Complexity of Networking - An Experimental Study of the Network Hawk Dove Game

Siegfried Berninghaus/Stephan Schosser/Bodo Vogt

OTTO VON GUERICKE
**UNIVERSITÄT
MAGDEBURG**

**FACULTY OF ECONOMICS
AND MANAGEMENT**

Complexity of Networking - An Experimental Study of the Network Hawk Dove Game

Siegfried Berninghaus, Stephan Schosser and Bodo Vogt*

November 30, 2011

Abstract

Complexity of strategies is central for human decision making and attracted interest of different game theorists in the recent years. Nevertheless, behavioral economists have neglected the importance of complexity in their analyses. In this paper, we analyze network formation and action selection in a Hawk Dove Game with focus on complexity aspects. We conduct experiments with three variants of the game which are equivalent from a game theoretic perspective, but differ from a complexity theoretic perspective. Our results show, that complexity of decision making has an impact on the strategies played and that efficiency is higher the less complex the decision problem is.

With the emerging popularity of game theory we put increasing interest in predicting human behavior with simple game theoretic models. We can reduce several everyday decision problems to the prisoner's dilemma, coordination games or other simple bimatrix games. At the same time, game theorists are aware of the fact that complexity of the decision situation can prevent players from resorting to certain equilibria. Therefore, they introduced extensions of traditional models, which capture the complexity of decision problems (see e.g. (Abraham Neyman 1985, Ariel Rubinstein 1986, Ehud Kalai & William Stanford 1988)). Behavioral sciences analogously show (Georg A. Miller 1956, Nelson Cowan 2001) that the human ability to process information is limited. Nevertheless, to date a gap between both results, namely game theoretic models capturing complexity and behavioral research on limits of human information processing, exists.

In this paper, we argue that the lack of complexity is central in networking problems. Here, participants simultaneously decide with whom to interact and what strategies to use. Resulting, as we will show, in difficult decision problems. The focus of this paper lies on several different aspects: (1) Will varying the complexity of decision problems influence the equilibria participants in behavioral experiments play? (2) Does complexity influence the overall payoffs reached? (3) Which complexity models capture human behavior best? By analyzing these aspects, we go a first step towards narrowing the gap between behavioral justification and game theoretic modeling of complexity.

Traditional evolutionary game theory focuses on the evolution of strategy configurations in populations of players who play the same two-person game with the whole population via random matching. Researchers investigate whether population members coordinate on one particular equilibrium of the one-shot game (see e.g. (H. Peyton Young 1993, Madjid Amir & Siegfried K. Berninghaus 1996)). Early game theoretic analyses of networks extend this work by externally imposing fixed network structures (see e.g. (Siegfried K. Berninghaus & Ulrich Schwalbe 1996, Siegfried K. Berninghaus 1996, Siegfried K. Berninghaus, Karl-Martin Ehrhart & Claudia Keser 2002)). In corresponding work on action selection, players are exclusively matched with a fraction of the population, their neighborhood, and not with any other member of the population. Furthermore, the neighborhoods are overlapping. I.e., each member of the population can belong to several neighborhoods. In analogy to traditional evolutionary game theory, analyses of action selection in networks investigate whether the structure of the network ensures that one equilibrium is spread across the whole population or that different equilibria coexist.

A drawback of models with a focus on action selection is that the model maker imposes the network structure exogenously. Literature on network formation overcomes this drawback. Here, the members of the population decide which network structure to establish by playing a non cooperative network game (see e.g. (Matthew O. Jackson & Asher Wolinsky 1996, Venkatesh Bala & Sanjeev Goyal 2000, Hans Haller & Sudipta Sarangiz 2003)). Network games are commonly interpreted as information exchange games in which the payoff of one member of the population depends on the number of other population members he is (directly or indirectly) connected to.

As models on action selection, models on network formation have their drawbacks. One central criticism concerns their applicability to real world problems. While network formation itself is important, it is unclear who will establish a link, if one cannot use this link for beneficial interaction. Therefore, models emerged which combine network formation and action selection (see e.g.(Brian Skyrms & Robin Pemantle 2000, Yann Bramoullé, Dunia López-Pintado, Sanjeev Goyal & Fernando Vega-Redondo 2004, Siegfried K. Berninghaus & Bodo Vogt 2006). In these models the members of the population simultaneously decide with whom they want to interact and what strategy they want to play in a two person game identical to all members in the population.

In contrast to bimatrix games, games on network formation or games on action selection in networks the simultaneous choice of actions and network links is complex. Each member of the population has to anticipate the links all other member establish and what strategies they play depending on their link choice. Only if this prediction is adequate, he can resort to the equilibrium strategy. While the corresponding decisions are rather simple in coordination games as all members can coordinate towards one common strategy and than link to each other, decision are difficult in anti-coordination games. Here, strategies can coexist within the population. Therefore, in this paper we focus our analysis on hawk-dove games, a popular variant of anti-coordination games.

A recent experimental study (Siegfried K. Berninghaus, Karl-Martin Ehrhart & Marion Ott 2010) analyzed the network formation and action selection using a Hawk Dove Game as base game. According to this study participants do not play the Nash equilibrium. Nevertheless, the authors observed that participant behavior is regular. The participants ended up in stable strategy configura-

tion that, also different to the Nash prediction, where similar across different participant groups.

In this paper, we experimentally analyze the behavior of participants in a game on network formation and action selection using a Hawk Dove Game. We play three different treatments of the game varying the complexity of the game: (1) In the first treatment, we let participants changes their neighborhoods, i.e. their links, and their actions in the Hawk Dove Game every period. (2) In the second treatment, we restrict participants choices by letting them only modify their action choices in all periods. (3) In the third treatment, participants can only modify their link choices throughout all periods. Also behavior is consistent within each treatment, it differs across the three treatments. This is especially surprising, as from a game-theoretic perspective participants face the same equilibria in all treatments.

The results get clearer, when analyzing the results with respect to complexity. In this way, we can show that all three treatments differ concerning the complexity of decision making for the participants, and that participants are closer to the game theoretic prediction the less complex their decision situation is. We deem this work an important first step towards understanding the boundaries of human decision making.

We introduce our game in the following section. In Section 2 we analyze the game from a theoretic perspective. In Section 3, we discuss our experimental design, before we describe the experimental results in Section 4. We discuss our results with respect to complexity prediction in Section 5, before we conclude in Section 6.

1   The Game

The remainder of this section formally introduces the Network Hawk Dove Game (Berninghaus & Vogt 2006) which builds the basis for our analyses. In the Network Hawk Dove Game players face two decisions. First, they choose to whom they want to establish a link at a certain cost. Second, they choose one action in a bimatrix game, namely the Hawk Dove Game, they use when interacting with the players they are linked to.

Formally, in a set $I = \{1, \dots, n\}$ of n players, each single player $i \in I$ takes part in Hawk Dove Games with all other players i is linked to, using a Network Game. The Hawk Dove Game is a symmetric 2×2 normal form game $G^H := \{\Sigma^H, \Pi^H(\cdot)\}$ with $\Sigma^H := \{X, Y\}^1$ and $\Pi^H(\cdot)$ represented by the payoff matrix in Table 1.

Table 1: Payoff Matrix Hawk Dove Game (with $a > b > c > d > 0$)

	Hawk (X)	Dove (Y)
Hawk (X)	d,d	a,c
Dove (Y)	c,a	b,b

Aside the Hawk Dove Game players participate in a Network Game $G^N :=$

---

[1] In the remainder of this paper, we call strategy X hawk strategy and Y dove strategy.

3

$\{\Sigma^N, \Pi^N(\cdot)\}$. In the Network Game the players decide with whom they want to interact. Hence, each strategy in the Network Game σ_i^N is a subset of all players $\sigma_i^N \subseteq \Sigma^N \setminus \{i\}$ with $\Sigma^N := I$. Establishing links to other players is not costless, but infers a cost k per established link. Hence, we define the payoff of the Network Game as $H^N(\sigma_i^N, \sigma_{-i}^N) := -k \cdot |\sigma_i^N|$, with $k > 0$. If at least one link, between two players i and j exists, i.e. if at least a unilateral link exists, both players participate in a Hawk Dove Game G^H.

Each strategy profile $\sigma^N = (\sigma_1^N, \ldots, \sigma_n^N)$ implies a directed graph $g(\sigma^N) = (V(\sigma^N), E(\sigma^N))$, with edges $E(\sigma^N)$ and vertices $V(\sigma^N)$. Each vertex $v_i \in V(\sigma^N)$ corresponds to one player $i \in I$. For each link a player i establishes to another player j, i.e. if $j \in \sigma_i^N$, an edge (i,j) from vertex v_i to v_j exists in $E(\sigma^N)$. We call all players j player i has outgoing edges to active neighbors, formally the active neighbors of i are σ_i^N. Aside his active neighbors, each player i has passive neighbors, i.e. players establishing links towards him. Formally, we define the set of passive neighbors as $\{j | (j, i) \in E(\sigma^N)\}$. We call all players linked to player i, be it active or passive, his neighbors $N_i(\sigma^N) := \sigma_i^N \cup \{j | (j, i) \in E(\sigma^N)\}$.

Using both, the Hawk Dove Game G^H and the Network Game G^N, we compose the non-cooperative Network Hawk Dove Game $\Gamma := \{S; H\}$ with a strategy set $S := \Sigma^H \times \Sigma^N$ and payoff function $P : S \to \mathcal{R}$. In this game, the strategy $s_i \in S$ of each player i consists of his action[2] in the Hawk Dove Game σ_i^H and the choice of his active neighbors σ_i^N. To simplify presentation, we define the number of neighbors playing the hawk strategy as $n_X^i(s) = \sum_{j \in N_i(\sigma^N)} 1_{\{\sigma_j^H = X\}}$ and the neighbors playing the dove strategy as $n_Y^i(s) = \sum_{j \in N_i(\sigma^N)} 1_{\{\sigma_j^H = Y\}}$. We formally represent the number of all neighbors by $n^i(s) := |N_i(\sigma^N)|$.

Depending on the action σ_i^H player i uses in the Hawk Dove Game, his payoff is defined as:

$$P_i(s_{-i}, \{X; \sigma_i^N\}) := d \cdot n_X^i(s) + a \cdot n_Y^i(s) - k |\sigma_i^N|$$

$$P_i(s_{-i}, \{Y; \sigma_i^N\}) := c \cdot n_X^i(s) + b \cdot n_Y^i(s) - k |\sigma_i^N|$$

In other words, we define the payoff of player i as the benefit from participating in a Hawk Dove Game with each of his neighbors minus the costs for establishing links to his active neighbors according to the Network Game.

Formally the Network Hawk Dove Game is a combination of two isolated games, the Hawk Dove Game and the Network Game. This allows us to introduce two versions of the Network Hawk Dove Game: (1) In the Fixed Link Game $\Gamma^{\overline{N}}$, we reduce the strategy set of the players to Σ^H and enforce a fixed set of links $\overline{\sigma^N}$ with $\overline{\sigma^N} \subseteq \Sigma^N \setminus \{i\}$. We leave everything else unchanged. I.e., each player i faces link costs for his active neighbors $\overline{\sigma_i^N}$ and plays a Hawk Dove Game with all neighbors $N_i(\overline{\sigma^N})$. (2) We derive the Fixed Action Game $\Gamma^{\overline{H}}$ by reducing the strategy set of the players to Σ^N. Here, we enforce a fixed set of actions $\overline{\sigma^H}$. Again, everything else is unchanged. I.e. each player i can modify his set of active neighbors σ_i^N, although his and all fellow players actions $\overline{\sigma^H}$ in the Hawk Dove Game are fixed. Notice, also the Network Hawk Dove Game

---

[2]To clarify presentation, in the remainder of this paper, we call strategies in the Hawk Dove Game actions, strategies in the Network Game links and strategies which combine actions and links in the Network Hawk Dove Game strategies.

has been analyzed both formally (Berninghaus & Vogt 2006) and experimentally (Berninghaus, Ehrhart & Ott 2010), this paper is the first to study the Fixed Link Game and the Fixed Action Game.

2 Game Theoretic Predictions

We now introduce equilibria predictions for the one-shot Network Hawk Dove Game Γ and both versions of the game, the Fixed Link Game $\Gamma^{\overline{N}}$ and the Fixed Action Game $\Gamma^{\overline{H}}$, before we discuss all three games concerning efficiency. All results concerning the Network Hawk Dove Game follow the formal introduction in the literature (Berninghaus & Vogt 2006), while this paper is the first to discuss theoretic predictions concerning the Fixed Link Game and the Fixed Action Game. We focus our analysis on a scenario in which $b > k > c$ holds as we deem this scenario to be the most interesting: (1) If $k > a$ holds, no links are established. (2) If $a > k > b$ holds, only links from hawks to doves pay. Hence, the link decision of doves is trivial. They should never establish any links. (3) If $c > k > d$ holds, all links except links between hawks always pay. Hence, doves should always establish all links, turning the link decision trivial again. (4) If $d > k$, all links pay. Aside these four scenarios, if $a > b > k > c > d$ holds, links from hawks to doves pay, while they do not pay between hawks, and links from doves to hawks do not pay, while they pay between doves, offering all players interesting link decisions.

2.1 Nash Equilibria

In the remainder of this section, we derive all stable strategy configurations of $\Gamma^{\overline{N}}$ and $\Gamma^{\overline{H}}$, before we discuss the equilibria in Γ. We base our considerations on the following extension of the Nash concept(Berninghaus & Vogt 2006).

Definition 1 *Each strategy configuration $s^* = (\sigma^{N*}, \sigma^{H*})$ in Γ is an equilibrium if*

$$\forall i : \quad P_i(s^*_{-i}, s^*_i) \geq P_i(s^*_{-i}, s_i) \text{ for } s_i \in S_i.$$

In an equilibrium no player has an incentive to deviate by either changing his links $\sigma_i^{N*} \in \Sigma^N$ or actions $\sigma_i^{H*} \in \Sigma^H$ unilaterally. Notice, this definition holds in all three versions of the game, namely $\Gamma^{\overline{N}}$, $\Gamma^{\overline{H}}$ and Γ.

Lemma 1 *Given a Fixed Action Game $\Gamma^{\overline{H}}$ an equilibrium s^* in $\Gamma^{\overline{H}}$ is established, if the following statements hold:*

   a) *Each hawk player builds links to all doves, but no links to other hawks.*

   b) *Each dove player has exactly one (active or passive) link to each other dove player, but no link to any hawk players.*

Both conditions, namely (a) and (b), directly follow from $a > b > k > c > d$. Players only establish links to other players, if their benefit from playing the Hawk Dove Game exceeds the cost of the link. The benefit of a hawk player facing a dove player, a, exceeds the link costs k, as does the benefit of a hawk player facing another dove player b. Hence, in equilibrium such links exist, while

links from hawks or doves to hawks do not. In addition, dove players connected via a bilateral connection can improve their payoff by dropping one link resulting in an unilateral connection.

Lemma 2 *Given a Fixed Link Network Hawk Dove Game $\Gamma^{\overline{N}}$ an equilibrium s^* in $\Gamma^{\overline{N}}$ is established, if the following statement holds:*

a) *Each player i uses the hawk action, if in his neighborhood the following condition holds*

$$n_X^i(s) < \frac{n^i(s)(a-b)}{c-d+a-b}. \tag{1}$$

b) *Each player i uses the dove action, otherwise.*

Conditions (a) and (b) follow from the payoff function $P(\cdot)$ of the Network Hawk Dove Game. The payoff of the hawk action exceeds the payoff of the dove action, if the following inequality holds: $P_i(s_{-i}^*, \{X; \sigma_i^{\overline{N}}\}) = d \cdot n_X^i(s) + a \cdot n_Y^i(s) - k\,|\sigma_i^N| > c \cdot n_X^i(s) + b \cdot n_Y^i(s) - k\,|\sigma_i^N| = P_i(s_{-i}^*, \{Y; \sigma_i^{\overline{N}}\})$. This inequality simplifies to the inequality of condition (a). A player resorts to the dove action, if the inequality does not hold (condition (b)).

Lemma 2 has two implications: (1) If no link exists, the left hand side of Equation 1 is undefined, as its denominator is 0. In this case player i can use any action in equilibrium. (2) If at least one link in the network exists, no trivial equilibria exist, in which the whole population chooses the hawk action or the dove action, as $0 < \frac{n_X^i(s)}{n^i(s)} < 1$ holds[3].

Lemma 3 *Given a Network Hawk Dove Game Γ an equilibrium s^* in Γ is established, if the following statements hold (Berninghaus & Vogt 2006):*

a) *Each hawk player builds links to all doves, but no links to other hawks.*

b) *Each dove player has exactly one (active or passive) link to each other dove player, but no link to any hawk players.*

c) *The number of hawk n_X^* in the population has to satisfy the condition*

$$n_X^* \geq \frac{(n-1)(a-b)}{a-b+c-d} > 0. \tag{2}$$

Following the same arguments as in Lemma 1 conditions (a) and (b) directly follow from $a > b > k > c > d$. Condition (c) follows from Conditions (a) and (b) and the payoff function $P(\cdot)$ of the Network Hawk Dove Game. According to (a) and (b) hawk players only have active neighbors and their neighbors are all doves. Hence, for unilateral deviation from hawk to dove to be beneficial, the following inequality has to hold: $P_i(s_{-i}^*, \{X; \sigma_i^{N*}\}) = n_Y^* a - k|\sigma_i^{N*}| < n_Y^* b - k|\sigma_i^{N*}| = P_i(s_{-i}^*, \{Y; \sigma_i^{N*}\})$ This inequality is always false, as $a > b$ holds. Each dove player should deviate from dove to hawk if $P_i(s_{-i}^*, \{Y; \sigma_i^{N*}\}) = n_X^* c + (n_Y^* - 1)b - k|\sigma_i^{N*}| < n_X^* d + (n_Y^* - 1)a - k|\sigma_i^{N*}| = P_i(s_{-i}^*, \{X; \sigma_i^{N*}\})$ holds. This inequality is false, if the inequality of condition (c) is met.

---

[3]This condition follows from comparing the right hand side of Equation 1 to 0 and 1 and the equations $a > b$ and $c > d$ from the payoff matrix.

Notice, that while conditions (a) and (b) in Lemma 3 and conditions (a) and (b) in Lemma 1 are equivalent, condition (c) in Lemma 3 seem to contradict Lemma 2. In the Fixed Link Game an upper bound for the number of hawk players per neighborhood exists, while in the Network Hawk Dove Game, equilibria exist, in which all players resort to the hawk action. At the same time, there is no lower bound for the number of hawk players in the Fixed Link Game which does exist in the Network Hawk Dove Game.

2.2 Efficiency

After characterizing the properties of the Nash Equilibria in the Network Hawk Dove Game, the Fixed Link Game and the Fixed Action Game, we now characterize efficiency in the games.

Fixed Action Game: According to Lemma 1, in the Fixed Action Game all links are established if and only if they increase the overall payoff of the population. Hence, neither adding nor removing any links can increase any payoffs. We conclude that in a Fixed Action Game $\Gamma^{\overline{H}}$ every equilibrium s^* in $\Gamma^{\overline{H}}$ is efficient.

Fixed Link Game: In the Fixed Link Game all links are fixed. Hence, we focus our analysis on the links in the network. Links between two hawks yield an overall payoff of $2 \cdot d - k$, links between two doves yield a payoff of $2 \cdot b - k$ and links between a hawk and a dove a payoff of $c + a - k$. Given that $2 \cdot d < 2 \cdot b$ and $2 \cdot d < c + a$ holds, all combinations of hawk and dove players that form an equilibrium are efficient, if $2 \cdot b = c + a$ holds. Hence, in a Fixed Link Game $\Gamma^{\overline{N}}$ equilibria s^* in $\Gamma^{\overline{N}}$ result in higher payoffs, the less links between hawks exist, if $2 \cdot b = c + a$ holds.

Network Hawk Dove Game: Given a Network Hawk Dove Game Γ equilibria s^* in Γ result in higher payoffs, the more players resort to the dove action, if $2 \cdot b = c + a$ holds (Berninghaus, Ehrhart & Ott 2010).

Notice, that all three games are different from each other concerning efficiency. While in the Fixed Action Game all equilibria are efficient, in Fixed Link Games the number of links between hawks needs to be minimized to be efficient. Finally, in the Network Hawk Dove Game players increase efficiency by increasing the number of players using the dove action. In contrast to the equilibrium predictions, the preconditions for efficient equilibria are inline for all three games: The less hawks, and therefore the less links between hawks, exist in the population the more likely are efficient outcomes.

2.3 Repeated Game

In finitely repeated versions of the three games, players will resort to the equilibrium of the stage game. Hence, the described predictions for the stage game will hold. The game becomes more interesting, if we consider finitely repeated versions of the stage games and mix different stage games. In the remainder of this section, we focus on mixing the Network Hawk Dove Game, with the Fixed Link Game and the Fixed Action Game respectively.

Lemma 4 *In a finitely repeated Network Hawk Dove Game all equilibria of the stage game exist. These equilibria persist, even if several stages of repeated play are replaced with Fixed Link Games or Fixed Action Games.*

As all equilibria of the stage game are equilibria of the finitely repeated game, the equilibria described in Lemma 3 also exist in the repeated version of the Network Hawk Dove Game. Replacing part of the stages with the Fixed Action Game or the Fixed Link Game, neither reduces nor extends the set of equilibria. In both modifications of the Network Hawk Dove Game, the strategy set is limited compared to the Network Hawk Dove Game. Hence, players will choose the equilibrium strategy of the Network Hawk Dove Game in corresponding stages and resort to this strategy if the strategy set is limited. This observation is obvious for the Fixed Action Game, as the equilibrium conditions for this stage game are inline with the equilibrium conditions of the Network Hawk Dove Game. Notice, that this even holds for the Fixed Link Game. If players are in equilibrium during the Network Hawk Dove stage, they have no interest in deviating from their current strategy: (1) All hawk players are connected to other dove players only. Hence, condition (a) of Lemma 2 is met, and they will continue playing the hawk action. (2) All dove players have incoming links from all hawks and are connected to all other dove players. I.e. each dove player is connected to every other player. In his neighborhood $n_X^i(s) = n_X^*$ and $n^i(s) = n$ hold. Hence, a dove will keep playing the dove action if $n_X^* > \frac{n(a-b)}{a-b+c-d}$ holds, which is satisfied, if condition (c) of Lemma 3 is met.

Also equilibrium conditions for all three games differ, in a repeated game mixing the stage games with each other results in the equilibrium predictions of the Network Hawk Dove Game.

3 Experimental Design

To evaluate the equilibrium predictions described above, we conducted laboratory experiments between September and November 2010 at the Karlsruhe Institute of Technology. Each experimental session lasted approximately 1.5 hours. For all sessions, we recruited a total of 162 participants using ORSEE (Ben Greiner 2004) from a pool of students in Karlsruhe. In the beginning of each experimental session, we randomly assigned the participants to groups of six. We handed out written instructions to each participant describing the experimental setup. After all participants had read the instructions, they played one treatment implemented using zTree (Urs Fischbacher 2007) at a computer terminal. Finally, we paid the participants in private depending on their success in the treatment.

The baseline treatment (Treatment Basic) is equivalent to the Network Hawk Dove Game and consisted of 50 periods. In every period, participants could specify other participants they wanted to establish links to and they could specify the action they wanted to play. In the end of each period, the experimental software calculated the payoff of the participants. For each link a participant had established, he had to pay $k = 50$ points. All linked participants then played the Hawk Dove Game shown in Table 2. I.e., if a participant played dove and his neighbor played hawk, he received 40 points, while his neighbor received 80 points. If both participants played hawk, they received 20 points. They received 80 points when coordinating towards dove action. After payoff calculation, the computer terminal showed the actions, all links and the individual performance to each participant.

We conducted two modifications of Treatment Basic: (1) Treatment Fixed

Table 2: Payoff Matrix Hawk Dove Game in the Experiment

	Hawk (X)	Dove (Y)
Hawk (X)	$20, 20$	$80, 40$
Dove (Y)	$40, 80$	$60, 60$

Link and (2) Treatment Fixed Action. In Treatment Fixed Link participants played the Network Hawk Dove Game during the first period and then every fifth period, i.e. in period 6, 11, 15, ... In all other periods participants played the Fixed Link Game. In Treatment Fixed Action, the Network Hawk Dove Game was played during the same periods as in Treatment Fixed Link. In all other periods, participants played a Fixed Action Game.

In the end of the experiment, all participants received a show up fee of 5.00 €. For 1,000 points earned during the experiment, a participant received 1.00 €. On average each participant earned 11.39 €.

4 Experimental Results

To get a first impression of our experimental results, we compare the behavior of all participants over time (see Figure 1). A first look at the data shows that the number of participants playing hawk per group (see Table 3 for averages over all periods) lies between 2 and 3 and remains stable throughout the experiment for all three treatments. An exception is the first and the last period (especially in Treatment Basic and Treatment Fixed Link). Here, the number of participants playing hawk is lower in the first periods and increases during the last periods. To reduce the impact of such start game and end game behavior respectively, we focus on Periods 10 to 40 in the rest of our analysis.
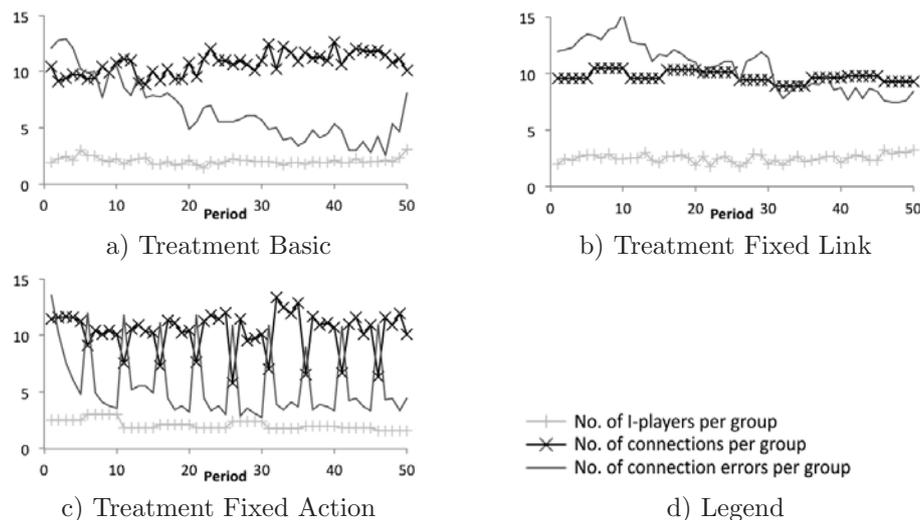

a) Treatment Basic

b) Treatment Fixed Link

c) Treatment Fixed Action

d) Legend

— No. of I-players per group
✕ No. of connections per group
— No. of connection errors per group

Figure 1: Development of behavior during the experiment

9

The number of links seems to be almost constant in Treatments Basic and Treatment Fixed Link. This may not be very surprising for Treatment Fixed Link where we did not allow link changes for five periods. But even if we allowed participants to adapt links in this treatment they did not vary the total number of links. In Treatment Fixed Action we observe a typical pattern of behavior. During the period in which we allowed participants to switch actions they drop links sharply expecting unprofitable links. This "crash" of links after every 5 periods observed in Treatment Fixed Action is perfectly correlated with the number of link errors, i.e. the number of existing links that have a negative impact on the payoff or non existing links which would increase payoffs, which sharply increases before action changes are allowed. Independent of the number of periods with fixed actions, we expect the same decrease to occur before participants can change their action. To reduce the impact of these crashes, we focus our analysis on the rounds before the crashes occur. Namely, we analyze rounds 5, 10, 15, ... in the remainder.

Table 3: Number of hawk players per group

Group	Treatment Basic	Treatment Fixed Link	Treatment Fixed Action
1	1.83	2.50	2.50
2	2.17	1.17	1.83
3	2.50	2.17	1.50
4	2.67	2.67	2.00
5	2.50	2.67	2.17
6	1.17	2.17	2.33
7	1.67	1.17	2.17
8	1.67	2.50	1.83
9	1.67	1.83	1.83
Average	1.98	2.09	2.02

In the following we will analyze our experimental data in more detail (in each treatment participated nine groups). In Table 3 we present separate for each group the average number of participants choosing the hawk action. The average number lies between 1 and 3 and does not significantly differ across treatments[4].

We find differences between the treatments, when we analyze the average number of links per group (see Table 4)[5]. Here, we find significant differences between Treatment Fixed Link and both other treatments.

[4]Comparison of average number of hawk players per group: Overall comparison (Kruskal-Wallis test): test statistic=0.707, degrees of freedom=2, p-value=0.702. Pairwise Comparisons (2-tailed Mann-Whitney-U test): Basic vs. Fixed Link: test statistic=33.500, p-value=0.531. Fixed Link vs Fixed Action: test statistic=31.500, p-value=0.420. Basic vs. Fixed Action: test statistic=37.500, p-value=0.789)

[5]Comparison of average number of links per group: Overall comparison (Kruskal-Wallis test): test statistic=4.716, degrees of freedom=2, p-value=0.095. Pairwise Comparisons (2-tailed Mann-Whitney-U test): Basic vs. Fixed Link: test statistic=19.500, p-value=0.063. Fixed Link vs. Fixed Action: test statistic=19.500, p-value=0.063. Basic vs. Fixed Action: test statistic=37.500, p-value=0.791)

Table 4: Avg. number of links per group

Group	Treatment Basic	Treatment Fixed Link	Treatment Fixed Action
1	10.67	10.83	10.50
2	11.17	9.00	13.50
3	10.83	8.33	10.00
4	7.00	10.17	11.00
5	11.67	10.83	11.83
6	13.17	9.50	9.50
7	9.83	11.17	12.17
8	13.33	7.17	11.00
9	11.83	10.00	10.00
Average	11.06	9.67	11.06

From Table 5 we see that link errors are significantly[6] different between all treatments. Link errors occur when either an existing link generates a loss or when a profitable link is not opened by anyone of the involved players. Obviously, participants in Treatment Fixed Action are performing best.

Table 5: Avg. link errors per period per group

Group	Treatment Basic	Treatment Fixed Link	Treatment Fixed Action
1	5.33	13.33	3.83
2	3.67	8.50	3.17
3	2.50	12.33	5.17
4	9.33	13.67	3.83
5	5.67	6.67	2.67
6	8.17	8.83	3.67
7	5.83	9.67	1.33
8	2.00	8.83	3.67
9	6.67	12.17	4.00
Average	5.46	10.44	3.48

Comparing the per capita payoffs (see Table 6) we do not see significant[7] differences between Treatment Basic and Treatment Fixed Action. But there exist significant differences between Treatment Fixed Link and the remaining

---

[6]Comparison of number of link errors per group: Overall Comparison (Kruskal-Wallis test): test statistic=16.862, degrees of freedom=2, p-value=0.000. Pairwise Comparisons (2-tailed Mann-Whitney-U test): Basic vs. Fixed Link: test statistic=5.500, p-value=0.002. Fixed Link vs Fixed Action: test statistic=0.000, p-value=0.000. Basic vs. Fixed Action: test statistic=21.000, p-value=0.084)

[7]Comparison of average payoff per group: Overall Comparison (Kruskal-Wallis test): test statistic=14.234, degrees of freedom=2, p-value=0.001. Pairwise Comparisons (2-tailed Mann-Whitney-U test): Basic vs. Fixed Link: test statistic=8.000, p-value=0.004. Fixed Link vs Fixed Action: test statistic=0.000, p-value=0.000. Basic vs. Fixed Action: test statistic=38.500, p-value=0.860)

treatments. On average the individual per period payoff is larger in Treatment Fixed Action than in the remaining treatments.

Table 6: Payoff per capita and per period

Group	Treatment Basic	Treatment Fixed Link	Treatment Fixed Action
1	112.22	71.95	110.28
2	124.72	85.00	130.83
3	123.06	72.78	103.33
4	71.67	75.28	111.67
5	111.67	97.50	128.06
6	112.50	84.17	100.83
7	105.83	89.17	141.95
8	147.78	68.06	118.33
9	112.50	74.45	113.33
Average	113.55	79.82	117.62

To sum up, all three treatments yield almost the same number of hawk players per population, namely about 2. First, given the theoretical predictions of the Network Hawk Dove Game, these are less hawk players than we expect in an Nash equilibrium. According to Lemma 3 $n_X^* > \frac{(n-1)(a-b)}{a-b+c-d} = 2.5$ has to hold in equilibrium. Second, we would expect less if we only considered efficiency. According to the discussion of efficiency in Section 2 the number of hawk players needs to be 0 or 1 in an efficient network. Aside this similarity, all three treatments are different concerning the number of links. In Treatment Fixed Link the number of links is significantly lower than in both other treatments. The picture gets even clearer, when we analyze link errors. While the number of link errors is maximal in Treatment Fixed Link, it is minimal in Treatment Fixed Action and it lies between both other treatments in Treatment Basic. The difference between the payoffs of the participants finally are only a consequence of these differences.

5   Complexity Considerations

The experimental results are not inline with the theoretical predictions we described in Section 2. In this section, we show that analyzing the game from a complexity motivated perspective can better describe the observed behavior. Therefore, we apply different complexity concepts to the Network Hawk Dove Game. In particular, we discuss the impact of limiting the choice set of the players on complexity. In our analysis, we apply different popular complexity concepts. First, we use the concept of state complexity as suggested by A. Rubinstein (Rubinstein 1986), before we apply space and time complexity the most popular complexity concepts of computer scientists.

5.1 State Complexity

In this section, we calculate the state complexity for the Network Hawk Dove Game based on the intuition described by Salant ((Yuval Salant 2011)). According to him, the "state compexity of a given choice behavior is the minimal number of states needed to implement that behavior". In this sense, we can calculate the minimal state complexity of a strategy playing the Nash Equilibrium in the Network Hawk Dove Game.

Proposition 1 *The state complexity of a machine playing the best response in a Network Hawk Dove Game cannot fall bellow $2^{2(n-1)}$.*

In the stage game of a Network Hawk Dove Game, a player has to consider two aspects of his fellow players: (1) The $n-1$ actions of all fellow players, as this defines which action the player has to play. (2) For all $n-1$ potential links from the fellow players towards him, he has to know whether they are established or not, to decide whether he should establish a link towards the fellow player or not. As the Hawk Dove Game is a bimatrix game, i.e. each player can choose from two actions, there exist 2^{n-1} possible action combinations (1). The number of link combinations (2) also is 2^{n-1} as each link has two possible states, i.e. existing and not existing. Therefore, a machine deriving best replies to both aspects has the state complexity of at least $2^{2(n-1)} = 2^{n-1} \cdot 2^{n-1}$. Any deviation from a strategy playing the best response to the behavior of the fellow players in the previous period, i.e. by considering the history of play or the links between fellow players, leads to a higher state complexity of the machine.

This result also holds, if the strategy set is limited for some periods, i.e. if in some periods participants only choose their actions or their links.

Proposition 2 *The state complexity of a strategy does not change, if the participants only choose links or actions in some periods.*

If players in one period only select their links, the player have to consider the $n-1$ action choices of their fellow players, as a player using a hawk action will only establish links to doves and a player using a dove action will only establish links to hawks. Aside this, the player has to consider for all their $n-1$ fellow players, whether they established a link towards him or not. To determine whether he should establish an additional link or not. This results in a state complexity of $2^{2(n-1)}$ following the arguments in the proof of Proposition 1. Players only selecting their actions have to consider the same aspects. They have to check whether a (incoming or outgoing) link to each of the $n-1$ fellow players exists and for each linked fellow player which action to use. Each player is linked to a maximum of $n-1$ fellow players yielding a state complexity of $2^{2(n-1)}$ following the arguments in the preceding proof.

To sum up, the state complexity of the Network Hawk Dove Game is 2^{n-1} and does not change, if the strategy set of a player is limited.

5.2 Space Complexity

We now apply the concept of space complexity to the decision situation of the Network Hawk Dove Game. In our analysis, we focus on the worst case scenario as is common when describing complexity theoretic results.

Proposition 3 *If participants in a Network Hawk Dove Game simultaneously choose their actions and their links, the space complexity of identifying an equilibrium is $O(2(n-1)) = O(n)$.*

According to Lemma 3, in equilibrium three conditions have to be met. From the perspective of each player i, he has to (1) establish links to fulfill conditions (a) and (b), and he has to (2) choose his action to fulfill condition (c). Choosing the action [2] depends on the number of other participants playing the hawk action (n_X). To derive n_X the player has to calculate a belief concerning the action of each other player. Building adequate links [1] depends on the actions the player chose. Given he plays the dove action, he needs to distinguish whether one of the n_Y other dove players opened a link towards him. If no link was established, he would have to establish the link himself. All other links are determined by the corresponding actions. To choose the own action [2] the player has to derive the action of each fellow player, i.e. whether the fellow player is a hawk or a dove. The space complexity is $O(n-1)$. For building adequate links[1] player i has to know whether an incoming link (j, i) exists for any player j using the dove action. In the worst case all other players are doves. Hence, the space complexity of this step is $O(n-1)$. The overall space complexity for playing the best response to a given strategy profile is $O(n-1) + O(n-1) = O(2(n-1)) = O(n)$

Proposition 4 *If players repeatedly participate in a Network Hawk Dove Game for one period and the Fixed Action Game for t periods afterwards, the space complexity of identifying an equilibrium is $O(\frac{1}{t+1}2(n-1) + \frac{t}{t+1}(n-1)) = O(\frac{t+2}{t+1}(n-1)) = O(n)$.*

In the equilibrium of the Fixed Action Network Hawk Dove Game conditions (a) and (b) of Lemma 1 are both fulfilled by choosing adequate links. As discussed in the proof of Proposition 3, the space complexity for choosing adequate links is $O(n-1)$. Hence, in $\frac{t}{t+1}$ of all periods the complexity of identifying an equilibrium is $O(n-1)$, while it is $O(2(n-1))$ in $\frac{1}{t+1}$ of all periods. In sum the space complexity is $O(\frac{2}{t+1}(n-1) + \frac{t}{t+1}(n-1)) = O(\frac{t+2}{t+1}(n-1)) = O(n)$.

Proposition 5 *If players repeatedly participate in a Network Hawk Dove Game for one period and the Fixed Link Game for t periods afterwards, the space complexity of identifying an equilibrium is $O(\frac{1}{t+1}(n)(n-1) + \frac{t}{t+1}n - 1) = O(n^2)$.*

In the equilibrium of the Fixed Link Game conditions (a) and (b) of Lemma 2 are both fulfilled by choosing the adequate action. To do so, each player has to consider all other players in his neighborhood and calculate beliefs concerning the strategies they play. As described in the proof of Proposition 3 a player can do this in $O(n-1)$, if an equilibrium was reached in the preceding Network Hawk Dove Game. If in the preceding Network Hawk Dove Game no equilibrium was reached, the complexity of this step increases. It is even possible, that no equilibrium inline with Lemma 3 can be reached. E.g. think of a network in which all players are organized in a circle with one incoming and one outgoing link. An equilibrium according Lemma 3 could not be reached, as all players would have to play the dove action (hawk players only have outgoing links in equilibrium) which contradicts condition (c) of Lemma 3. Hence, in the

Network Hawk Dove Game, we expect players to focus on their links, as they cannot correct action errors during subsequent periods. As we have discussed in conditions (a) and (b) of Lemma 3, establishing links to other players is the immediate consequence of the actions all other players and oneself have chosen. As each player i knows this, he has to consider action optimizations during all subsequent t periods. As this action optimizations are the consequence of the links of all other players, he has to predict all links between all other players. Per link (j, k) player i with $\{j, k | i \neq j \vee i \neq k \vee j \neq k\}$ has to derive one belief. In the whole network up to $(n-1)(n-1)$ links between players j, k can exist. In addition, player i has to consider all incoming links (j, i) from players j playing the dove action. In the worst case $n-1$ such links exist. Hence the space complexity of this period is $O(n(n-1))$, while the overall complexity is $O(\frac{1}{t+1}(n)(n-1) + \frac{t}{t+1}n - 1) = O(n^2)$.

To summarize, concerning space complexity the three games differ. Fixing the action choices of the players reduced space complexity, while fixing link choices increases space complexity compared to the unmodified Network Hawk Dove Game.

5.3 Time Complexity

Next, we derive time complexity of the decision situation of the Network Hawk Dove Game and combinations of the Network Hawk Dove Game with both versions of the game.

Proposition 6 *All results concerning the space complexity of the different scenarios also hold for time complexity. I.e., simultaneous link and action choice has a time complexity of $O(n)$, determining links only has a time complexity of $O(n)$ and determining actions only has a time complexity of $O(n^2)$*

This result directly follows from the observation, that we deem deriving strategy parameters has a complexity of $O(1)$. As every parameter derived is also memorized and therefore increases space complexity, both measures yield equivalent results. Additional aggregation steps are independent of the size of the input and are not considered in O-notation.

5.4 Discussion

Humans who face a list of different alternatives can choose their favored item either by (1) satisficing or by (2) optimizing (H.A. Simon 1955). When optimizing (2) humans analyze all possible alternatives, compare them to each other and choose the best, while they resort to one option which satisfies their expectations when resorting to satisfying (1). Both behavioral paradigms are well known from marketing literature. As consequence from these "consumer types", namely optimizers and satisficers, marketing studies find that an increase of alternatives leads to a decrease of well-being of the optimizers (Barry Schwartz, Andrew Ward, John Monterosso, Sonja Lyubomirsky, Katherine White & Darrin R. Lehman 2002). An online study (Tilottama G. Chowdhury, S. Ratneshwar & Praggyan Mohanty 2008), confirms these results. In the study, participants were asked to choose one gift from a list of different items within a limited time frame. Participants played in two treatments, in one treatment with only

few alternatives and one with many alternatives. Although both optimizers and satisficers felt time pressure, when they faced many alternatives, maximizers evaluated more alternatives and felt regret for their choice more often than satisfiers.

From the perspective of behavioral economists, humans find their choice by resorting to satisficing. They have an expectation concerning an adequate alternative and choose the alternative beating this expectation, see e.g. (Reinhard Selten 1998). In behavioral experiments participants choose one element from a list of different alternatives. Therefore, they are informed concerning the value of the alternative after choosing it. They can then decide whether to analyze a next alternative or not. Each inspection has a fixed cost, representing the costs for analyzing the corresponding alternative. Early work of the corresponding decision situation shows (Anatol Rapoport & Amos Tversky 1970, Mark Pingle 1992) that some participants stop analyzing alternatives earlier than predicted by theory. A reason for this behavior might be the adaptation of expectations: Participants who face alternatives which monotonically increase in value analyze fewer alternatives, than participants who face alternatives monotonically decreasing (P Brickman 1972, Z Shapira & Itzhak Venezia 1981). That humans search until their expectations are beaten is also supported by an experiment in which human participants can return to an alternative they rejected in an earlier period (C.A. Kogut 1990). In these experiments several participants chose alternatives they have rejected in earlier periods. They probably adapted their expectations concerning an adequate alternative making an alternative rejected in earlier periods acceptable.

In other experiments this model was applied to job search (JamesC. Cox & RonaldL. Oaxaca 1989). Here, in contrast to preceding work, a decision maker explicitly faces a loss the longer he searches for an adequate wage as his wages are paid starting in the first period after the acceptance of an offer. As the decision maker earns the selected wage alternative multiplied with the remaining periods a fixed period exists after which further search is not beneficial. The optimal decision is to stop searching before this "cut-off period" period is reached. Experimental investigations of this model show that the time humans search for new jobs is inline with theoretical predictions for risk neutral decision makers. The fact that (some) participants stop their searches to early (Rapoport & Tversky 1970, Pingle 1992) could be attributed to such hidden utility participants receive when having chosen one alternative earlier.

A more recent study (J Bearden & T Connolly 2007) discusses experiments in which participants choose from a bundle of two goods instead of choosing just one good in all previous studies. After finding out the quality of the first good by investing part of their endowment, they can choose whether to investigate the second good at an additional cost. Human participants played in two different treatments: (1) when paying for finding out the quality of one good they were informed about the exact value of the good (2) participants specified a threshold value when interested in the quality of one good and where only informed whether the value lie above or below the cut-off value. Participants who where informed about the exact value (1) searched significantly less periods until choosing their alternative than participants who had to specify thresholds (2). This difference between treatments can be attributed to complexity aspects of both alternatives. For the human decision maker memorizing exact values for two goods is more complex then remembering two thresholds only he than

16

compares to new alternatives and adapts from time to time. We argue that complexity implies costs for the decision maker for each calculation. These "search costs" also result in the shorter search observed in earlier experimental studies (Rapoport & Tversky 1970, Pingle 1992).

As Salant (Salant 2011) discussed in his recent work, state complexity can be used to justify that participants searching for certain elements of lists resort to satisficing rather than optimizing. We argue that space complexity justifies the behavior as well, and is inline with the experimental work cited above. According to this work, humans tend to have certain expectations concerning an adequate outcome of an experiment. They adapt their expectations according to the alternatives they have seen and choose the first alternative which justifies their expectations. Salant has shown that the state complexity of satisficing is lower than the state complexity of optimizing. The space complexity of satisficing is 1: The decision maker first analyzes the first alternative. According to this alternative, he modifies his expectation concerning the value he can expect and remembers it. Remembering exactly one expectation has a space complexity of 1. Afterwards, he chooses the next alternative, compares it to his expectation (without the need of additional space) and decides whether to continue or not. If he continues, he modifies his expectation, i.e. he overwrites his current expectation needing no more memory, and starts over again. Otherwise he stops. Optimizing behavior on the other hand has a space complexity of 2. When traversing through the list, the decision maker has to remember both the value of the best alternative seen to date and the position of the alternative in the list, requiring two memory slots.

6 Summary

In this paper, we present theoretical and experimental results concerning the Network Hawk Dove Game. Our treatments differ in the stage games participants play. While in all treatments periods exist in which all participants can choose both actions and links, in one treatment we let participants only choose actions in some periods and in another treatment we let participants only choose links in some periods. Although this modification has no impact on theoretical equilibrium predictions, the behavior of participants changes dramatically. If participants only choose links, they faster coordinate towards the equilibrium, while they find it more difficult to coordinate towards an equilibrium if they only choose actions compare to the baseline treatment, in which both aspects are chosen simultaneously. We attribute the observed differences to the space complexity of the corresponding decision making processes. Therefore, we derive the space complexity of all three treatments and show that the experimental results are inline with the complexity theoretic predictions.

References

Amir, Madjid, and Siegfried K. Berninghaus. 1996. "Another approach to mutation and learning in games." *Games and Economic Behavior*, 14(1): 19–43.

Bala, Venkatesh, and Sanjeev Goyal. 2000. "A Noncooperative Model of Network Formation." *Econometrica*, 68(5): 1181–1229.

Bearden, J, and T Connolly. 2007. "Multi-attribute sequential search." *Organizational Behavior and Human Decision Processes*, 103(1): 147–158.

Berninghaus, Siegfried K. 1996. "Evolution, interaction, and Nash equilibria." *Journal of Economic Behavior & Organization*, 29(1): 57–85.

Berninghaus, Siegfried K., and Bodo Vogt. 2006. "Network formation in symmetric 2x2 games." *Homo Oeconomicus*, 23: 421–466.

Berninghaus, Siegfried K., and Ulrich Schwalbe. 1996. "Conventions, Local Interaction, and Automata." *Evolutionary Economics*, 6: 297–312.

Berninghaus, Siegfried K., Karl-Martin Ehrhart, and Claudia Keser. 2002. "Conventions and Local Interaction Structures: Experimental Evidence." *Games and Economic Behavior*, 39(2): 177–205.

Berninghaus, Siegfried K., Karl-Martin Ehrhart, and Marion Ott. 2010. "Cooperation and Forward-Looking Behavior in Hawk-Dove Games in Endogenous Networks: Experimental Evidence." Working Paper.

Bramoullé, Yann, Dunia López-Pintado, Sanjeev Goyal, and Fernando Vega-Redondo. 2004. "Network formation and anti-coordination games." *International Journal of Games Theory*, 33(1): 1–19.

Brickman, P. 1972. "Optional stopping on ascending and descending series." *Organizational Behavior and Human Performance*, 7(1): 53–62.

Chowdhury, Tilottama G., S. Ratneshwar, and Praggyan Mohanty. 2008. "The time-harried shopper: Exploring the differences between maximizers and satisficers." *Marketing Letters*, 20(2): 155–167.

Cowan, Nelson. 2001. "The magical number 4 in short-term memory: a reconsideration of mental storage capacity." *The Behavioral and brain sciences*, 24(1): 87–114.

Cox, JamesC., and RonaldL. Oaxaca. 1989. "Laboratory experiments with a finite-horizon job-search model." *Journal of Risk and Uncertainty*, 2(3).

Fischbacher, Urs. 2007. "z-Tree: Zurich toolbox for ready-made economic experiments." *Experimental Economics*, 10(2): 171–178.

Greiner, Ben. 2004. "The Online Recruitment System ORSEE 2.0 - A Guide for the Organization of Experiments in Economics." University of Cologne - Working Paper Series in Economics 10.

Haller, Hans, and Sudipta Sarangiz. 2003. "Nash Networks with Heterogeneous Agents." DIW Discussion Paper 337.

Jackson, Matthew O., and Asher Wolinsky. 1996. "A Strategic Model of Social and Economic Networks." *Journal of Economic Theory*, 71(1): 44–74.

Kalai, Ehud, and William Stanford. 1988. "Finite rationality and interpersonal complexity in repeated games." *Econometrica: Journal of the Econometric Society*, 56(2): 397–410.

Kogut, C.A. 1990. "Consumer search behavior and sunk costs." *Journal of Economic Behavior & Organization*, 14(3): 381–392.

Miller, Georg A. 1956. "The magical number seven, plus or minus two: some limits on our capacity for processing information." *Psychological Review*, 63: 81–97.

Neyman, Abraham. 1985. "Bounded complexity justifies cooperation in the finitely repeated prisoners' dilemma." *Economics Letters*, 19: 227–229.

Pingle, Mark. 1992. "Costly optimization: an experiment." *Journal of Economic Behavior & Organization*, 17(1): 3–30.

Rapoport, Anatol, and Amos Tversky. 1970. "Choice behavior in an optional stopping task*1." *Organizational Behavior and Human Performance*, 5(2): 105–120.

Rubinstein, Ariel. 1986. "Finite automata play the repeated prisoner's dilemma." *Journal of Economic Theory*, 39(1): 83–96.

Salant, Yuval. 2011. "Procedural analysis of choice rules with applications to bounded rationality." *American Economic Review*, 101(April): 724–748.

Schwartz, Barry, Andrew Ward, John Monterosso, Sonja Lyubomirsky, Katherine White, and Darrin R. Lehman. 2002. "Maximizing versus satisficing: Happiness is a matter of choice." *Journal of Personality and Social Psychology*, 83(5): 1178–1197.

Selten, Reinhard. 1998. "Aspiration Adaptation Theory." *Journal of mathematical psychology*, 42(2/3): 191–214.

Shapira, Z, and Itzhak Venezia. 1981. "Optional stopping on nonstationary series*1." *Organizational Behavior and Human Performance*, 27(1): 32–49.

Simon, H.A. 1955. "A behavioral model of rational choice." *The quarterly journal of economics*, 69(1): 99.

Skyrms, Brian, and Robin Pemantle. 2000. "A dynamic model of social network formation." *Proceedings of the National Academy of Sciences of the United States of America*, 97(16): 9340–6.

Young, H. Peyton. 1993. "The Evolution of Conventions." *Econometrica: Journal of the Econometric Society*, 57–84.