered deal, or reject it and leave the EU anyway.

The second model presented by McCulloch (2017, Figure 3) introduces a third strategy: reject the deal and stay in the EU. **Figure 17** presents this second game, in which the EU has two strategies, whereas the UK has three strategies. According to McCulloch, the payoffs represent qualitative benefits, not monetary outcomes.

If the UK accepts a bad deal proposed by the EU, then both parties are assumed to suffer heavy losses. Accepting a good deal obviously Pareto-dominates the bad deal. Leaving without a deal is assumed to lead to intermediate losses for both sides. If the UK stays in the EU after having been offered a good deal preserves the status quo for the EU, so its payoff is normalized to zero. For many in the UK, however, this outcome is not preferred, so there is some loss involved on the part of the UK. Keeping the UK in the EU after tough negotiations would be a political win for the EU; hence, McCulloch assigns a payoff of +5 here. In the evaluation of the game, the author concludes that "the equilibrium of this situation" is for the EU to offer a bad deal and for the UK to stay.

EU UK	offer "a really bad" deal	offer a good deal
accept deal	-15 -15	**-2** **-2**
reject and exit anyway	**-10** -10	**-10** -10
reject and stay in EU	**5** **-8**	0 -8

Figure 19: Second game in McCulloch (2017)

Although this is not wrong, it is a very incomplete analysis. Indeed, if the table in **Figure 19** is interpreted as the strategic form of the game between players EU and UK, then the action combination "offer a bad deal" with "reject and stay" would be a Nash equilibrium of this game. However, the action combination "offer a good deal" with "accept deal" is a Nash equilibrium as well (best replies are underlined in **Figure 19**). There is no compelling criterion to select one of the Nash equilibria. Moreover, if a game has two Nash equilibria in pure strategies, it will have another one in mixed strategies. Hence, the analysis of McCulloch is definitely not complete.

The main objection, however, against this superficial evaluation of the interaction between the players EU and UK relates to the assumption that the players make their moves simultaneously. In game theory, "simultaneous" does not necessarily mean "at the same time." Rather, the term refers to the transmission of information. If no player is able to observe the other players' behavior when making her own decision, then the game is simultaneous, even if the decisions take place at different points in time. An example for such an information structure is the game of poker, in which one player after another makes decisions regarding his hand of cards, but no player is permitted to observe the others' decisions. In chess, the information structure is different: Each player, when planning her next move, can observe what moves her opponent has made up to this point.

The verbal description of the players' actions in the game under scrutiny insinuate that the interaction takes place sequentially. If player UK decides whether to accept the EU's proposal, the UK has certainly observed whether the proposal was a "good" or a "bad" one. **Figure 20** shows this interaction as an "extensive form" or "game tree." The interaction starts at the decision node labeled "EU" at the left hand side. The EU may choose either to offer a "bad" deal – and move upwards in the game tree – or a "good" deal (downwards). In both cases, the UK can observe its opponents choice before choosing its reply. As the UK can distinguish between the two possible proposals, it has the opportunity to react differently at its two decision nodes. At the upper decision node labeled "UK," this player chooses the reaction to the bad deal. At the lower decision node, player UK determines its reaction to the "good" offer.

The game tree can be evaluated in two different ways: First, it is possible to set up the adequate normal form (strategic form) taking into account the information structure conveyed by the game tree. Second, the game tree (extensive form) can be directly solved by "backwards induction." The normal form of a game consists of the players, their strategies, and their payoff functions, as explained above. The set of players contains the EU and the UK. Player EU's strategy set is easy to determine. It contains simply the two strategies "good" and "bad."

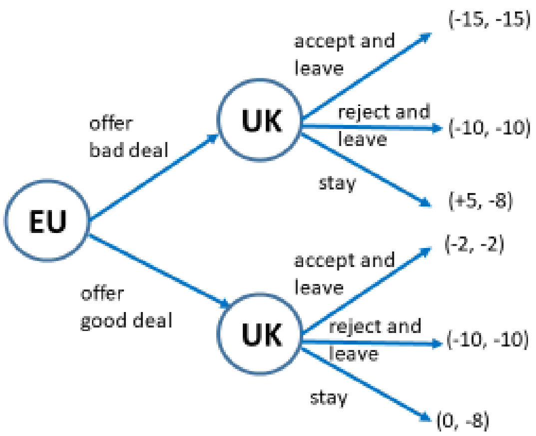

Figure 20: Extensive form of the game proposed by McCulloch (2017)

More problematic is the strategy set of player UK. This player has two decision nodes. At each decision node, three actions are feasible. A "strategy" is a plan how to behave at each position of the game where a player has to make a decision. Hence, a strategy of player UK consists of two decisions, one at the upper node and another one at the lower node. Of course, the player can choose similar actions at the two nodes. An example for such a strategy is "stay, stay." The two actions are similar, but not identical, since the first one is the plan how to behave at the upper node, and the second entry denotes the plan for the bottom node. Alternatively, player UK could choose different actions at his two different decision nodes. An example would be "accept, stay" – the plan to carry out accept at the upper node, and stay at the bottom node.

Player UK's strategy set, therefore, consists of nine strategies, which are listed in the first column of **Figure 21**. If the UK chooses, for example, the strategy "accept, reject" it plans to accept a bad proposal (at the upper node) and to reject a good one (at the lower node). If the EU replies by submitting a bad proposal (moves upwards), this would lead to payoffs of -15 for both players. In the game tree, the players would move along the path (bad, accept). Player UK's plan to choose "reject" at the bottom node will not be carried out, since this node is offside this path. If UK chooses their strategy "accept, reject" and the EU chooses a "good" deal, then the players receive -10 each as payoffs. They bring about the path (good, reject) and the decision how to react to a bad deal is offside this path.

The strategic form can be analyzed for Nash equilibria in the standard way explained above. The best replies of player UK are underlined (and printed in bold): Should the EU offer a bad deal, it is optimal to choose stay, regardless of what the UK's choice at the other decision node would be. Therefore, player UK is indifferent between three best replies to "bad", namely (stay, accept), (stay, reject), and (stay, stay). Under the hypothesis that EU offers a "good deal", the best reaction of UK would be to accept it; hence, this player is indifferent between (accept, accept), (reject, accept), and (stay, accept).

EU UK	offer bad deal	offer good deal
(accept, accept)	-15 -15	**-2** -2
(accept, reject)	-15 -15	-10 -10
(accept, stay)	-15 -15	0 -8
(reject, accept)	-10 -10	-2 -2
(reject, reject)	-10 -10	-10 -10
(reject, stay)	-10 -10	0 -8
(stay, accept)	**+5** **-8**	-2 -2
(stay, reject)	**+5** **-8**	-10 -10
(stay, stay)	**+5** **-8**	0 -8

Figure 21: The normal of the extensive form game inspired by McCulloch (2017)

In the second step, these best replies are examined as to whether player EU's action is a best reply as well. This is true for the first case: If the UK chooses a strategy that starts with "stay", regardless of the second entry, it is indeed optimal for the EU to reply with "bad". This includes player UK's strategy (stay, accept). If the UK chooses one of the other two strategies that show "accept" as the second entry, namely (accept, accept) and (reject, accept), then it is optimal for the EU to reply with "good". Hence, we have identified five Nash equilibria in the game, which lead to the following two of equilibrium paths:

- <bad, stay>, regardless of how the UK would react to a “good” proposal, with payoffs -8 for the UK and 5 for the EU;
- <good, accept>, regardless of whether player UK would react to a “bad” proposal with accept or reject, which brings the players -2 each.

McCulloch’s analysis, according to which “the equilibrium of the game” is <bad, stay> is, hence, not only methodologically flawed but also incomplete. It is flawed because **Figure 19** is not the adequate normal form of the interaction under scrutiny, if the interaction is a sequential game. Then, **Figure 21** is the adequate normal form. However, this is only a technical caveat.

More important is that even the game depicted in **Figure 19**, and even more so the game in **Figure 21**, has more than one Nash equilibrium. In both models, the simultaneous version as well as the sequential version, the normal form reveals that two patterns of behavior can be predicted: “bad” proposal, combined with “stay,” and a “good” offer that is accepted.

The equilibrium path <bad, stay> brings the players -8 for the UK and +5 for the EU, whereas (good, accept) brings -2 for both players. Hence, in this case even the (questionable) criterion of Pareto-dominance does not allow for a decision which equilibrium is more plausible. The German economist Reinhard Selten (who, in 1994, shared the Nobel Prize with John F. Nash and John C. Harsanyi) introduced “subgame perfectness” as a simple selection criterion that helps to eliminate implausible Nash equilibria, see Selten (1965, 1978). A Nash equilibrium is subgame perfect if it consists only of optimal decisions in each part of the game tree.

The easiest way to identify the subgame perfect equilibrium of a game is “backwards induction.” In the game tree depicted in **Figure 18**, the backwards induction starts at the final decision nodes, those of player UK. At the upper decision node of the UK (after the bad offer), this player would clearly choose “stay”, because that secures the lowest possible loss of -8. At the lower node, player UK would choose “accept” so as to secure -2.

According to the analysis of the normal form in **Figure 21**, player UK has five Nash equilibrium strategies: all three that start with “stay” and, additionally, the other two that end with “accept.” According to the first step of the backwards induction, only one of these five strategies is subgame perfect, namely the strategy (stay, accept).

If player EU anticipates that UK will choose nothing but the subgame perfect strategy, then the best reply (EU’s subgame perfect strategy choice) is to offer a “bad” deal. Hence, the subgame perfect equilibrium of the game depicted in **Figure 20** consists of the strategy combination “bad” with (stay, accept), leading to the equilibrium path <bad, stay>. The other Nash equilibrium strategy combinations are not subgame perfect.

In the end, the result of McCulloch’s analysis appears to be correct – the most plausible of the many Nash equilibria of the sequential game in **Figure 18** is for the EU to offer a bad deal, and for the UK to stay. However, the derivation of this result was questionable. The game presented in the paper (see **Figure 19** above) has two Nash equilibria, and the equilibrium selection criterion of subgame perfectness is not applicable as the game is simultaneous anyway. The sequential version, in its normal form, has five Nash equilibria, but the application of subgame perfectness would then allow for selecting one of them as plausible. The original game presented by McCulloch does not allow for the conclusion that the game has “one equilibrium;” only the sequential version leads to this result.

It is a different question whether this result bears any empirical relevance. This is, of course, highly questionable, given the fact that Britain has indeed carried out Brexit on January 31, 2020.

Literature

Arrow, Kenneth J. (1951) Social Choice and Individual Values, Yale University Press, 2nd ed. 1963.

Becker, Sascha O. / Fetzer, Thiemo / Novy, Dennis (2017) Who Voted for Brexit? A Comprehensive District-Level Analysis. In: Economic Policy 32(92), 601–650.

Busch, Berthold / Diermeier, Matthias / Goecke, Henry / Hüther, Michael (2016) Brexit and Europe's Future – a Game Theoretical Approach. IW policy paper 18/2016, Institut der deutschen Wirtschaft, Köln.

Chu, Ben (2017) The Brexit Negotiations Are a Game of Chicken between a Juggernaut and a Mini – and Britain is the Mini. In: The Independent, October 15, 2017. Downloaded from https://www.independent.co.uk/voices/brexit-negotiations-no-deal-europe-trade-economics-a8001341.html (last access on March 2, 2020).

Corbett, Richard (2017) The 'no deal is better than a bad deal' fallacy. In: The Parliament Magazine March 31. Downloaded from https://www.theparliamentmagazine.eu/articles/opinion/no-deal-better-bad-deal-fallacy (last access on April 15, 2020).

Elgot, Jessica (2019) May's deal, no deal or Brexit delay: what happens now? The Guardian, February 26, 2019. Downloaded from https://www.theguardian.com/politics/2019/feb/26/mays-deal-no-deal-or-brexit-delay-how-will-mps-vote (last access on March 2, 2020).

Elgot, Jessica / Weaver, Matthew (2017) Brexit Talks: UK Must Prepare to Leave Without Deal, Say Ex-ministers. In: The Guardian, October 19, 2017. Downloaded from https://www.theguardian.com/politics/2017/oct/19/brexit-talks-uk-must-prepare-to-leave-without-deal-say-former-ministers (last access on March 2, 2020).

Exton, Guy (2016) Brexit and Game Theory: A Single-case Analysis. In: Rogers, Matthew (ed.), Merici, vol. 2, ANU eView, The Australian National University, Canberra.

Fairchild, Richard (2019) Opinion: Decoding Brexit through Game-theoretic Approach. Published online by China Global Television Network (CGTN), updated January 17, 2019. Downloaded from https://news.cgtn.com/news/3d3d674e3045444d32457a6333566d54/index.html (last access on March 2, 2020).

Gibbard, Allan (1973) Manipulation of voting schemes: a general result. In: Econometrica 41(4), 587–601.

Gongloff, Mark (2019) We Might Have Found the Least-Hated Brexit Option. Theresa May's much-disparaged plan might be the one most Brits could at least stomach. Bloomberg Opinion, January 23, 2019. Downloaded from https://www.bloomberg.com/opinion/articles/2019-01-23/brexit-options-one-is-surprisingly-least-hated (last access February 10, 2020).

Holm, Håkan J. (1995) The Prisoners' Dilemma or the Jury's Dilemma? A Popular Story with a Dubious Name: Note. In: Journal of Institutional and Theoretical Economics (JITE) / Zeitschrift für die gesamte Staatswissenschaft 151(4), 699-702.

Kirstein, Roland (2009) Optimal Delegation in Nash Bargaining. FEMM discussion paper 2009-01; Magdeburg. Downloaded from http://dx.doi.org/10.2139/ssrn.1353054 (last access on April 13, 2020).

McCulloch, Neil (2017) The Game Theory of Brexit. Published online on politics.co.uk. Downloaded from https://www.politics.co.uk/comment-analysis/2017/03/15/the-game-theory-of-brexit (last access on March 2, 2020).

Muthoo, Abhinay / Siobhan, Benita (2017) Game Theory Experts: Credibility is Key for a Successful "No Deal" Brexit Strategy. Published online in The Conversation. Downloaded from http://theconversation.com/game-theory-experts-credibility-is-key-for-a-successful-no-deal-brexit-strategy-85919 (last access on March 2, 2020).

Nash, John F. (1950a) The Bargaining Problem. In: Econometrica 18(2), 155-162.

Nash, John F. (1950b) Equilibrium Points in n-Person Games. In: Proceedings of the National Academy of Sciences of the USA 36(1), 48-49.

Nash, John F. (1951) Non-cooperative Games. In: Annals of Mathematics 54(2), 286-295.

Nash, John F. (1953) Two-Person Cooperative Games. In: Econometrica 21(1), 128-140.

Palfrey, Thomas R. / Rosenthal, Howard (1983) A strategic calculus of voting. In: Public Choice 41, 7–53.

Rasmusen, Eric (2001) Games and Information. An Introduction to Game Theory. Third edition. Blackwell Publishing, Malden, Oxford, Carlton.

Reny, Philip J. (2001) Arrow's Theorem and the Gibbard-Satterthwaite Theorem: a Unified Approach. In: Economics Letters 70(1), 99-105.

Riker, William H. / Ordeshook, Peter C. (1968) A Theory of the Calculus of Voting. In: American Political Science Review 62(1), 25-42.

Satterthwaite, Mark A. (1975) Strategy-proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions. In: Journal of Economic Theory 10(2), 187–217.

Schwuchow, Soeren C. / Pitsoulis, Athanassios (2019) Though this Be Madness: A Game-Theoretic Perspective on the Brexit Negotiations. Paper presented during the annual conference of the Comittee for Economic Systems of the Verein für Socialpolitik, Potsdam, July 2019.

Selten, Reinhard (1965) Spieltheoretische Behandlung eines Oligopolmodells mit Nachfrageträgheit. In: Zeitschrift für die gesamte Staatswissenschaft (Journal of Institutional and Theoretical Economics), part 1: 121(2), 301-324; part 2: 121 (4), 667-689.

Selten, Reinhard (1978) The Chain Store Paradox. In: Theory and Decision 9, 127–159.

The Balance of Trade (2017) Game Theory and Why Early Brexit Discussions Were Destined to Fail. Published online by Seeking Alpha.
Downloaded from https://seekingalpha.com/article/4104319-game-theory-and-why-early-brexit-discussions-destined-to-fail (last access on March 2, 2020).

Otto von Guericke University Magdeburg
Faculty of Economics and Management
P.O. Box 4120 | 39016 Magdeburg | Germany

Tel.: +49 (0) 3 91 / 67-1 85 84
Fax: +49 (0) 3 91 / 67-1 21 20

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